Paper- 4: FUNDAMENTALS OF BUSINESS MATHEMATICS AND STATISTICS

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Full Marks: 100 **Time Allowed: 3 Hours**

Section – A
(Fundamentals of Business Mathematics)
PART A

1. **Answer All objective questions.**
   (a) **Answer Multiple Choice Question** **[9×2= 18]**

(i) **A certain amount was divided between X and Y in the ratio 4 : 3. If B's share was ₹4,800, the total amount was:**

(a) ₹11,200

(b) ₹6,400

(c) ₹19,200

(d) ₹39,200

(ii) **At what rate converted semi-annually will the present value of a perpetuity of ₹450 payable at the end of each 6 months be ₹20,000.**

(a) 5.4%

(b) 5%

(c) 4.59

(d) 4%

(iii) **In how many ways can 15 things be divided into three groups containing 8, 4 and 3 things respectively.**

(a) $\frac{15!}{8!.\ 4!.\ 3!}$

(b) 15!

(c) 7!

(d) None of these

(iv) **How many combinations can be formed of 8 counters marked 1, 2, 3, 4, 5, 6 7, 8 taking them 4 at a time, there being at least one odd and one even counter in each combination**

(a) 80

(b) 86

(c) 68

(d) None of these

(v) **$\log_{100}(0.1)$ = ?**

(a) -2

(b) $\frac{1}{2}$

(c) $-\frac{1}{2}$

(d) 2

(vi) **What will be the difference between simple and compound interest on ₹8,000 at the rate of 5 percent per annum at the end of 3 years?**

(a) ₹61.00

(b) ₹122.00

(c) ₹91.50

(d) ₹152.50

(vii) **In how many ways can 8 books can be arranged, so that the best and worst books never come together**

(a) 8!

(b) 7!. 2!

(c) 7!

(d) None of these

(viii) **If the sum of an infinitely decreasing G.P. is 3 and the sum of the squares of its terms is (9/2), then the sum of the cubes of these terms is –**

(a) $\frac{105}{13}$

(b) $\frac{108}{13}$

(c) $\frac{729}{8}$

(d) None

(ix) **[n (n + 1)/2 : n is a positive integer] is**

(a) a finite set

(b) An infinity set

(c) is an empty set

(d) None of these

(b) Answer the following Question True or False **[6×1=6]**

(i) The g.c.d of the equations $2x^2-x-1 = 0$ and $4x^2+8x+3 = 0$ is $3x+1$ **( )**

(ii) The total number of 9 digits numbers which have all different digits id 9×9 **( )**

(iii) The numbers of different number of 6 digits (without repetition) can be formed form the digits 3,1,7,0,9,5 is 120 **( )**

(iv) The logarithms with base e are called comm. Logarithm **( )**

(v) If $9\times81^x = \frac{1}{27^x - 3}$ then the value of x is **( )**

(vi) The difference between S.I and C.I on ₹1,000 for 1 years at 4% payable quarterly is 0.40 **( )**

Answer: 1 (a)

(i) (a)

(ii) (c)

(iii) (a)

(iv) (c)

(v) (c)

(vi) (a)

(vii) (b)

(viii) (b)

(ix) (b)

Answer: 1 (b)

(i) False

(ii) True

(iii) False

(iv) False

(v) True

(vi) False

PART B

4 Questions to be answered out of 6 questions [4×4=16]

2. A locomotive engine without a train can go 24 Km./hr. and its speed is diminished by a quantity which varies as the square root of the number of wagons attached. With 4 wagons its speed is 20 Km./hr. Find the greatest number of wagons with which the engine can move. [4]

Answer:

Let v Km./hr. be the speed of the engine when x wagons are attached. Then the decrease in speed = $(24 - v) \propto \sqrt{x}$.

or, $24 - v = k\sqrt{x}$ (k is a constant).

Given: v = 20 when x = 4. This gives $4 = k\sqrt{4}$ or k = 2. Now, let y be the least number of wagons attached with which the engine just fails to move (i.e. v = 0).

Then $24 - 0 = 2\sqrt{y}$ or y = 144.

$\therefore$ the greatest number of wagons with which the engine can move.

3. In what time will the C.I. on ₹ 1,200 be ₹ 124.60 p. at 8% p.a. payable quarterly? [4]

Answer:

Let n = reqd. no. of years.

Here, P = ₹1,200. $i = \frac{8}{100 \times 4} = 0.02$

C.I. = A – P = $P\left[(1+i)^n - 1\right]$ = ₹ 1,200 $\left[(1+0.02)^{4\times n} - 1\right]$

$\therefore$ ₹ 124.60 = ₹ 1,200 $\left[(1.02)^{4n} - 1\right]$.

or, $\frac{124.60}{1200} + 1 = (1.02)^{4n}$, or $\frac{1324.60}{1200} = (1.02)^{4n}$,

or, taking log of both sides, log 1324.60 – log 1200 = 4n log 1.02;

log 1324.60	=	3.1221
log 1200	=	<u>3.0792</u>
diff.,	=	0.0429
4 × log 1.02	=	4 × 0.0086
	=	0.0344

$$\therefore \quad n = \frac{\log 1324.60 - \log 1200}{4 \times \log 1.02} = \frac{0.0429}{0.0344}$$

$= \frac{429}{344}$ = **1 year 3 months** (approx.)

4. Which term of the G.P. 36, 24, 16, ….. is 512/81? [4]

Answer:

Take $a = 36,\ r = \frac{24}{36} = \frac{2}{3},\ a_n = \frac{512}{81}$

$a_n = a\ r^{n-1}$

$\Rightarrow\ 512/81 = 36 \times (2/3)^{n-1}$

$\Rightarrow\ \frac{512}{81 \times 36} = (2/3)^{n-1}$

$\Rightarrow\ \frac{128}{729} = (2/3)^{n-1}$

$\Rightarrow\ (2/3)^7 = (2/3)^{n-1}$

$\Rightarrow\ n - 1 = 7$

$\Rightarrow\ n = 8$

$\therefore\ \frac{512}{81}$ is 8^{th} term.

5. A If $x = 2 + 2^{2/3} + 2^{1/3}$, prove that $x^3 - 6x + 6x - 2 = 0$. [4]

Answer:

From the given condition

$x - 2 = 2^{2/3} + 2^{1/3}$

Cubing both sides [Remember: $(a + b)^3 = a^3 + b^3 + 3ab\ (a + b)$]

$(x - 2)^3 = (2^{2/3} + 2^{1/3})^3$

$= (2^{2/3})^3 + (2^{1/3})^3 + 3 \,.\, 2^{2/3} \,.\, 2^{1/3}\ (2^{2/3} + 2^{1/3})$

$= 4 + 2 + 3.2 \times (x - 2)$

or, $x^3 - 3x^2 . 2 + 3 . x . 2^2 - 2^3 = 6 + 6\ (x - 2)$

or, $x^3 - 6x^2 + 12x - 8 = 6x - 6$

or, $x^3 - 6x^2 + 6x - 2 = 0.$

6. If ${}^nC_8 = {}^nC_6$, find ${}^nC_2$. [4]

Answer:

${}^nC_8 = {}^nC_6$

$\Rightarrow\ {}^nC_{n-8} = {}^nC_6$ $\quad (\because\ {}^nC_{n-r} = {}^nC_r)$

$\Rightarrow\ n - 8 = 6$

$\therefore\ n = 6 + 8 = 15$

∴ $^{n}C_2 = {}^{15}C_2 = \frac{15!}{2!(15-2)!}$

$\frac{15!}{2\times1\times13!}$

$= \frac{15\times\overset{7}{\cancel{14}}\times13!}{\cancel{2}\times13!}$

$= 15 \times 7$

$= 105$

7. Solve $x^2+7x+\sqrt{x^2+7x+9}=3$ **[4]**

Answer:

Adding 9 to both sides, we have $x^2+7x+9+\sqrt{x^2+7x+9} = 12$.

Now putting $u = \sqrt{x^2+7x+9}$, the equation reduces to

$U^2 = u + 12$ or $U^2 + U - 12 = 0$

Or, $U^2 + 4U - 3U - 12 = 0$ or, $u(u+4) - 3(u+4) = 0$

Or, $(u-3)(u+4) = 0$ $u = 3, -4$

Since is not negative, we reject the value – 4 for u

$u = 3$

$\sqrt{x^2+7x+9} = 3$

Or, $x^2 + 7x + 9 = 9$

Or, $x^2 + 7x = 0$

Or, $x^2 = -7x$

Or, $x = -7$

$u = -4$

$\sqrt{x^2+7x+9} = -4$

Or, $x^2 + 7x + 9 = 16$

Or, $x^2 + 7x - 7 = 0$

$x = \frac{-7\pm\sqrt{49+28}}{2}$

Section – B

PART A

8. Answer All objective questions.

(a) Answer Multiple Choice Question **[12×2= 24]**

(i) If r = 0.7, then the value of coefficient of determination is

(a) 0.51

(b) 0.7

(c) 0.49

(d) 0.50

(ii) The two regression lines are 3x – y = 0 and 3x – 4y = 0. If variance of x is 4 then variance of y is

(a) 4

(b) 2

(c) 3

(d) 9

(iii) The mode of 5, 5, 5, 7, 9, 10, 10, 10 is

(a) 5

(b) 10

(c) 5 and 10

(d) None of these

(iv) In the method of concurrent deviations, only the changes of signs in the values of the variables are taken in account for the calculation of

(a) Coefficient of standard deviation

(b) Coefficient of determination

(c) Coefficient of regression

(d) Coefficient of correlation

(v) Mean deviation from the mean for the observations 0, -1, 4 is

(a) 2

(b) 2/5

(c) 3/5

(d) None of these

(vi) For three mutually exclusive and exhaustive events A, B and C, 2P(A) = 3P (B=) = P (C). What is P(BUC)?

(a) $\frac{6}{11}$

(b) $\frac{5}{11}$

(c) $\frac{9}{11}$

(d) $\frac{8}{11}$

(vii) If r = -0.9, it indicates that

(a) There is high degree of correlation between two variables and changes are in opposite direction

(b) The assumption of liner correlation is valid

(c) The correlation between population variables is significant

(d) All of these

(viii) N = 10, ∑x = 55, ∑y = 88, ∑x2 = 385, ∑y2 = 1114, ∑xy = 586. The regression equation of y on x is

(a) 1.98x- y + 1.24 = 0

(b) 1.24x-y + 2 = 0

(c) 124x – 100y + 198 = 0

(d) 12.4x – 10y + 1.94 = 0

(ix) A frequency distribution

(a) Arranges observations on an increasing order

(b) Arranges observation in terms of a number of groups

(c) Relates to a measurable characteristic

(d) All these

(x) Median of 2, 4, 5, 6, 7, 8 and 9 is

(a) 9

(b) 6

(c) 3

(d) 5

(xi) Three families consist of 3 boys and 2 girls, 2 boys and 2 girls, and 2 boys and 3 girls respectively. A family is selected at random and from it two children are selected. What is the probability that both of them are girls?

(a) 0.19

(b) 0.12

(c) 0.04

(d) None

(xii) The variance of standard normal distribution is

(a) 1

(b) μ

(c) 02

(d) 0

(b) Answer the following Question True or False **[12×1=12]**

(i) If P(A) = 7/8 then P(A^C) is equal to 0

(ii) Initially, probability was a branch of Mathematics

(iii) If P(A) = 1, then the event A is known as Importable event

(iv) BAYE's Theorem is not associated with the name of Reverend Thomas Bayes.

(v) Two regression lines coincide when r = D

(vi) If r = 0.6 then the coefficient of non-determination is 0.64

(vii) 10th percentile is equal to 1st decile

(viii) Quartile deviation is based on the Highest 50%

(ix) An ideal measure of central tendency is Moving average

(x) Pooled mean is also called Grouped mean

(xi) The colour of a flower is an example of a variable

(xii) Weights are generally called Frequencies

Answer: 1 (a)

(i) (c)

(ii) (c)

(iii) (c)

(iv) (d)

(v) (a)

(vi) (d)

(vii) (d)

(viii) (c)

(ix) (d)

(x) (b)

(xi) (a)

(xii) (a)

Answer: 1 (b)

(i) False

(ii) True

(iii) True

(iv) False

(v) False

(vi) True

(vii) True

(viii) False

(ix) True

(x) True

(xi) False

(xii) True

PART B

4 Questions to be answered out of 6 questions [6×4=24]

9. Explain the Importance and Scope of Statistics. [6]

Answer:

Importance and Scope of Statistics:

Statistics and Economics: According to Prof. Alfred Marshall, "Statistics are the straws out of which I like every other economist, have to make bricks." The following are some of the fields of economics where statistics is extensively used.

(a) **Consumption**: Statistical data of consumption enable us to find out the ways in which people in different strata of society spend their incomes.

(b) **Production**: The statistics of production describe the total productivity in the country. This enables us to compare ourselves with other countries of the world.

(c) **Exchange**: In the field of exchange, an economist studies markets, laws of prices are determined by the forces of demand and supply, cost of production, monopoly, competition, banking etc. A systematic study of all these can be made only with the help of statistics

(d) **Econometrics**: With the help of econometrics, economics has become exact science. Econometrics is the combination of economics, mathematics and statistics.

(e) **Public Finance:** Public finance studies the revenue and expenditure activities of a country. Budget, (a statistical document), fiscal policy, deficit financing, etc., are the concepts of economics which are based on statistics.

(f) **Input-Output Analysis**: The input-output analysis is based on statistical data which explain the relationship between the input and the output. Sampling, Time series, Index numbers, Probability, Correlation and Regression are some other concepts which are used in economic analysis.

10. From the following table, find the median time taken by 40 male students to solve a problem. [6]

Table

Time (S)	Frequency
118-126	3
127-135	5
136-144	9
145-153	12
154-162	5
163-171	4
172-180	2
	Total = 40

Answer:

Median = the size of $n/2$th item. Here Median = the size of 40/2th item = the size of 20th item. Since the sum of the first three and first four class frequencies are 3 + 5 + 9 = 17 and 3 + 5 + 9 + 12 = 29 respectively, it is clear that the median lies in the fourth class, which is therefore the median class.

And L_1 = Lower class boundary of median class = 144.5

N = number of items in the data = 40

cf = Sum of all classes lower than the median class = 3+ 5 + 9 = 17
(the cumulative frequency of the class preceding the median class)

f = frequency of median class = 12

c = Size of median class interval = 9

and thus

Median = $L_1 + \frac{N/2 - cf}{f} \times c \qquad = 144.5 + \frac{40/2 - 17}{12} \times 9$

$= 146.75$

11. Calculate (a) mean coefficient of dispersion from the following data: [6]

Table

Marks:	**10**	**15**	**20**	**30**	**40**	**50**
Frequency:	**8**	**12**	**15**	**10**	**3**	**2**

Answer:

Table : Calculation of Mean Coefficient of Dispersion

Marks x	***f***	***fx***	**Deviation of x from 21.6 ignoring signs |D|**	***f* |D|**
10	8	80	11.6	92.8
15	12	180	6.6	79.2
20	15	300	1.6	24.0
30	10	300	8.4	84.0
40	3	120	18.4	55.2
50	2	100	28.4	56.8
	N = 50	Σfx = 1080		$\Sigma f\|D\|$ = 392

$\overline{x} = \frac{\Sigma fx}{n} = \frac{1080}{50} = 21.6$

M.D. = $\frac{\Sigma f|D|}{N} = \frac{392}{50} = 7.84$

Mean Coefficient of dispersion = $\frac{\text{Mean Deviation}}{\text{Mean}} = \frac{7.84}{21.6} = 0.363$

12. Consider the following table. Calculate the product moment correlation coefficient.

Table

1998	**Mean Temperature (°C)**	**Beer Production (million barrels)**
January	**6**	**2.5**
February	**5**	**2.4**
March	**5**	**3.3**
April	**8**	**3.3**
May	**12**	**3.5**

June	**17**	**3.7**
July	**19**	**3.9**
August	**18**	**3.6**
September	**14**	**3.4**
October	**11**	**3.1**

[6]

Answer:

Table

X	Y	X^2	Y^2	XY
6	2.5	36	6.25	15
5	2.4	25	5.76	12
5	3.3	25	10.89	16.5
8	3.3	64	10.89	26.4
12	3.5	144	12.25	42
17	3.7	289	13.69	62.9
19	3.9	361	15.21	74.1
18	3.6	324	12.96	64.8
14	3.4	196	11.56	47.6
11	3.1	121	9.61	34.1
$\Sigma X = 115$	$\Sigma Y = 32.7$	$\Sigma X^2 = 1585$	$\Sigma Y^2 = 109.07$	$\Sigma XY = 395.4$

$$r = \frac{n\Sigma XY - \Sigma Y\,\Sigma X}{\sqrt{(n\Sigma X^2 - (\Sigma X)^2\,(n\Sigma Y^2 - (\Sigma Y)^2)}}$$

$$= \frac{10 \times 395.4 - 32.7 \times 115}{\sqrt{(10 \times 1585 - (115)^2)\,(10 \times 109.07 - (32.7)^2)}}$$

$$= \frac{193.5}{\sqrt{2625 \times 21.41}} = \frac{193.5}{237.1} = 0.816$$

13. Compute the regression coefficients from the data given below and find the value of 'r' (the correlation coefficient) using the same:

Table

X	**7**	**4**	**8**	**6**	**5**
Y	**6**	**5**	**9**	**8**	**2**

[6]

Answer:

Table

X	x	Y	y	x^2	Y^2	xy
7	1	6	0	1	0	0

4	-2	5	-l	4	1	2
8	2	9	3	4	9	6
6	0	8	2	0	4	0
5	1	2	-4	1	16	4
$\Sigma X = 30$		$\Sigma Y = 30$		$\Sigma x^2 = 10$	$\Sigma y^2 = 30$	$\Sigma xy = 12$

$\bar{X} = \frac{30}{5} = 6$ and $\bar{Y} = \frac{30}{5} = 6$ and $x = (X - \bar{X})$

And $y = (Y - \bar{Y})$

Regression coefficient of X on Y is

$$b_{xy} = \frac{\Sigma xy}{\Sigma y^2} = \frac{12}{30} = 0.4$$

Regression coefficient of Yon X is

$$b_{yx} = \frac{\Sigma xy}{\Sigma x^2} = \frac{12}{10} = 1.2$$

$$r = \pm\sqrt{b_{xy} \cdot b_{yx}}$$

$$= \sqrt{0.4 \times 1.2}$$

$$= 0.693.$$

14. Four cards are drawn at a time from a pack of 52 playing cards. Find the probability of getting all the four cards of the same suit. [6]

Answer:

There are four suits, namely, clubs, spades, diamonds and hearts. Each suit contains 13 cards out of total 52 cards.

P (all 4 cards of the same suit)

= P (4 spades or 4 clubs or 4 hearts or 4 diamonds)

= P (4 spades) + P (4 clubs) + P (4 hearts) + P (4 diamonds)

$$= \frac{^{13}C_4}{^{52}C_4} + \frac{^{13}C_4}{^{52}C_4} + \frac{^{13}C_4}{^{52}C_4} + \frac{^{13}C_4}{^{52}C_4}$$

$$= 4 \times \frac{^{13}C_4}{^{52}C_4}$$

$$= 4 \times \frac{13 \times 12 \times 11 \times 10}{52 \times 51 \times 50 \times 49}$$

$$= \frac{44}{4165}.$$