Paper- 4: FUNDAMENTALS OF BUSINESS MATHEMATICS AND STATISTICS

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Full Marks: 100 **Time Allowed: 3 Hours**

Section – A

(Fundamentals of Business Mathematics)

PART – A

1. (a) Choose the correct answer from the given four alternatives: [9×2 = 18]

(i) An alloy contains zinc and copper in the ratio 5 : 8 and another alloy contains zinc and copper in the ratio 5 : 3. If equal amounts of both the alloys are melted together, then the ratio of zinc and copper in the resulting alloy is:

(a) 25 : 24

(b) 3 : 8

(c) 103 : 105

(d) 105: 103

(ii) Divide 581 among A, B, C so that 4A = 5B = 7C.

(a) ₹ 245, 196, 140

(b) ₹ 140, 160, 240

(c) 200, 250, 280

(d) None

(iii) If $\log_e 2 \log_n 625 = \log_{10} 16 \log_e 10$, then the value of n will be :

(a) 16

(b) 10

(c) 5

(d) 4

(iv) $3x^2 + 6x + 3 = 0$ then the roots of the equations are –

(a) (3,3)

(b) (–1, –1)

(c) (2,4)

(d) (4,1)

(v) The difference between compound interest and simple interest on a sum for 3 years at 5% per annun is ₹ 122. The sum is –

(a) ₹ 15,000

(b) ₹ 16,000

(c) ₹ 12,000

(d) ₹ 18,000

(vi) Out of 7 gents and 4 ladies a committee of 5 is to be formed. The number of committees such that each committee include at least one lady is

(a) 240

(b) 144

(c) 441

(d) None of these

(vii) In total number of ways in which six '+' and four '–' signs occur together is

(a) $\frac{7!}{3!}$

(b) 45

(c) 35

(d) None of these

(viii) If p^{th}, q^{th} **and** r^{th} **terms of a G.P. be a, b, c respectively, then** $a^{(q-r)}\, b^{(r-p)}\, c^{(p-q)} = ?$

(a) 0

(b) 1

(c) –1

(d) None

(ix) The set {0, 2, 4, 6, 8, 10} can be written as

(a) $\{2x/0 \leq x \leq 5\}$

(b) $\{x : 0 > x > 5\}$

(c) $\{2x : 0 < x < 5\}$

(d) None of these

Answer:

(i) (d) 105: 103

(ii) (a) ₹ 245, 196, 140

(iii) (d) 4

(iv) (b) (–1, –1)

(v) (b) ₹ 16,000

(vi) (c) 441

(vii) (c) 35

(viii) (b) 1

(ix) (a) {2x/0 ≤ × ≤ 5}

(b) State whether the following statements are True (or) False. [6×1=6]

(i) If the ratio of two positive numbers is 4:5 and their L.C.M is 140 then the number are 35, 45

(ii) A sum of money amounts to ₹ 720 in 2 years and ₹ 783 in 3 years the rate of interest is 12%

(iii) The statement $(A \cap B)' = A'UB'$ is true (or) False

(iv) If $x = 5+2\sqrt{6}$ and xy = 1 then $\frac{1}{x^2}+\frac{1}{y^2}$ is 89

(v) The integral part of the value of logarithm of a number is called characteristic

(vi) The roots of the equation $(x-4)^2(x-2)(x+4)$ are 4,4,2,-2

Answer:

(i) False

(ii) True

(iii) True

(iv) False

(v) True

(vi) False

PART - B

Answer any *four* questions out of six questions: [4×4=16]

2. The ratio of the no. of boys to the no. of girls in a school of 720 students is 3:5. If 18 new girls are admitted in the school, find how many new boys may be admitted so that the ratio of the no. of boys to the no. of girls may change to 2:3. [4]

Answer:

1. Ratio of No. of Boys to No. of Girls = 3:5.

Total Parts in the Ratio = Sum of the Ratios = 3 + 5 = 8

2. No. of the Boys in the School = $\frac{3}{8}$ × 720 = 270.

   No. of Girls in the school = $\frac{5}{8}$ × 720 = 450

3. The above data may be summarized in a table as follows –

Particulars	Boys	Girls	Total
1. Existing Strength	270	450	720
2. Additions	X (Assumed)	18	X + 18
3. New Strength	270 + X	468	738 + X
4. Required New Ratio	2	3	5

4. As per the requirements of the questions, $\frac{270+X}{468}=\frac{2}{3}$.

   On Cross Multiplication, we have 3 × (270 + X) = 2 × 468

   Solving, 810 + 3X = 936 or, 3X = 126 or, X = 42.

   Hence the no. of boys admitted = 42.

3. What is the present value of ₹ 1,000 due in 2 years at 5% compound interest, according as the interest is paid (a) yearly, (b) half-yearly? [4]

Answer:

(a) Here A = ₹ 1,000, i = $\frac{5}{100}$ = 0.05, n = 2, P = ?

$A = P(1+i)^n$ or $1000 = P(1 + .05)^2 = P(1.05)^2$

$\therefore$ P = $\frac{1000}{(1.05)^2}=\frac{1000}{1.1025}$ = 907.03

$\therefore$ Present value = ₹ 907.03

(b) Interest per unit per half-year $\frac{1}{2}\times 0.05$ = 0.025

From $A = P\left(1+\frac{i}{2}\right)^{2n}$ we find.

$1,000 = P\left(1+\frac{0.05}{2}\right)^{2\times 2} = P(1+ .025)^4 = P(1.025)^4$

Or, $P = \frac{1000}{(1.025)^4}$

$\therefore$ log P = log 1000 – 4 log (1.025) = 3 – 4 (0.0107) = 3 – 0.0428 = 2.9572

$\therefore$ P = antilog 2.9572 = 906.1

Hence the present amount = ₹ 906.10

4. If a, b, c, d are in G.P., prove that, $(b-c)^2 + (c-a)^2 + (d-b)^2 = (a-d)^2$

[4]

Answer:

a, b, c, d are in G.P.

$\therefore \frac{b}{a} = \frac{c}{b} = \frac{d}{c} = r$

$b/a = r \Rightarrow b = r\,a$

$\frac{c}{b} = r = c = r\,b = r(ra) = r^2a$

$d/c = r \Rightarrow d = r\,c = r(r^2 a) = r^3 a$

Mow, L.H.S. $= (b-c)^2 + (c-a)^2 + (d-b)^2$

$= (r\,a - r^2 a)^2 + (r^2 a - a)^2 + (r^3 a - r\,a)^2$

$= (r^2 a^2 + r^4 a^2 - 2\,r^3 a^2) + (r^4 a^2 + a^2 - 2\,r^2 a^2) + (r^6 a^2 + r^2 a^2 - 2\,r^4 a^2)$

$= a^2\,(r^2 + r^4 - 2\,r^3 + r^4 + 1 - 2\,r^2 + r^6 + r^2 - 2\,r^4]$

$= a^2\,[r^6 - 2r^3 + 1]$

$= a^2\,(r^3 - 1)^2$

R.H.S. $= (a-d)^2 = (a - r^3 a)^2 = [a\,(1-r^3)]^2$

$= a^2\,(1 - r^3)^2$

$= a^2\,(r^3 - 1)^2$

L.H.S. = R.H.S.

5. In a class of 50 students appearing for an examination of ICWA, from a centre, 20 failed in Accounts, 21 failed in Mathematics and 27 failed in Costing, 10 failed both in Accounts and Costing, 13 failed both in Mathematics and Costing and 7 failed both in Accounts and Mathematics. If 4 failed in all the three, find the number of

(i) Failures in Accounts only.

(ii) Students who passed in all the three subjects. [4]

Answer:

A = Accounts, M = Mathematics, C = Costing [No. of students failed (say)]

Now $n(A) = 20$, $n(M) = 21$, $n(C) = 27$, $n(A \cap C) = 10$, $n(M \cap C) = 13$, $n(A \cap M) = 7$

$n(A \cap M \cap C) = 4$.

(i) $n\left(A \cap \bar{M} \cap \bar{C}\right) = n(A) - n(A \cap M) - n(A \cap C) + n(A \cap M \cap C) = 20 - 7 - 10 + 4 = 7$

(ii) Total no of students failed

$= n(A) + n(M) + n(C) - n(A \cap M) - n(M \cap C) - n(A \cap C) + n(A \cap M \cap C)$

$= 20 + 21 + 27 - 7 - 13 - 10 + 4 = 42$

$\therefore$ reqd. no. of pass = 50 – 42 = 8.

6. Number of permutations of *n* objects taken 4 at a time is twice the number of permutations of 5 objects taken 3 at a time. Find the value of *n*. **[4]**

Answer:

Number of permutations of *n* objects taken 4 at a time = ${}^nP_4$.

Number of permutations of 5 objects taken 3 at a time = ${}^5P_3$

According to the given condition, ${}^nP_4 = 2 \times {}^5P_3$

$$\Rightarrow \quad \frac{n!}{(n-4)!} = 2 \times \frac{5!}{(5-3)!}$$

$$\Rightarrow \quad \frac{n!}{(n-4)!} = 2 \times \frac{5!}{2!}$$

$$\Rightarrow \quad \frac{n(n-1)(n-2)(n-3)\cdot(n-4)!}{(n-4)!} = 5!$$

$$\Rightarrow \quad n(n-1)(n-2)(n-3) = 5 \times 4 \times 3 \times 2$$

Comparing both sides, we get

$$n = 5$$

7. $2^{x-2} + 2^{3-x} = 3$ **[4]**

Answer:

$$2^{x-2} + 2^{3-x} = 3$$

$$2^x \cdot 2^{-2} + 2^3 \cdot 2^{-x} = 3$$

$$\frac{2^x}{2^2} + \frac{2^3}{2^x} = 3$$

$$\frac{t}{4} + \frac{8}{t} = 3 \quad \text{when } t = 2^x$$

$$t^2 + 32 = 12t$$

$$t^2 - 12t + 32 = 0$$

$$t^2 - 8t - 4t + 32 = 0$$

$$t(t-8) - 4(t-8) = 0$$

$(t-4)(t-8) = 0$

$\therefore\ t = 4,\ 8$

When, $t = 4$ $\quad 2^x = 4 = 2^2$ i.e. $x = 2$

When, $t = 8$ $\quad 2^x = 8 = 2^3$ i.e. $x = 3$

Section – B

PART - A

8. Answer All objective questions.

(a) Answer Multiple Choice Question **[12×2= 24]**

(i) The Row heading is also known as

(a) Title

(b) Stub

(c) Caption

(d) Body of table

(ii) Horizontal bar diagrams is used for

(a) Qualitative data

(b) Data varying over time

(c) Data varying over space

(d) a or c

(iii) The third quartile of the following observations 10, 19, 22, 16, 15, 18, 20, 18, 14, 18, 23 is

(a) 17.55

(b) 18

(c) 15

(d) 20

(iv) For any two numbers SD is always

(a) Twice the range

(b) Half of the range

(c) Square of the range

(d) none of these

(v) If the coefficient of correlation between two variables is 0.7. Then the percentage of variation unaccounted for is

(a) 70%

(b) 30%

(c) 51%

(d) 49%

(vi) If r = 1, the angle between two regression equation is

(a) 0^0

(b) 90^0

(c) 60^0

(d) 45^0

(vii) If y = $ax^2 + bx + c$, where c > 0, b > 0 and a ≠ 0, then Karl Pearson's correlation coefficient between x and y is

(a) +1

(b) – 1

(c) 0

(d) none of these

(viii) If letters of the word "PENCIL" are arranged in a random order, the probability that N is always next to E is

(a) 4/6

(b) 3/6

(c) 1/6

(d) none of these

(ix) 8 persons are to be arranged in a r ow. What is the probability that there are exactly 3 persons between two particular persons A and B.

(a) 1/7

(b) 1/6

(c) 1/5

(d) none of these

(x) A man and his wife appear for an interview for two posts. The probability of husband's selection is 1/7 and the wife's selection is 1/5. What is the probability that only one of them will be selected?

(a) 2/7

(b) 1/35

(c) 12/35

(d) None

(xi) The mean of first 10 even number is –

(a) 5.5

(b) 55

(c) 11

(d) None of these

(xii) Mode depends on change of –

(a) Origin

(b) Scale only

(c) Both origin and scale

(d) Neither origin nor scale

Answer:

(i) (b) Stub

(ii) (a) Qualitative data

(iii) (d) 20

(iv) (b) Half of the range

(v) (c) 51%

(vi) (a) 0^0

(vii) (d) none of these

(viii) (c) 1/6

(ix) (a) 1/7

(x) (a) 2/7

(xi) (c) 11

(xii) (c) Both origin and scale

(b) State whether the following statements are True (or) False. **[12×1= 12]**

(i) **If each item reduced by 15, AM is increased by 15.**

(ii) **The greater of the two numbers where arithmetic mean is 34 and the geometric mean is 16 is 64.**

(iii) **If the first and third quartiles are 22.16 and 56.36, then the quartile deviation is 17.1.**

(iv) **"Root – mean square deviation from Mean's" is Quartile deviation and Standard deviation.**

(v) **In ogive, abscissa corresponding to ordinate KN/10 is K^{th} percentile.**

(vi) **Scatter diagram helps us to find the nature of correlation between two variables.**

(vii) **When one regression coefficient is positive, the other would be negative.**

(viii) **If P(A) = 1, then the event A is known as improbable event.**

(ix) **If events are mutually exclusive, then both events cannot occur at some time.**

(x) **As the sample increases, range tends to decrease.**

(xi) **The positive average is harmonic means.**

(xii) **Difference between the maximum and minimum value of a given data is range.**

Answer:

(i) False

(ii) True

(iii) True

(iv) False

(v) True

(vi) True

(vii) False

(viii) True

(ix) True

(x) False

(xi) False

(xii) True

PART – B

4 Question to be answered out of 6 questions [6×4=24]

9. Explain the Methods of collecting Primary Data. [6]

Answer:

The primary data can be collected by the following methods:

1. **Direct personal observation**: In this method, the investigator collects the data personally and, therefore, it gives reliable and correct information.
2. **Indirect oral investigation**: In this method, a third person is contacted who is expected to know the necessary details about the persons for whom the enquiry is meant.
3. **Estimates from the local sources and correspondence**. Here the investigator appoints agents and correspondents to collect the data
4. **Data through questionnaires**. The data can be collected by preparing a questionnaire and getting it filed by the persons concerned.
5. **Investigations through enumerators**. This method I generally employed by the Government for population census, etc.

10. An incomplete frequency distribution is given as follows:

Variable	Frequency	Variable	Frequency
10-20	12	50-60	?
20-30	30	60-70	25
30-40	?	70-80	18
40-50	65	Total	229

Given that the median value is 46, determine the missing frequency using the median formula. [6]

Answer:

Let the frequency of class 30-40 be f_1, and that of 50-60 be f_2.

Then $f_1 + f_2 = 229 - (12 + 30 + 65 + 25 + 18) = 79$

Since median is given to be 46, the class 40-50 is the median class. Hence using median formula we get,

$$46 = 40 + \frac{114.5-(12+30+f_1)}{65}\times 10$$

$$46 - 40 = \frac{72.5-f_1}{65}\times 10 \quad \text{or } 6 = \frac{72.5-f_1}{6.5}$$

$f_1 = 72.5 - 39 = 33.5 = 34$ [Since frequency is never fraction]

$f_2 = 79 - 34 = 45.$ [Since $f_1 + f_2 = 79$]

11. From the following table giving height of students calculate the semi-interquartile Range and the co-efficient of Quartile Deviation.

Height in inches	No. of students
53	**25**
55	**21**
57	**28**
59	**20**
61	**18**
63	**24**
65	**22**
67	**18**
69	**23**

[6]

Answer:

Computation of the Semi-interquartile Range

Height in inches	No. of Students	Cumulative Frequencies
53	25	25
55	21	46
57	28	74
59	20	94
61	18	112
63	24	136
65	22	158
67	18	176
69	23	199

First Quartile or. Q_1 = the height of the $\frac{199+1}{4}$ i.e. 50^{th} student = 57 inches.

Third Quartile or Q_3 = the height of the $\frac{3(199+1)}{4}$ i.e., 150^{th} student = 65 inches.

Semi-interquartile Range or the Quartile Deviation = $\frac{Q_3 - Q_1}{2} = \frac{65-57}{2}$ = 4 inches

Quartile Co-efficient of dispersion = $\frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{65-57}{65+57}$ = .066 inches

12. Find Pearson's co-efficient of correlation from the following data: **[6]**

(i) **n = 50, $\Sigma X = 75$, $\Sigma Y = 80$, $\Sigma X^2 = 130$, $\Sigma Y^2 = 140$, $\Sigma XY = 120$**

(ii) **n = 10, $\Sigma X = 140$, $\Sigma Y = 150$, $\Sigma(X - 10)^2 = 180$.**

$\Sigma(Y - 15)^2 = 215$, $\Sigma(X - 10)(Y - 15) = 60$.

Answer:

(i) $r = \dfrac{50\times120-75\times80}{\sqrt{50\times130-(75)^2}\cdot\sqrt{50\times140-(80)^2}}$

$= \dfrac{6000-6000}{\sqrt{6500-5625}\cdot\sqrt{7000-6400}} = \dfrac{0}{\sqrt{875}\cdot\sqrt{600}} = 0$

(ii) $\Sigma d_x = 140 - 10 \times 10 = 140 - 100 = 40$ (as differences are taken from A = 10).

Again $\Sigma d_y = 150 - 10 \times 15 = 150 - 150 = 0$

(in this case B = 15, from which difference are taken)

By question, $\Sigma d_x^2 = 180$, $\Sigma d_y^2 = 215$, $\Sigma d_x d_y = 60$.

$r = \dfrac{10\times60-40\times0}{\sqrt{10\times180-(40)^2}\,\sqrt{10\times215-0}} = \dfrac{600-0}{\sqrt{1800-1600}\,\sqrt{2150}}$

$= \dfrac{600}{\sqrt{200}\,\sqrt{2150}} = \dfrac{600}{14.14\times46.37} = \dfrac{600}{655.67} = 0.915 = 0.92.$

13. Fit a linear regression of marks in University examination to the same in College test.

Serial No.	**1**	**2**	**3**	**4**	**5**	**6**
Marks in College Test:	**35**	**42**	**20**	**50**	**72**	**64**
Marks in University examination:	**40**	**48**	**24**	**60**	**84**	**68**

Let X = marks in College test, Y = marks in University examination. Here we are to determine the equation of best fitted regression line of Y on X. [6]

Answer:

Calculation of Regression Equations

Sr. No.	X	Y	X^2	Y^2	XY
1	35	40	1225	1600	1400
2	42	48	1764	2304	2016
3	20	24	400	576	480
4	50	60	2500	3600	3000
5	72	84	5184	7056	6048
6	64	68	4096	4624	4352
Total	**283**	**324**	**15169**	**19760**	**17296**

Regression equation of Y on X is $Y - \overline{Y} = b_{YX}\ (X - \overline{X})$

where byx = $b_{YX} = \dfrac{\frac{\Sigma XY}{n} - \frac{\Sigma X}{n}\frac{\Sigma Y}{n}}{\frac{\Sigma X^2}{n} - \left(\frac{\Sigma X}{n}\right)^2} = \dfrac{\frac{17296}{6} - \frac{283}{6}\cdot\frac{324}{6}}{\frac{15169}{6} - \left(\frac{283}{6}\right)^2}$ (here n = 6)

$= \dfrac{2882.67 - 47.17 \times 54}{2528.17 - (47.17)^2} = \dfrac{2882.67 - 2547.18}{2528.17 - 2250.01} = \dfrac{335.49}{278.16} = 1.206$

Again $\overline{Y} = \dfrac{\Sigma Y}{n} = \dfrac{324}{6} = 54$, $\quad \overline{X} = \dfrac{\Sigma X}{6} = \dfrac{283}{6} = 47.17$ (n = 6 given)

$\therefore$ reqd. equation is Y – 54 = 1.206 (X – 47.17)

or Y – 54 = 1.206 (X – 47.17) or Y = 1.206 X – 2.89.

Note. Mean of X is not an integer, so calculation of regr. eqn. by taking deviations from A.M. is avoided.

14. A bag contains 5 red and 4 black balls, and the second one 3 red and 5 black balls. One of these is selected at random and a draw of two balls is made from it. What is the probability that one of them is red and other is black? [6]

Answer:

Let A_1 = event of selecting 1st bag

A_2 = event of selecting 2nd bag

B_1 = event of drawing 2 balls of different colours from 1st bag

B_2 = event of drawing 2 balls from 2nd bag

Now we have $(A_1 \cap B_1)$ or $(A_2 \cap B_2)$ which are mutually exclusive and equally likely.

So $P[(A_1 \cap B_1) \text{ or } (A_2 \cap B_2)] = P[(A_1 \cap B_1) \cup (A_2 \cap B_2)]$

$= P(A_1 \cap B_1) + P(A_2 \cap B_2)$

$= P(A_1)\, P(B_1/A_1) + P(A_2)\, P(B_2/A_2)$ by compound probability

Again $P(A_1) = \dfrac{1}{2}$, $P(B_1/A_1) = \dfrac{5 \times 4}{{}^9C_2} = \dfrac{20}{9.8/2} = 20 \times \dfrac{2}{9 \times 8} = \dfrac{5}{9}$

And $P(A_2) = \dfrac{1}{2}$, $P(B_2/A_2) = \dfrac{3 \times 5}{{}^8C_2} = \dfrac{3 \times 5}{8.7/2} = 15 \times \dfrac{2}{8 \times 7} = \dfrac{15}{28}$

$\therefore$ reqd. probability $\dfrac{1}{2} \cdot \dfrac{5}{9} + \dfrac{1}{2} \cdot \dfrac{15}{28} = \dfrac{1}{2}\left(\dfrac{5}{9} + \dfrac{15}{28}\right) = \dfrac{1}{2} \times \dfrac{275}{252} = \dfrac{275}{504}$