CA Final

Paper-2

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SRIRAM SOMAYAJULA

CA, CFA, ISB

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DIKSHA GOYAL (AIR 1)

I took AFM classes from 1FIN for my CA Final preparation I liked Sriram Sir's teaching on Excel a lot. I used 1FIN material fully and made short notes while watching the videos. I used 1FIN app for MCQs both at Inter & Final level - they were extremely value adding. The animation feature is something I really loved. I still remember the Brinjal Farmer example for Forwards. I recommended 1FIN's classes to several of my friends.

Sindhuri

Every concept was explained with crystal clarity, I feel way more confident now. Soo glad that I found your classes!!

Maanu

The way of presentation is absolutely beautiful. It's easy to understand the concepts from the classes. Thanks for the session

Priya Namburi

I have cleared my CA final both groups in my first attempt. I would like to thank Suraj and Sriram sir. Because of their teaching it was easy to understand the core of the subject and I gained confidence. I scored 50+ in both. Their teaching style is the best

Shivaji Hari

Thank you so much for sharing your tips. You've completely changed my perspective and thought process. You are my true guru in every aspect since 3 years.Expressing my gratitude in words doesn't feel enough .

Devika

Watching 1FIN classes was the best choice. I love how you break things down so well and explain clearly Sir.

Sathya K

I cannot thank Indigo learn enough" The practical insights provided during lectures prepared me for the real exam...where I cleared the group with exemption in AFM with 82 marks. Special thanks to Sriram sir who cautioned me in each and every sum which really helped me to understand the concept. Thank you for the wonderful classes. I owe a lot!

Ravi Pulavarthi

Sir nice very good explanation of theory

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Rama Sesha Gopal
I have scored 70 marks in AFM. I want to thank Sriram sir for this. Secondly, the P600+ has been a solid practice material and aided my preparation with detailed answers, questions from immediately ended exams. Thanks for making an impact in our lives... very grateful sir

Nikhil M
Classes are amazing! Thankyou for explaining everything so clearly. I actually get the concepts now.

Prashant
Superb classes! Perfect to achieve a strong grasp of the subject.I liked how they made learning fun.

Gayatri maniyam
Honestly, these classes made a huge difference for me. I feel more confident with many topics now.

Painedi Adharsh
Hi Sir, Your teaching was excellent. I watched your revision lectures whenever I had doubts. I'm pleased to share that I scored 71 marks in AFM. Thank you for your valuable guidance and support.

Srinivas P
These were one of the best classes I have ever experienced. Great explanation, covering every topic right from basics to advanced. Glad I found 1FIN and your classes Sir.

Bala
AFM I got exemption sir 62. Thanks for your support sir ur videos helped me a lot in my preparation.

Kavya S
Omg Amazing classes thank you so much Sriram Sir

Shiva R
Each and every topic was covered soo clearly. Thankyou soo much for the classes Sir, they were really very helpful!!

Nanditha
The best class I have ever seen in my career. Satisfied

Asalamsha
Classes were very useful for conceptual understanding

Our Rankers

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CA Final Jan-26

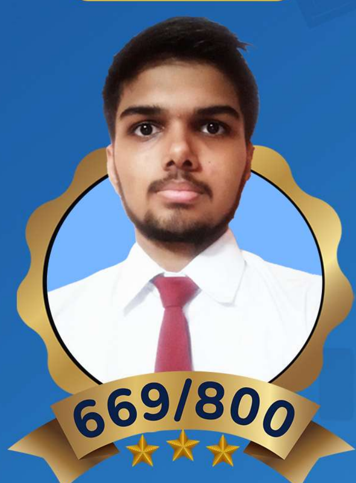
AIR 49

Hamsa Gayatri
CA Final Sep-25

AIR 27

Sudeepta Benya
CA Final Nov-24

AIR 5



Sarthak
CA Inter May-23

AIR 19

Aman Mahajan
CA Inter Dec-21

AIR 33

Sundar B
CA Inter Dec-21

1.

ADVANCED FINANCIAL MANAGEMENT

Question Bank - Volume 1

V 6.0 |April2026| 600 + Illustrations

Dear Students,

This compiler has been designed to help students appearing for the CA Final Advanced Financial Management (AFM) Paper in the ***Nov'26 Exams*** and thereafter to **SUCCEED LIKE NEVER BEFORE**. The Notes are updated with Questions and suggested solutions till Jan 2026 Exams

This Compiler is unique and special for following reasons,

a) This compiler is **NOT** a copy paste of ICAI solutions – all the solutions as per what our faculty has solved in the class. You will not have issues of steps / answers being incorrect (like some other compilers / ICAI resources)
b) It is designed in such a way that problems of all varieties are covered. There are no unnecessary duplication of similar variety of probelms being solved multiple times – so using this compiler **saves time**
c) The compiler has **600 + questions and answers** from all practical chapters and covers Past Exam papers, ICAI Study Material, RTPs, MTPs and even the Old Syllabus Practice Manual.
d) We are also providing you with a quick reference **Formula Sheet** for your revision purposes

In the process of creating this compiler, despite our best intentions, some errors could have crept in. If you find any such errors, do reach out to us at 1fin.in/support and we would be happy to correct them in the next version

This document is the **Sixth** version of compilation of our efforts. We will be coming out with the next version in late 2026 or early 2027 with updated RTPs, MTPs, and of course relevant updates for subsequent attempts including corrections if any 😊

We look forward to your success!!

Yours

Sriram Somayajula CA, CFA, ISB
1FIN

CA FINAL PAPER 2:ADVANCED FINANCIAL MANAGEMENT (100M)
CHAPTER WISE WEIGHTAGE BASED ON PAST EXAM & ANALYSIS THEREOF

S. No.	CHAPTER NAME	Jan-26	Sep-25	May-25	Nov-24	May-24	Avg. of category	ICAI Weightage
1	Financial Policy & Corporate Strategy	4	4	4	4	4	13.20 Marks	8-15 Marks
2	Advanced Capital Budgetting Decisions	6	6	6	8	8		
3	Risk Management	4	4	4	-	-		
	TOTAL	**14**	**14**	**14**	**12**	**12**		
4	Security Analysis	4	4	4	-	6	26.00 Marks	20-30 Marks
5	Security Valuation	6	12	6	18	6		
6	Portfolio Management	14	6	16	14	14		
	TOTAL	**24**	**22**	**26**	**32**	**26**		
7	Securitization	4	4	4	-	4	28.60 Marks	20-30 Marks
8	Mutual Funds	12	17	17	6	8		
9	Derivatives, Analysis & Valuation	18	10	13	20	6		
	TOTAL	**34**	**31**	**34**	**26**	**18**		
10	Forex Exposure & Risk Management	16	17	12	14	-	25.80 Marks	20-25 Marks
11	International Financial Management	6	7	6	6	8		
12	Interest Rate Risk Management	6	4	7	10	10		
	TOTAL	**28**	**28**	**25**	**30**	**18**		
13	Business Valuation	4	13	11	8	-	14.40 Marks	10 - 15 marks
14	M&A & Corporate Restructuring	10	6	4	6	10		
	TOTAL	**14**	**19**	**15**	**14**	**10**		
15	Startup Finance	4	4	4	4	4	4.00 Marks	2-5 marks
	TOTAL	**4**	**4**	**4**	**4**	**4**		
16	Removed Topics / MCQS				-	30		
	Grand Total	**118**	**118**	**118**	**118**	**118**		

NOTE: 1: Weightage of Optional questions has been taken in calculations;
MCQ Breakup is not known for May-24 as Q paper is not publicly shared

CA FINAL PAPER 2: ADVANCED FINANCIAL MANAGEMENT

QUESTION WISE TOPICS BASED ON PAST EXAM PATTERN

Q.	Jan-26		Sep-25		May-25		Nov-24		May-24	
	Topic	M	Topic	M	Topic	M	Topic	M	Topic	M
Part 1 - MCQs										
Set 1	Sec Val Bonds - Repor / Rev Repo	6	Adv Cap Budgetting NPV sensitivity	6	Forex - Evaluation of various choices for hedging	8	Security Valuation - Equities DDM	10		
Set 2	Forex - fate of Forward	6	MF NAV computation	6	Portfolio Management - Beta, Co Variance & SD	10	Security Valuation - Bonds - Price & Duration	8		
Set 3	Portfolio Management - CAPM	6	Derivatives - Options Pay off	6	MFs - NAV Computation	6	Derivatives - Options Call & Put Pay off	6		
Set 4	M&A Swap Ratio	6	Forex - Broken period forward	6	Devivatives - Futures - Gain Loss	6	M&A - Swap Ratio & Shares Issues	6		
Set 5	Adv Cap Bud Dec - product Mix	6	M&A Post Acq EPS etc	6				8		
COMPULSORY										
1 (a)	MF - compute ope NAV after bonus	6	Business Valuation - FCFF / FCFE based valuation	6	Sec Valuation Eq - Earnings Growth Model	6	Portfolio Management - Risk Return Using SD	6	Portfolio Management - Action to be done on shares based on next years financials	6
1 (b)	M&A - Free Float Market Cap	4	Forex - Gain loss on hedging	4	Business Valuation - EVA	4	Real options - Abandonment Option	4	Mutual Funds - Compute NAV based on certain data	4
1 (c)	Options - Call + Put Pay off	4	Succession Planning (T)	4	Types of risks in Imports & Exports (T)	4	Dividend Decisions (Fin Policy & Corp Startegy) - (T)	4	Interface of Strategic Management & Financial Policy (T)	4
OPTIONAL (4 of 5)										
2(a)	Forex - Supplier Credit / Local credit	6	Sec Val Eq - GGM valuation	6	Portfolio management - Port var with Correl	6	Mutual Funds - Compute NAV based on Holding Period Returns	6	Security Analysis - Exponential Moving Average	6
2(b)	Fin Policy (EFR)	4	IRRM - FRA payoff	4	Forex - Buy & sell currency rates	4	ACBD - Compute Investmet based on CF to arrive at 0 NPV	8	International Financial Management - GDR floatation cost	4
2(c)	Real Options Theory Diff with Fin options (T)	4	Factors that made Organization Financially Sustainable (T)	4	Role of CFO (T)	4			Mutual Fund NAV Computation	4
3(a)	Derivatives - Futures + Options Portfolio P&L	6	Sec Val Bonds - Deb Redemption	6	Adv Cap Budgetting Decision, Machine NPV evaluation	6	Interest Rate Risk Management - FRA - compute int rate based on payoffs	6	Interest Rate Risk Management - Cap computation	10
3(b)	Forex - cross currency arbitrage	4	MF - Chosing best return option	4	MFs - Return computation	4	Forex Nostro Account Balance	8	Start ups - Why India preferred for startups (or) Tokenization & Securitization (T)	4
3(c)	Risk Management - political Risks (T)	4	Risks with Block chain (T)	4	Sec Analysis - Weak Mkt efficiency tests (T)	4				
4(a)	Int Rate Risk Managemeent - Cap + Floor strategy	6	PM - Portfolio Risk computation	6	IFM - Investment into india	6	Derivatives - Futures - compute Portfolio Beta, Hende & Nifty Futures Value	6	Derivatives - Options - Compute Gain / loss on Options portfolio	6
4(b)	Biz Valuation - EVA	4	Derivatives - NIFTY futures and portfolio loss	4	M&A - Breakevern swap ratio	4	Biz Valuation - EVA back work to NP	4	M&A	4
4(c)	Charts & Patterns Sec Analysis (T)	4	Technical analysis disadvantages (T)	4	Securitization - Tokenization (T)	4	Succession planning (T)	4	GIFT City & IFC meaning (T)	4
4 (d)			Currency risk identification (T)	4	Stages of VC funding (T)	4	IRRM - MIBOR OIS Swap Fixed rate	4		
5(a)	Portfolio Management - Sharpe's optimal portfolio	8	Intl Fin Mgmt - Investment NPV	7	MF Div Reinvestment	7	IFM / Forex - Derive forward rate based on Cashflows	6	Portfolio Management (Rsik & Return of Two Securities)	8
5(b)	Ifm - GDRs	6	MFs - NAV computation backwards	7	Biz Valuation - EV computation	7	Biz Valuation - Share buy back missing count & price	4	Security Valuation Intrinsic Value	6
5(c)							Derivatives - Portfolio Gain loss incl Futures gain loss	4		
6(a)	MF - Aum 7 NAV computation	6	Business Valuation - FCFF / FCFE	7	Futures - MTM computation	7	PM - Sharpe & Treynor Ratio based back working	8	Advanced Capital Budgeting Decisions	8
6(b)	Derivatives - Abandonment option	4	Forex Hedging Comparison money Market Vs forward	7	Int Rate Risk mgmt - Collar	7	Forex -Project profitability based on swap rate	6	M&A - Maximum price for acquisition	6
6(c)	Role of GOI in Startup India (T)	4								
6 (d)	SPV structure (PTS, PTC, IO, PO) (T)	4								
MCQs										30
		118		**118**		**118**		**126**		**114**

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1. Illustration

The Balance Sheet of M/s. Sundry Ltd. as on 31-03-2020 is follows:

(₹ in lakhs)

Liabilities	₹	Assets	₹
Share Capital	3,000	Fixed Assets	6,000
Reserves	2,000	Inventory	5,000
Long Term Loan	4,000	Receivables	2,400
Short Term Loan	3,000	Cash	600
Payables & Provisions	2,000		
Total	14,000	Total	14,000

Sales for the year was ₹ 6000 lakhs. The sales are expected to grow by 20% during the year. The profit margin and dividend pay-out ratio are expected to be 4% and 50% respectively.

The company further desires that during the current year Sales to Short Term Loan and Payables and Provision should be in the ratio of 4: 3. Ratio of fixed assets to Long Term Loans should be 1.5. Debt Equity Ratio should not exceed 1.5.

You are required to determine:

(i) The amount of External Fund Requirement (EFR)

(ii) The amount to be raised from Short Term, Long Term and Equity funds.

[ICAI SM (New Question), Jan'21 QP 8 marks, MTP Mar'23]

Solution:

Method 1

Year / Amounts in ₹ Lacs	0	1	Increase	Remarks
Sales	6,000	7,200	1,200	(Growth of 20%)
PAT	240 (6,000 x 4%)	288 (7,200 x 4%)	48	PAT Margin 4%
Dividend	120	144	24	Pay out Ratio 50%
Retained Earnings	120	144	24	PAT – Dividend
ST Loans + Payables + Provisions	5,000	5,400	400	(WN 1)
Fixed Assets	6,000	7,200	1200	(WN2)
LT Loans	4,000	4,800	800	(WN 3)
Current Assets	8,000	9,600	1,600	(WN 4)
Payables & Provisions	2,000	2,400	400	(WN 5)
ST Loans	3,000	3,000	-	WN1 – WN5
Total Debt / Equity	7,000/5,000 = 1.4x	7,800 / 6,600 = 1.18x		Max 1.5x for Y 1
LT Debt / Equity	4,000 / 5,000 = 0.80	4,800 / 6,600 = 0.727		Max 1.5x for Y 1

Balance Sheet for Year 1

Equity Capital + Reserves (Bal Figure)	**6,600**	**Fixed Assets**	**7,200**
LT Loans	4,800	CA	9,600
ST Loans	3,000		
Prov & Payables	2,400		
Total	**16,800**	**Total**	**16,800**

Equity + Reserves Total = ₹6,600 Lacs

Less: Opening Equity Share Capital + Reserves Y 0 = ₹ 5,000 Lacs

Less: Profits retained for Y1 = ₹144 Lacs

⇨ **Capital Raised in Year 1 = ₹ 1,456 Lacs**

External Funds Raised = ₹1,456 Lacs Equity + LT Debt ₹800 Lacs= ₹2,256 Lacs

Working Notes 1:

Sales / (ST Loans + Payables + Provisions) = 4/3 = 1.33

Sales for year 1 = ₹7200 Lacs

⇨ : ST Loans + Payables + Provisions = ₹7,200 Lacs / 1.33 = ₹5400 Lacs

Working Notes 2:

Sales / Fixed Assets for Year 0 = ₹6,000 Lacs /₹ 6,000 Lacs

Applying same ratio on Year 1 Sales of ₹7,200 Lacs => FA of ₹7,200 Lacs

Working Notes 3:

Fixed Assets / LT Loans for Year 0 = ₹6,000 Lacs /₹ 4,000 Lacs = 1.5x

Applying same ratio on Year 1 FA of ₹7,200 Lacs => LT Loans= ₹7,200Lacs/1.5 = ₹4,800 Lacs

Working Notes 4:

CA / Sales for Year 0 = ₹8,000 Lacs /₹ 6,000 Lacs = 1.33

Sales for Year 1 = ₹7,200 Lacs => CA = ₹7,200 Lacs x 1.33 = ₹ 9,600 Lacs

Working Notes 5:

Payables & Provisions / Sales for Year 0 = ₹2,000 Lacs /₹ 6,000 Lacs = 0.33

Sales for Year 1 = ₹7,200 Lacs => CA = ₹7,200 Lacs x 0.33 = ₹ 2,400 Lacs

Method 2:

EFR = TA x Increase in sales / Old Sales – Payables x Increase in Sales / Old Sales

- Net Margin x New Sales x (1- Payout ratio)

= 14,000 x 1,200 / 6,000 - 2,000 x 1200/ 6000 – 4% x 7200 x (1-50%)

= 2800- 240 – 144

= ₹ 2256 Lacs

2. Illustration

MNC Limited company's financial statements for FY 2024-25 are provided:

Income Statement	(₹ in Cr.)
Sales revenues	7500
Costs and expenses	7300
Income before taxes	200
Taxes (30%)	60
Net income	**140**

MNC Limited's Balance Sheet as at 31st March, 2025

Liabilities	(₹ in Cr.)	Assets	(₹ in Cr.)
Equity	2000	Net Fixed Assets	4000
Long term Debt	2500	Current Assets	2000
Current Liabilities	1500		
	6000		**6000**

Additional Information:

(i) The company expects a 40% sales growth next financial year.

(ii) The company will have a 25% dividend payout ratio next year.

(iii) All costs, current assets and current liabilities are expected to increase with sales.

(iv) Except retained earnings no new Equity is to be raised.

Required:

Compute External Funding Requirement through raising Long-term Debt:

(1) If the company is operating at 65% capacity usage for fixed assets.

(2) If the company is operating at 95% capacity usage for fixed assets.

(Q2b Jan 26 4 Marks)

Solution:

EFR = F0x S1/S0 – F0 + NCA0x (S1-S0) /S0 – Net profit (1- Dividend)

EFR under normal circumstances = 4000 x 10500/7500 – 4000 + 500x (7500x 0.4)/7500 – 147 (WN1)

EFR = 1600 +200-147 ------ (1)

EFR = 1653

(A)

If company is currently operating at 65% capacity and there is a 40% growth in Sales same will get reflected in FA, NCA & PL

Revised fixed assets utilization = 65% x 1.4 = 91%

Since it is less than 100% there no requirement to ad FA

Revised EFR = 0 +200 -147 = 53

Money raised by LT debt = ₹ 53 Cr

Revised Balance sheet

Liabilities		(₹ in Cr.)	Assets		(₹ in Cr.)
Equity	2000 + 147	2147	Net Fixed Assets	4000+0	4000
Long term	2500+53	2553	Current Assets	2000+800	2800

Debt					
Current Liabilities	1500+600	2100			
Total		**6800**			**6800**

(B)

If company is currently operating at 95% capacity and there is a 40% growth in Sales same will get reflected in FA, NCA & PL

Revised fixed assets utilization = 95% x 1.4 = 133%

Additional requirement for FA = 0.33 x 4000 = 1320

Revised EFR = 1320 +200 -147 = 1373

Money raised by LT debt = ₹ 1373 Cr

Revised Balance sheet

Liabilities		**(₹ in Cr.)**	**Assets**		**(₹ in Cr.)**
Equity	2000 + 147	2147	Net Fixed Assets	4000+1320	5320
Long term Debt	2500+1373	3873	Current Assets	2000+800	2800
Current Liabilities	1500+600	2100			
Total		**8120**			**8120**

Working Note 1:

Revised P&L

	Old	**Increase**	**Revised**
Sales	7500	7500 x 40% = 3000	10500
Cost	7300	7300 x 40% = 2920	10220
PBT	200	200 x 40% = 80	280
Tax @ 30%	60	30% of 80 =24	84
PAT	140	56	196
Less: Dividend			25% of 196 = 49
Retained Earnings			**196-49 = 147**

1. Illustration

Neel holds ₹ 1 crore worth shares of XY ltd., whose market price standard deviation is 2% per day. Assume there are 252 days. Determine the maximum loss level over the period of 1 trading day and 10 trading days at 99% confidence level assuming:

- Share prices are distributed normally
- For 99% level the equivalent Z score from the normal table of cumulative area is 2.33

(May 18 QP 4 marks)

Solution :

Particulars	Amount (INR)
Value of Shares Held	1,00,00,000
Share Price Standard Deviation Per Day	2%
99% Probability Cumulative Area	2.33
Loss Over 1 Day	= 1,00,00,000 × 2.33 × 2% = 4,66,000
Loss over 10 Days	= 466,000 × $\sqrt{10}$ = 14,73,621.39

Note : *In order to calculate loss of any number of days, you multiply **Square Root** of number of days with Loss for 1 Day.*

2. Illustration

Following is the information about Mr. J's portfolio

Particulars	Prices
Investment in shares of ABC Ltd.	200 Lakhs
Investment in shares of XYZ Ltd.	200 Lakhs
Daily standard deviation of both shares	1%
Co-efficient of correlation between both shares	0.3

Determine the 10 days 99% Value at Risk (VAR) for Mr. J's portfolio. Given: The Z score from the normal table at 99% confidence level is 2.33 (Show your calculations upto four decimal points)

(Nov'19 QP 4 marks)

Solution :

Details	INR Lakhs	%	SD	SD in INR Lakhs
ABC Ltd	200	50%	1%	2
XYZ Ltd	200	50%	1%	2
Total Portfolio	400			
Correlation Coefficient			0.3	

Portfolio Variance (σ_P) = $\sqrt{W1^2\sigma1^2 + W2^2\sigma^2 + 2W1W2\sigma1\sigma2\rho}$

Portfolio Variance	$= 2^2 + 2^2 + 2 \times 2 \times 2 \times 0.3$ $=10.40$	$=(1\% \times 50\%)^2 + (1\% \times 50\%)^2 +$ $2 \times 50\% \times 1\% \times 50\% \times 1\% \times 0.3$ $= 0.000065$
Portfolio Standard Deviation		$=\sqrt{0.000065}$ $= 0.0081$
Portfolio Standard Deviation in INR Lakhs	$=\sqrt{10.40}$ $=3.2249$	$= 0.0081 \times 400$ $= 3.2249$
Z Value at 99% Confidence Level	2.33	2.33
Number of Days	10	10
Value at Risk in INR Lakhs	$= 3.2249 \times 2.33 \times \sqrt{10}$ $= 23.7614$	$= 3.2249 \times 2.33 \times \sqrt{10}$ $= 23.7614$

3. Illustration

Suppose you hold ₹ 2 crore worth shares of X Ltd. whose market price standard deviation is 2% per day. Assuming 252 trading days a year, determine maximum loss level over the period of 1 trading day and 10 trading days with 99% confidence level.

(ICAI SM Example)

Solution :

Details	**INR Lakhs**
Value of Shares of X Ltd	200
Standard Deviation	2%
Confidence Level 99%	2.33
VAR 1 Day	$= 200 \times 2\% \times 2.33$ $= 9.3200$
VAR 10 Days	$=$ VAR 1 Day $\times \sqrt{10}$ $= 9.3200 \times \sqrt{10}$ $= 29.4724$

4. Illustration

On Tuesday morning (before opening of the capital market) an investor, while going through his bank statement, has observed that an amount of ₹ 7 lakhs is lying in his bank account. This amount is available for use from Tuesday till Friday. The Bank requires a minimum balance of ₹1000 all the time. The investor desires to make a maximum possible investment where Value at Risk (VaR) should not exceed the balance lying in his bank account. The standard deviation of market price of the security is 1.5 per cent per day. The required confidence level is 99 per cent.Given

Standard Normal Probabilities										
Z	0.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9998	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9923	.9925	.9929	.9931	.9932	.9934	.9936

You are required to determine the maximum possible investment.

(Nov'20 QP 8 marks)

Solution :

Particulars	Amount
Current Balance	7,00,000
Minimum Balance	1,000
Utilisable Balance	= 7,00,000 - 1,000 = 6,99,000
Standard Deviation (SD)	1.5%
Confidence Level	99%
Z Score at 99%	2.3300
Assuming 1 Day VAR is INR 6,99,000	
VAR	= SD × Portfolio × Z Score
	6,99,000 = 1.5% × Portfolio × 2.33 Portfolio = 6,99,000/(2.33 × 1.5%) Portfolio = 2,00,00,000
Assuming 4 Day VAR is INR 6,99,000	
VAR	= SD × Portfolio × Z Score × $\sqrt{4}$
	6,99,000 = 1.5% × Portfolio × 2.33 × $\sqrt{4}$ Portfolio = 6,99,000/(2.33 × 1.5% × 2) Portfolio = 1,00,00,000
Maximum Investment for a Day is	INR 2 Crores
Maximum Investment for 4 Days is	INR 1 Crores

5. Illustration

Consider a portfolio of shares of P Ltd. and Q Ltd.

Particulars	Amount (₹)
P Ltd.	5 Crs
Q Ltd	12 Crs

The daily Standard deviation of the shares of P Ltd. is 0.5% and of Q Ltd. is 0.2%. The correlation co-efficient is 0.3. You are required to determine 1 week VAR at 99% confidence level for the portfolio (Assuming week is 5 trading days)

Solution :

Details	INR Crores	Standard Deviation	Weights	SD in INR Crores
P Ltd	5	0.5%	=5/17 = 29.41%	= 5 × 0.5% = 0.0250
Q Ltd	12	0.2%	= 12/17 = 70.59%	= 12 × 0.2% = 0.0.240
Total	17			
Correlation		0.3		

Portfolio Variance (σ_P): $\sqrt{W1^2\sigma1^2 + W2^2\sigma^2 + 2W1W2Cov1,2}$			Alternatively,	
	=(29.41% × 0.50%)^2 + (70.59% × 0.20%)^2 + 2 × 0.5% × 0.2% × 29.41% × 70.59% × 0.3 = 0.00000540		= 0.0250 ^2 + 0.0240 ^2 + (2 × 0.0250 × 0.0240 × 0.3) = 0.0016	
Portfolio SD	= $\sqrt{0.00000540}$ = 0.00232409		=$\sqrt{0.0016}$ = 0.0395	
Portfolio SD in INR Crores	= 0.00232409 × 17 = 0.0395			
Portfolio SD in INR	= 0.0395 × 1,00,00,000 = 3,95,094.93		= 0.0395 × 1,00,00,000 = 3,95,094.93	
5 Day VAR	**= 3,95,094.93 × $\sqrt{5}$ × 2.33 = 20,58,459.73**		**= 3,95,094.93 × $\sqrt{5}$ × 2.33 = 20,58,459.73**	

6. Illustration

Mr. Bull is a rational risk taker. He takes his position in a single stock for 4 days a week. He does not take a position on Friday to avoid the weekend effect and takes position only for four days in a week i.e., Monday to Thursday. He transfers the amount on Monday morning and withdraws the balance on Friday morning. He desires to make a maximum investment where Value at Risk (VAR) should not exceed the balance lying in his bank account. The position by his manager, as per standing instructions, is taken on the free balance lying in the bank account in the morning on each Monday.

On Monday morning (before the opening of the capital market) he transferred an amount of 11 Crore to his bank account. A fixed deposit also matured on this Monday. The maturity amount of ₹63,42,560 was also credited to his account by the bank in the morning of the Monday. However, Mr. Bull received the intimation of the same in the evening. The bank needs a minimum balance of ₹1,000 all the time. The value of Z score, at the required confidence level of 99 percent is 2.33. The other information with respect to stocks X and Y, which are under consideration for this week, is as under:

X		Y	
Return	**Probability**	**Return**	**Probability**
6	0.10	4	0.10
7	0.25	6	0.20
8	0.30	8	0.40
9	0.25	10	0.20
10	0.10	12	0.10

You are required to recommend a single stock, where maximum investment can be made and compute the amount of maximum investment. *(May'23 QP 8 marks)*

Solution:

Expected Return $\bar{x} = \sum P_i x_i$

VAR $= \sigma * \bar{z} * Portfolio\,Value$

For Security X:

x_i	P_i	$\bar{x} = x_i * P_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$(x_i - \bar{x})^2 * P_i$
6	0.10	0.6	-2	4	0.4
7	0.25	1.75	-1	1	0.25
8	0.30	2.4	0	0	0
9	0.25	2.25	1	1	0.25
10	0.10	1	2	4	0.4
		8			**1.3**

$\sigma^2 = 1.3$

$\sigma = \sqrt{1.3}$ = **1.14018**

For Security Y:

y_i	P_i	$\bar{y} = y_i * P_i$	$y_i - \bar{y}$	$(y_i - \bar{y})^2 * P_i$	$(y_i - \bar{y})^2 * P_i$
4	0.10	0.4	-4	16	1.6
6	0.20	1.2	-2	4	0.8
8	0.40	3.2	0	0	0
10	0.20	2.0	2	4	0.8
12	0.10	1.2	4	16	1.6
		8			**4.8**

$\sigma^2 = 4.8$

$\sigma = \sqrt{4.8}$ = **2.1909**

Mr. Bull **should invest in security X** as it has same return i.e., 8% but at lower risk i.e., S.D of 1.1402%

Maximum Value of Investment

Particulars	**₹**
Balance Available	11,00,00,000
Fixed Deposit Maturity	63,42,560
Total	11,63,42,560
Less: Minimum Balance	1000
Maximum Loss	**11,63,41,560**

Assuming, Investment is for 4 days and the Manager invests Fixed Deposit amount also despite Mr. Bull not being aware till Monday evening

VAR (maximum loss @99% confidence) $= Portfolio^* \sigma * z * \sqrt{t}$

$11{,}63{,}41{,}560 = P * 1.1402\% * 2.33 * \sqrt{4}$

$P = \frac{11{,}63{,}41{,}560}{1.1402\% * 2.33 * 2}$

P = **₹ 2,18,96,15,857**

The maximum investment is **₹ 218.96 Crores.**

1. Illustration

Determine NPV of the project with the following information:

Initial Outlay of project	₹ 40,000
Annual revenues (Without inflation)	₹ 30,000
Annual costs excl depreciation (Without inflation)	₹ 10,000
Useful life	4 years
Salvage value	Nil
Tax Rate	50%
Cost of Capital (Including inflation premium of 10%)	12%

[ICAI SM, Nov'19 QP (Old)]

Solution:

Depreciation:

Initial Investment = 40,000

Life = 4 Years

Salvage Value = Nil

Depreciation per Annum = $\frac{40000}{4}$ = 10,000

Calculation of Profit After Tax:

Particulars	₹ without inflation
Revenue	30,000
Less: Cost	10,000
EBITDA	20,000
Less: Depreciation	10,000
EBIT/ PBT	10,000
Tax@ 50%	5,000
Profit After Tax (PAT)	5,000

Cash Flow After Tax = (Revenue – Cost- Depreciation) (1– Tax) + Depreciation
= 5,000+ 10,000 = **15,000**

OR

CFAT = (Revenue – Cost) (1–Tax)+ D*T
= (30,000–10,000) (1-50%)+ 10,000* 50% = **15,000** (without inflation)

k_c (incl. inflation) = 12%

Method 1: Discounting Nominal Cash Flows

Year	Real Cash Flow	Inflation	Nominal Cash Flow	Discount Rate @12%	Present Value
1	15,000	=(1+10%)^1= 1.1	16,500	0.892	14,718.000
2	15,000	=(1+10%)^2= 1.21	18,150	0.797	14,465.550
3	15,000	=(1+10%)^3= 1.331	19,965	0.712	14,210.693
4	15,000	=(1+10%)^4= 1.4641	21,961.5	0.636	13,967.514
			76,576.5		**57,361.76**

PVCIF = ₹ 57,362

Initial Investment = ₹ 40,000
NPV = **₹ 17,362**

Method 2: Discounting Real Cash Flows with Real Rate

Real Discount Rate,
1+ Nominal Rate = (1+ Real Rate) (1+ Inflation Rate)

(1+ Real Rate) = $\frac{1+Nominal\ Rate}{1+Inflation\ Rate}$

Real Rate = $\frac{1+Nominal\ Rate}{1+Inflation\ Rate}$ -1

= $\frac{1+12\%}{1+10\%}$ -1

= 1.82%

Year	Real Cash Flow	PVF @ 1.82%
1	15,000	0.982
2	15,000	0.965
3	15,000	0.947
4	15,000	0.930
		3.824

PVCIF (real) = 15,000* 3.824
= ₹ 57,366
PVCOF (real) = ₹ 40,000
NPV = **₹ 17,366**

2. Illustration

XYZ Ltd. requires ₹ 8,00,000 for a unit.
Useful life of project - 4 years.
Salvage value - Nil.
Depreciation Charge ₹ 2,00,000 p.a.
Expected revenues & costs (excluding depreciation) ignoring inflation.

Year	1	2	3	4
Revenues	₹ 6,00,000	₹ 7,00,000	₹ 8,00,000	₹ 8,00,000
Costs	₹ 3,00,000	₹ 4,00,000	₹ 4,00,000	₹ 4,00,000

Tax Rate 60% cost of capital 10% (including inflation premium).
Calculate NPV of the project if inflation rates for revenues & costs are as follows:

Year	Revenues	Costs
1	10%	12%
2	9%	10%
3	8%	9%
4	7%	8%

[ICAI SM, Nov'20 QP (Old)]

Solution:

Year	CF	PVF @10%	PVCF
1	2,49,600	0.909	2,26,909
2	2,58,600	0.826	2,13,719
3	3,19,514	0.751	2,40.055
4	3,31,331	0.683	2,26,303

PVCIF = 9,06,987
Cash Outflow = 8,00,000
NPV = ₹ 1,06,987

Working Notes

Cash Flow After Tax = (Revenue – Cost – Depreciation) (1– Tax) + Depreciation

Year	(1+i)	Revenue	R_N
1	(1.10)	6,00,000	6,60,000
2	(1.10) (1.09)	7,00,000	8,39,300
3	(1.10) (1.09) (1.08)	8,00,000	10,35936
4	(1.10) (1.09) (1.08) (1.07)	8,00,000	11,08,451.52
			36,43,687.52

Year	(1+i)	Cost	C_N
1	(1.12)	3,00,000	3,36,000
2	(1.12) (1.1)	4,00,000	4,92,800
3	(1.12) (1.1) (1.09)	4,00,000	5,37,152
4	(1.12) (1.1) (1.09) (1.08)	4,00,000	5,80,124.16
			19,46,076.61

R_N	C_N	B = (R_N -C_N) * (1-T)	A = B + DT DT= 2,00,000* 60%	PVF @10%	A *PVF
6,60,000	3,36,000	1,29,600	2,49,600	0.909	2,26,909
8,39,300	4,92,800	1,38,600	2,58,600	0.826	2,13,719
10,35936	5,37,152	1,99,514	3,19,514	0.751	2,40.055
11,08,451.52	5,80,124.16	2,11,331	3,31,331	0.683	2,26,303
					9,06,987

3. Illustration

A firm has projected the following cash flows from a project under evaluation:

Year	₹ lakhs
0	(70)
1	30
2	40
3	30

The above cash flows have been made at expected prices after recognizing inflation. The firm's cost of capital is 10%. The expected annual rate of inflation is 5%. Show how the viability of the project is to be evaluated. *(ICAI SM, Old PM)*

Solution:

Method 1: Cash Flow and Discount Rate is Nominal

Year	CF	PVF @ 15.5% (WN 1)	PVCF
0	(70)	1	(70)
1	30	0.865	25.97
2	40	0.749	29.98
3	30	0.649	19.47
		NPV	**5.42889 lakhs**

Project is viable as NPV Is positive.

Method 2: Cash Flow is converted to Real and Discount Rate is Real

Year	CF	Inflation Adjustment @5%	Real CF	PVF @10%	PVCF
0	(70)	1	(70)	1	(70)
1	30	0.952	28.57	0.909	25.97
2	40	0.907	36.28	0.826	29.98
3	30	0.8638	25.915	0.751	19.47
				NPV	**5.426 lakhs**

Project is viable as NPV Is ₹ **5.426 lakhs.**

Working Notes:

1. Assuming given Discount rate is real, 10%. Inflation rate is 5%.
   Nominal Discount Rate,

$$(1+n) = (1+r)\ (1+i)$$
$$(1+n) = (1+10\%)\ (1+5\%)$$
$$(1+n) = 1.155$$
$$n = 15.5\%$$

2. Inflation is 5%, so all nominal cash flows are adjusted for inflation to convert them to real.

4. Illustration

Shashi Co. Ltd has projected the following cash flows from a project under evaluation:

Year	0	1	2	3
₹ (in lakhs)	(72)	30	40	30

The above cash flows have been made at expected prices after recognizing inflation. The firm's cost of capital is 10%. The expected annual rate of inflation is 5%. Show how the viability of the project is to be evaluated. PVF at 10% for 1 -3 years are 0.909, 0.826 and 0.751.

(ICAI SM, MTP Oct'19 Old, Old PM)

Solution:

Cash Flow is converted to Real and Discount Rate is Real

Year	CF	Inflation Adjustment @5%	Real CF	PVF @10%	PVCF
0	(72)	1	(72)	1	(72)
1	30	0.952	28.57	0.909	25.97
2	40	0.907	36.28	0.826	29.98
3	30	0.8638	25.915	0.751	19.47
					3.426 lakhs

5. Illustration

KLM Ltd. requires ₹ 15,00,000 for a new project. The useful life of the project is 3 years. Salvage value - NIL. Depreciation is ₹ 5,00,000 p.a. Given below are projected revenues and costs (excluding depreciation) ignoring inflation:

Year	1	2	3
Revenues in ₹	10,00,000	13,00,000	14,00,000
Costs in ₹	5,00,000	6,00,000	6,50,000

Applicable tax rate is 35%. Assume cost of capital to be 14% (after tax). The inflation rates for revenues and costs are as under:

Year	Revenues %	Costs %
1	9	10
2	8	9
3	6	7

PVF at 14%, for 3 years =0.877, 0.769 and 0.675 Show amount to the nearest rupee in calculations. You are required to calculate net present value of the project.

(ICAI SM, Old PM)

Solution:

Year	Rev_R	$Cost_R$	Inf_R	Inf_C	Inf Factor (Rev)	Inf Factor (Cost)
1	10	5	9%	10%	= 1.09	=1.1
2	13	6	8%	9%	=(1.09)(1.08) = 1.1772	(1.1)(1.09) =1.199
3	14	6.5	6%	7%	=(1.09)(1.08)(1.06) = 1.247	(1.1)(1.09)(1.07) = 1.283

Inflation Factor		Adjustment	
Revenue	Cost	Revenue	Cost
1.09	1.1	10.9	5.5
1.1772	1.199	15.303	7.194
1.247	1.283	17.458	8.3395

Cash Flow After Tax = (Revenue – Cost) (1– Tax) + Depreciation* Tax

Year	CFAT	PVF @ 14%	PVCF
1	5.26	0.877	4.613
2	7.02085	0.769	5.399
3	7.677	0.675	5.182

PVCIF = 15,19,400
Cash Outflow = 15,00,000
NPV = 19,400

Year	Revenue	Revenue Nominal	Cost	Cost Nominal	(R –C) Nominal
1	10,00,000	10,90,000	50,00,000	5,50,000	5,40,000
2	13,00,000	15,30,360	60,00,000	7,19,400	8,10,960
3	14,00,000	17,46,965	65,00,000	8,33,905	9,13,060

Cash Flow After Tax

(Revenue – Cost)	*PV Factor	*(1-Tax)	+Depreciation* Tax	= CFAT
5,40,000	*0.877	*0.65	+ 1,75,000	= 4,61,302
8,10,960	*0.769	*0.65	+ 1,75,000	= 5,39,933
9,13,060	*0.675	*0.65	+ 1,75,000	= 5,18,730
			PVCIF	= 15,19,965.43
			Investment	= 15,00,000
			NPV	**= ₹ 19,965.43**

6. Illustration

Possible net cash flows of Projects A and B at the end of first year and their probabilities are given below.
The discount rate is 10 per cent. For both the projects, initial investment is ₹ 10,000. Calculate the expected net present value for each project. State which project is preferable?

Possible Event	Project A		Project B	
	Cash Flow (₹)	Probability	Cash Flow (₹)	Probability
A	8,000	0.10	24,000	0.10
B	10,000	0.20	20,000	0.15
C	12,000	0.40	16,000	0.50
D	14,000	0.20	12,000	0.15
E	16,000	0.10	8,000	0.10

(ICAI SM)

Solution:

Project A

Probability	0.1	0.2	0.4	0.2	0.1
Cash Flow	8,000	10,000	12,000	14,000	16,000
Prob* CF	800	2,000	4,800	2,8000	1,600

Expected Value = 800+ 2,000+ 4,800+ 2,800+ 1,600 =12,000
PV of Project A (@10% discount rate) = $\frac{12,000}{1.1}$ = 10,909
NPV (A) = 10,909 – 10,000 = **₹ 909**

Project B

Probability	0.1	0.15	0.5	0.15	0.1
Cash Flow	24,000	20,000	16,000	12,000	8,000
Prob* CF	2,400	3,000	8,000	1,800	800

Expected Value = 2,400+ 3,000+ 8,000+ 1,800+ 800 =16,000

PV of Project B (@10% discount rate) = $\frac{16,000}{1.1}$ = 14,545

NPV (B) = 14,545 – 10,000 = ₹ **4,545**

NPV of Project B is greater than NPV of Project A, hence Project B should be preferred.

7. Illustration

Probabilities for net cash flows for 3 years of a project are as follows:

Year 1		Year 2		Year 3	
Cash Flow (₹)	**Probability**	**Cash Flow (₹)**	**Probability**	**Cash Flow (₹)**	**Probability**
2,000	0.1	2,000	0.2	2,000	0.3
4,000	0.2	4,000	0.3	4,000	0.4
6,000	0.3	6,000	0.4	6,000	0.2
8,000	0.4	8,000	0.1	8,000	0.1

Calculate the expected net present value of the project using 10 per cent discount rate if the Initial Investment of the project is ₹ 10,000.

(ICAI SM)

Solution:

Year 1			Year 2			Year 3		
Cf	P	Ex Val	Cf	P	Ex Val	Cf	P	Ex Val
2,000	0.1	200	2,000	0.2	400	2,000	0.3	600
4,000	0.2	800	4,000	0.3	1,200	4,000	0.4	1,600
6,000	0.3	1,800	6,000	0.4	2,400	6,000	0.2	1,200
8,000	0.4	3,200	8,000	0.1	800	8,000	0.1	800
		6,000			4,800			4,200

Year	**1**	**2**	**3**
A **Expected Value**	6,000	4,800	4,200
B **PVF @10%**	$\frac{1}{1.1^1}$	$\frac{1}{1.1^2}$	$\frac{1}{1.1^3}$
PV of Cash Inflow C= A*B	$\frac{6000}{1.1}$	$\frac{4800}{1.1^2}$	$\frac{4200}{1.1^3}$

PVCIF = 12,577

PVCOF = 10,000

NPV = 12,577– 10,000 = ₹ **2,577**

Expected NPV of the investment is ₹ **2,577**.

8. Illustration

Cyber Company is considering two mutually exclusive projects. The investment outlay of both the projects is ₹ 5,00,000 and each is expected to have a life of 5 years. Under three possible situations their annual cash flows and probabilities are as under:

		Cash Flow (₹)	
Situation	**Probabilities**	**Project A**	**Project B**
Good	0.3	6,00,000	5,00,000
Normal	0.4	4,00,000	4,00,000
Worse	0.3	2,00,000	3,00,000

Cost of Capital is 7%. Which project should be accepted and why?

(ICAI SM, Old PM)

Solution:

Project A

Expected Cash Flow = 0.3*6,00,000 + 0.4* 4,00,000 + 0.3*2,00,000

= ₹ 4,00,000 p.a for 5 yrs

PV of Cash Inflow for Project A for 5 yrs

= PVAF (7%, 5 yrs)

= 4.1

PVIF = 4.1* 4,00,000 = ₹ 16,40,000

PVOF = ₹ 5,00,000

NPV = **₹ 11,40,000**

Project B

Expected Cash Flow = 0.3*5,00,000 + 0.4* 4,00,000 + 0.3*3,00,000

= ₹ 4,00,000 p.a for 5 yrs

Since Expected value of Cash Inflow

= **₹ 4,00,000**

NPV of Project B will also be ₹ 11,40,000. When Outflow =₹ 5,00,000, and k_c= 7%.

Project A

Variance = $\sum P_i(x-\bar{x})^2$

$\bar{x}$ = Expected Cash Flow

σ^2 = 0.3* $(6,00,000- 4,00,000)^2$ + 0.4* $(4,00,000- 4,00,0000)^2$ + 0.3$(6,00,000- 4,00,000)^2$

= 2.4

σ = 1.549 = **₹ 1,54,919**

Project B

σ^2 = 0.3* $(5,00,000- 4,00,000)^2$ + 0.4* $(4,00,000- 4,00,0000)^2$ + 0.3$(3,00,000- 4,00,000)^2$

= 0.6

σ = 0.775 = **₹ 77,459.67**

Since the S.D of cash flow of Project B is ₹ 77,459.67 and is much lower than the S.D of Cash Inflow of Project a which is ₹ 1,54,919, **Project B is less risky**. Even though they both have same NPV is ₹ 11,40,000, we choose Project B as it is less risky.

9. Illustration

Calculate Variance and Standard Deviation of Project A and Project B based on following:

Possible Event	Project A		Project B	
	Cash Flow (₹)	Probability	Cash Flow (₹)	Probability
A	8,000	0.10	24,000	0.10
B	10,000	0.20	20,000	0.15
C	12,000	0.40	16,000	0.50
D	14,000	0.20	12,000	0.15
E	16,000	0.10	8,000	0.10

(ICAI SM)

Solution:

Variance, σ^2 $= \sum Pi\,(x - \bar{x})^2$

Project A

Cash Flows (Cf)	Probability (Pi)	$Pi * Cf$	Variance Computation
8,000	0.10	800	$0.10\,(8,000 - 12,000)^2$
10,000	0.20	2,000	$0.20\,(10,000 - 12,000)^2$
12,000	0.40	4,800	$0.40\,(12,000 - 12,000)^2$
14,000	0.20	2,800	$0.20\,(14,000 - 12,000)^2$
16,000	0.10	1,600	$0.10\,(16,000 - 12,000)^2$
		$\bar{x}$ = 12,000	**48,00,000**

σ^2 = **48,00,000**
σ = **₹ 2,190.89**

Project B

Cash Flows (Cf)	Probability (Pi)	$Pi * Cf$	Variance Computation
24,000	0.10	2,400	$0.10\,(24,000 - 16,000)^2$
20,000	0.15	3,000	$0.15\,(20,000 - 16,000)^2$
16,000	0.50	8,000	$0.50\,(16,000 - 16,000)^2$
12,000	0.15	1,800	$0.15\,(12,000 - 16,000)^2$
8,000	0.10	800	$0.10\,(8,000 - 16,000)^2$
		$\bar{x}$ = 16,000	**1,76,00,000**

σ^2 = **1,76,00,000**
σ = **₹ 4,195.23**

10. Illustration

Calculate Coefficient of Variation of Project A and Project B based on the following information:

Possible Event	Project A		Project B	
	Cash Flow (₹)	Probability	Cash Flow (₹)	Probability
A	10000	0.10	26,000	0.10
B	12,000	0.20	22,000	0.15
C	14,000	0.40	18,000	0.50
D	16,000	0.20	14,000	0.15
E	18,000	0.10	10,000	0.10

(ICAI SM)

Solution:

	Expected Cash Flow	σ	CV of ECF
Project A (WN 1)	14,000	2,190.89	0.1565
Project B (WN 2)	18,000	4,195.23	0.2331

Project A has lower Coefficient of Variation, hence from a risk perspective project A is chosen.

WORKING NOTES:

1. Expected Cash Flows of the project A

Pi	Cash Flow	Pi * Cash flows	$\sigma^2 = \sum Pi\,(x-\bar{x})^2$
0.1	10,000	1,000	$0.1\,(10{,}000 - 14{,}000)^2$
0.2	12,000	2,400	$0.2\,(12{,}000 - 14{,}000)^2$
0.4	14,000	5,600	$0.4\,(14{,}000 - 14{,}000)^2$
0.2	16,000	3,200	$0.2\,(16{,}000 - 14{,}000)^2$
0.1	18,000	1,800	$0.1\,(18{,}000 - 14{,}000)^2$
		14,000	**48,00,000**

σ^2 = **48,00,000**

σ = **2,190.89**

2. Expected Cash Flows of the project B

Pi	Cash Flow	Pi * Cash flows	$\sigma^2 = \sum Pi\,(x-\bar{x})^2$
0.1	26,000	2,600	$0.1\,(26{,}000 - 18{,}000)^2$
0.15	22,000	3,300	$0.15\,(22{,}000 - 18{,}000)^2$
0.5	18,000	9,000	$0.5\,(18{,}000 - 18{,}000)^2$
0.15	14,000	2,100	$0.15\,(14{,}000 - 18{,}000)^2$
0.1	10,000	1,000	$0.1\,(10{,}000 - 18{,}000)^2$
		18,000	**1,76,00,000**

σ^2 = **1,76,00,000**

σ = **4,195.235**

11. Illustration

Skylark Airways is planning to acquire a light commercial aircraft for flying class clients at an investment of ₹ 50,00,000. The expected cash flow after tax for the next three years is as follows:

Year 1		Year 2		Year 3	
CFAT	Probability	CFAT	Probability	CFAT	Probability
14,00,000	0.1	15,00,000	0.1	18,00,000	0.2
18,00,000	0.2	20,00,000	0.3	25,00,000	0.5
25,00,000	0.4	32,00,000	0.4	35,00,000	0.2
40,00,000	0.3	45,00,000	0.2	48,00,000	0.1

The Company wishes to take into consideration all possible risk factors relating to airline operations. The company wants to know:

(i) The expected NPV of this venture assuming independent probability distribution with 6 per cent risk free rate of interest.

(ii) The possible deviation in the expected value.

(iii) How would standard deviation of the present value distribution help in Capital Budgeting decisions?

(ICAI SM, Old PM, RTP Nov'18 Old, RTP May'20 Old, MTP Mar'19 Old4)

Solution:

i) Expected NPV @ 6%

Year	0	1	2	3
Cash Flow (WN1)	(50)	27	29.3	27.9
PVF @6%	1	0.943	0.890	0.840
	(50)			74.97
NPV				**24.974**

ii) Variance and SD of Cash Flow

Year	1	2	3
σ^2	85.4	98.61	74.29
σ	9.2412	9.9302	8.619

iii) Expected Value of the Deviation

$$= \sqrt{\frac{85.4}{(1.06)^2} + \frac{98.61}{(1.06)^4} + \frac{74.29}{(1.06)^6}}$$

$$= \sqrt{206.4855}$$

= **₹ 14.3696 lakhs**

iv) Standard Deviation identifies risk in cash flows. Using this, coefficient of variation can be computed and per unit of cashflows, how much risk is taken can be measured and decision on capital budgeting can be taken.

Working Notes:

1.

Year 1			Year 2			Year 3		
CF	Pi	CF* Pi	CF	Pi	CF* Pi	CF	Pi	CF* Pi
14	0.1	1.4	15	0.1	1.5	18	0.2	3.6
18	0.2	3.6	20	0.3	6	25	0.5	12.5
25	0.4	10	32	0.4	12.8	35	0.2	7
40	0.3	12	45	0.2	9	48	0.1	4.8
		27			29.3			27.9

2. Standard Deviation and Variance

σ^2 = $\sum P_i(x-\bar{x})^2$

Year 1

0.1* $(14-27)^2$ = 16.9
0.2* $(18-27)^2$ = 16.2
0.4* $(25-27)^2$ = 1.6
0.3* $(40-27)^2$ = 50.7
σ^2 = 85.4
σ = 9.2412

Year 2

0.1* $(15-29.3)^2$ = 20.449
0.3* $(20-29.3)^2$ = 25.947
0.4* $(32-29.3)^2$ = 2.916
0.2* $(45-29.3)^2$ = 49.298
σ^2 = 98.61
σ = 9.9302

Year 3

0.2* $(18-27.9)^2$ = 19.602
0.5* $(25-27.9)^2$ = 4.205
0.2* $(35-27.9)^2$ = 10.082
0.1* $(48-27.9)^2$ = 40.401
σ^2 = 74.29
σ = 8.619

12. Illustration

A company is considering Projects X and Y with following information:

Project	Expected NPV (₹)	Standard deviation
X	1,22,000	90,000
Y	2,25,000	1,20,000

(i) Which project will you recommend based on the above data?
(ii) Explain whether your opinion will change, if you use coefficient of variation as a measure of risk.
(iii) Which measure is more appropriate in this situation and why?

(ICAI SM, MTP Apr'18 Old)

Solution:

i) Based on the given data, if risk is the sole criterion, Project X has lower risk i.e., S.D and it will be chosen.However, if NPV were to be the criteria, Project Y which has higher NPV will be chosen.

ii) If Coefficient of variation is used then C.V will be as follows;

	X	Y
NPV	1,22,000	2,25,000
S.D	90,000	1,20,000
C.V = σ/ NPV	$\frac{90}{122}$	$\frac{120}{225}$
C.V	0.7377	0.533

Per unit of Expected NPV, **Project Y has lower risk** i.e., lower C.V of 0.533 and hence it will be chosen.

iii) Project Y is significantly better from both NPV and C.V perspectives and hence **Project Y should be recommended.**

iv) The most appropriate method would depend on an organisation's investment policy on the face of it considering NPV of Project Y which is 85% higher, **the NPV method would be the better method** to utilise in this scenario

13. Illustration

KLM Ltd., is considering taking up one of the two projects-Project-K and Project-S. Both the projects having same life require equal investment of ₹ 80 lakhs each. Both are estimated to have almost the same yield. As the company is new to this type of business, the cash flow arising from the projects cannot be estimated with certainty. An attempt was therefore, made to use probability to analyse the pattern of cash flow from other projects during the first year of operations. This pattern is likely to continue during the life of these projects. The results of the analysis are as follows:

Project K		Project S	
Cash Flow (in ₹)	Probability	Cash Flow (in ₹)	Probability
11	0.10	09	0.10
13	0.20	13	0.25
15	0.40	17	0.30
17	0.20	21	0.25
19	0.10	25	0.10

Required:

(i) Calculate variance, standard deviation and co-efficient of variance for both the projects.

(ii) Which of the two projects is riskier?

[ICAI SM, Old PM, May'18 QP (Old)]

Solution:

i)

Project K				Project S			
C.F	Pi	CF* Pi	$Pi\ (x-\bar{x})^2$	C.F	Pi	CF* Pi	$Pi\ (x-\bar{x})^2$
11	0.1	1.1	$0.1\ (11\text{-}15)^2$	9	0.1	0.9	$0.1\ (9\text{-}17)^2$
13	0.2	2.6	$0.2\ (13\text{-}15)^2$	13	0.25	3.25	$0.25\ (13\text{-}17)^2$
15	0.4	6.0	$0.4\ (15\text{-}15)^2$	17	0.3	5.1	$0.3\ (17\text{-}17)^2$
17	0.2	3.4	$0.2\ (17\text{-}15)^2$	21	0.25	4.25	$0.25\ (21\text{-}17)^2$
19	0.1	1.9	$0.1\ (19\text{-}15)^2$	25	0.1	2.5	$0.1(25\text{-}17)^2$
	1.0	15.0	**4.8**		1.0	17	**20.8**

Project K

σ^2 = 4.8

σ = **2.1909**

Project S

σ^2 = 20.8

σ = **4.5607**

Coefficient of Variation

$= \frac{\sigma}{ENPV}$

K $= \frac{2.1909}{15} =$ **0.14606**

S $= \frac{4.5607}{17} =$ **0.26827**

ii) **Project S is risky** considering per unit of Cash Flow, it has higher element of Standard Deviation. Also on an absolute basis, Project S has higher Standard Deviation.

14. Illustration

Project X and Project Y are under the evaluation of XY Co. The estimated cash flows and their probabilities are as below:

Project X : Investment (year 0) ₹ 70 lakhs

Probability weights	0.30	0.40	0.30
Years	₹lakhs	₹lakhs	₹lakhs
1	30	50	65
2	30	40	55
3	30	40	45

Project Y: Investment (year 0) ₹ 80 lakhs

Probability weighted	Annual cash flows through life ₹lakhs
0.20	40
0.50	45
0.30	50

(i) Which project is better based on NPV, criterion with a discount rate of 10%?

(ii) Compute the standard deviation of the present value distribution and analyse the inherent risk of the projects.

(ICAI SM, Old PM, RTP May'18 Old)

Solution:

Assumption: Both projects X and Y have same time frame for Cash Inflows i.e., 3 years.

i) Expected Value of Cash flows across 3 years

Project X

Probability	0.3	0.4	0.3	Expected CF
Year 1	30	50	65	48.5
Year 2	30	40	55	41.5
Year 3	30	40	45	38.5

ECF $= \sum Pi * CF$

Year	0	1	2	3
Cash Flow	(70)	48.5	41.5	38.5
PVF @ 10%	1	0.909	0.826	0.757
PV	(70)	44.09	34.29	28.925
NPV				**₹ 37.314 Lakhs**

Project Y

ECF (for each year) = 0.2*40 + 0.5*45 + 0.3*50

= 8+ 22.5+ 15 = **45.5 lakhs**

Year	0	1	2	3
Cash Flow	(80)	45.5	45.5	45.5
PVF @ 10%	1	0.909	0.826	0.757
PV	(80)	41.36	37.60	34.84
NPV				**₹ 33.15 Lakhs**

Project X is better on NPV criterion as it has higher NPV of ₹ 37.314 lakhs as compared to Project Y with NPV ₹ 33.15 lakhs

ii) Hillers Model,

$\sigma^2 = \sum_{i=0}^{n}(1+r)^{-2i}\sigma_i^2$

Variance: Project X

Year 1, $\sigma^2 = \sum P_i(x-\bar{x})^2$

= 0.3 (30-48.5)^2+ 0.4 (50-48.5)^2+ 0.3 (65-48.5)^2

= 185.25

σ = **13.6107**

Year 2, $\sigma^2 = \sum P_i(x-\bar{x})^2$

= 0.3 (30-41.5)^2+ 0.4 (40-41.5)^2+ 0.3 (55-41.5)^2

= 95.25

σ = **9.7596**

Year 3, $\sigma^2 = \sum P_i(x-\bar{x})^2$

= 0.3 (30-38.5)^2+ 0.4 (40-38.5)^2+ 0.3 (45-38.5)^2

= 35.25

σ = **5.93717**

Standard deviation of the expected Value/ Standard Deviation of the PV Distribution

$\sigma^2 = \sum_{i=0}^{n}(1+r)^{-2i}\sigma_i^2$

$\sigma = \sqrt{\frac{185.25}{(1.1)^2}+\frac{95.25}{(1.1)^4}+\frac{35.25}{(1.1)^6}}$

$= \sqrt{238.05}$

= **15.43**

Variance: Project Y

Year 1, $\sigma^2 = \sum P_i(x-\bar{x})^2$

= 0.2 (40-45.5)^2+ 0.5 (45-45.5)^2+ 0.3 (50-45.5)^2

= 12.25

σ = **3.5**

Standard deviation of the expected Value/ Standard Deviation of the PV Distribution

$\sigma^2 = \sum_{i=0}^{n}(1+r)^{-2i}\sigma_i^2$

$\sigma = \sqrt{\frac{12.25}{(1.1)^2}+\frac{12.25}{(1.1)^4}+\frac{12.25}{(1.1)^6}}$

$= \sqrt{25.4056}$

= **5.04**

Project X has higher Standard Deviation about the Expected Value and hence **has the higher risk** than Project Y which has Standard Deviation of 5.04.

15. Illustration

Shivam Ltd. is considering two mutually exclusive projects, A and B. Project A costs ₹ 36,000 and project B ₹30,000. You have been given below the net present value probability distribution for each project.

Project A		Project B	
NPV estimates (₹)	Probability	NPV estimates (₹)	Probability
15,000	0.2	15,000	0.1
12,000	0.3	12,000	0.4
6,000	0.3	6,000	0.4
3,000	0.2	3,000	0.1

(i) Compute the expected net present values of projects A and B.
(ii) Compute the risk attached to each project i.e. standard deviation of each probability distribution.
(iii) Compute the profitability index of each project.
(iv) Which project do you recommend? State with reasons

(ICAI SM, Old PM, RTP Nov'19 Old)

Solution:

i)

A			B		
P_i	CF	P_i *CF	P_i	CF	P_i *CF
0.2	15,000	3,000	0.1	15,000	1,500
0.3	12,000	3,600	0.4	12,000	4,800
0.3	6,000	1,800	0.4	6,000	2,400
0.2	3,000	600	0.1	3,000	300
1.0		**9,000**			**9,000**

Expected NPV of Project A and B are ₹ 9,000 each.

ii)

σ_A^2 = 0.2(15,000-9,000)^2 + 0.3(12,000-9,000)^2 +0.3(6,000-9,000)^2 +0.2(3,000-9,000)^2

= 1,98,00,000

σ = **4,449.72**

σ_B^2 = 0.1(15,000-9,000)^2 + 0.4(12,000-9,000)^2 +0.4(6,000-9,000)^2 +0.1(3,000-9,000)^2

= 1,44,00,000

σ = **3,794.73**

iii) Profitability Index of each project

	A	B
NPV	9,000	9,000
Add: Outflow	36,000	30,000
Inflow	45,000	39,000

$$\text{Profitability Index} = \frac{Discounted\ Cash\ Inflow}{Intial\ Outflow}$$

A $= \frac{45,000}{36,000} = \mathbf{1.25}$ B $= \frac{39,000}{30,000} = \mathbf{1.30}$

iv) Project B has lower standard Deviation of Cash Flow at ₹ 3,794.73 for the same NPV of ₹ 9,000 and for a lower investment of ₹ 30,000 implying a higher Profitability Index of 1.3, hence **project B should be chosen**.

16. Illustration

An enterprise is investing ₹100 lakhs in a project. The risk-free rate of return is 7%. Risk premium expected by the Management is 7%. The life of the project is 5 years. Following are the cash flows that are estimated over the life of the project:

Year	Cash flows (₹ in lakhs)
1	25
2	60
3	75
4	80
5	65

Calculate Net Present Value of the project based on Risk free rate and also on the basis of Risks adjusted discount rate.

(ICAI SM, RTP May '19, RTP Nov '19, RTP Nov '23, MTP April'22 and MTP Aug' 18)

Solution:

R_f = 7%

RADR = R_f + Risk Premium

= **14%**

Year	Cash Flow	PVF @7%	PVF@14%	Discount @7%	Cash Flow @14%
1	25	0.9346	0.8772	23.265	21.93
2	60	0.8734	0.7695	52.404	46.17
3	75	0.8163	0.6750	61.2225	50.625
4	80	0.7629	0.5921	61.032	47.368
5	65	0.7130	0.5194	46.345	33.761
				244.3685	**199.854**

Projected NPV	R_f Rate	RADR
PVCIF	244.3685	199.854
Less: PVCOF	(100)	(100)
NPV	**144.3685**	**99.54**

17. Illustration

If Investment proposal costs ₹ 45,00,000 and risk-free rate is 5%, calculate net present value under certainty equivalent technique:

Year	Expected cash flow (₹)	Certainty Equivalent coefficient
1	10,00,000	0.90
2	15,00,000	0.85
3	20,00,000	0.82
4	25,00,000	0.78

(ICAI SM)

Solution:

Cash outflow for Year 0 is ₹ 45 Lakhs

Year	0	1	2	3	4
Cash Flow (in ₹ Lakhs)	(45)	10	15	20	25
PVF @5%	1	0.952	0.907	0.864	0.823
DCF		9.524	13.605	17.277	20.568
∝		0.90	0.85	0.82	0.78
DCF * ∝		8.5716	11.56425	14.16714	16.04304

PVCIF = 50.34603 Lakhs
Less: COF = 45 Lakhs
NPV = **₹ 5.34603 Lakhs**

18. Illustration

X Ltd. is considering its new project with the following details:

Sr. No.	Particulars	Figures
1	Initial capital cost	₹ 400 Cr
2	Annual unit sales	5 Cr
3	Selling price per unit	₹ 100
4	Variable cost per unit	₹ 50
5	Fixed costs per year	₹ 50 Cr
6	Discount Rate	6%

Required:
1. Calculate the NPV of the project.
2. Compute the impact on the project's NPV considering a 2.5 per cent adverse variance in each variable. Which variable is having maximum effect? Consider Life of the project as 3 years

(ICAI SM, RTP May'24)

Solution:

1. Project NPV

Particulars	
Units Sold	5 Cr
Selling Price/ Unit	100
Variable Cost/ Unit	50
Contribution Per Unit	50
Total Contribution (D*A)	250 Cr
Less: Fixed Cost	50 Cr
Profit	200 Cr

Cash Flows

Year	0	1	2	3
Cash Flow	(400)	200	200	200
PVF @ 6%	1	0.943	0.8899	0.8396
PVCOF	(400)			
PVCIF	534.60			
NPV	**₹ 134.602 Cr**			

2. **Impact on NPV for 2.5% adverse variance in each factor**

Initial Investment	400	410	400	400	400	400	400
Units Sold	5 Cr	5 Cr	5 Cr	5 Cr	4.875 Cr	5 Cr	5 Cr
Selling Price/ Unit	100	100	97.5	100	100	100	100
Variable Cost/ Unit	50	50	50	51.25	50	50	50
Contribution / Unit	50	50	47.5	48.75	50	50	50
Total Contribution (D*A)	250 Cr	250 Cr	237.5 Cr	243.75 Cr	243.75 Cr	250 Cr	250 Cr
Less: Fixed Cost	50 Cr	50 Cr	50 Cr	50 Cr	50 Cr	51.25 Cr	50 Cr
Profit	200 Cr	200 Cr	187.5 Cr	193.75	193.75	198.75	200
Discount	6%	6%	6%	6%	6%	6%	6.15%
PVF	2.673	2.673	2.673	2.673	2.673	2.673	2.6656
PVIF	534.6	534.6	501.875	517.893	517.893	531.25	533.122
NPV	134.60	124.6	101.18	117.89	117.89	131.35	133.122
Sensitivity		7.43%	24.82%	12.41%	12.41%	2.48%	1.1%

Factors	**Sensitivity of 5% Adverse Change on NPV**
Investment	7.43%
Selling Price	24.82%
Units	12.41%
VC	12.41%
FC	2.48%
Discount Rate	1.1%

Selling Price variable has the maximum impact, and is the most sensitive factor.

19. Illustration

XYZ Ltd. is considering a project "A" with an initial outlay of ₹ 14,00,000 and the possible three cash inflow attached with the project as follows:

Particulars	**Year 1**	**Year 2**	**Year 3**
Worst case	**450**	**400**	**700**
Most likely	**550**	**450**	**800**
Best case	**650**	**500**	**900**

Assuming the cost of capital as 9%, determine NPV in each scenario. If XYZ Ltd is certain about the most likely result in first two years but uncertain about the third year's cash flow, analyze what will be the NPV expecting worst scenario in the third year.

(ICAI SM)

Solution:

Cash Flows

Year	**PVF**	**Worst**	**Most Likely**	**Best**	**Mix**
0	1	(14)	(14)	(14)	(14)
1	0.9174	4.5	5.5	6.5	5.5
2	0.8417	4	4.5	5	4.5
3	0.7722	7	8	9	7

Scenario	1	2	3	4
PVCIF	12,90,044.5	15,01,089.94	17,12,135.40	14,23,871.59
PVCOF	(14,00,000)	(14,00,000)	(14,00,000)	(14,00,000)
NPV	**(1,09,955.5)**	**1,01,089.94**	**3,12,135.40**	**23,871.59**

Assuming cash inflows of most likely case in Year 1 and 2, and worst case in Year 3.

20. Illustration

Following are the estimates of the net cash flows and probability of a new project of M/s X Ltd.:

	Year	P = 0.3	P = 0.5	P = 0.2
Initial investment	**0**	**4,00,000**	**4,00,000**	**4,00,000**
Estimated net after tax cash inflows per year	**1 to 5**	**1,00,000**	**1,10,000**	**1,20,000**
Estimated salvage value (after tax)	**5**	**20,000**	**50,000**	**60,000**

Required rate of return from the project is 10%. Find:

(i) The expected NPV of the project.

(ii) The best case and the worst case NPVs.

(iii) The probability of occurrence of the worst case if the cash flows are perfectly dependent overtime and independent overtime.

(iv) Standard deviation and coefficient of variation, assuming that there are only three streams of cash flow, which are represented by each column of the table with the given probabilities.

(v) Coefficient of variation of X Ltd. on its average project which is in the range of 0.95 to 1.0. If the coefficient of variation of the project is found to be less risky than average, 100 basis points are deducted from the Company's cost of Capital

Should the project be accepted by X Ltd?

(ICAI SM, MTP Mar'18 Old, Old PM)

Solution:

Initial Investment = ₹ 4,00,000

Cash Flows (Year 1-5)
(Expected)
0.3* 1,00,000
0.5* 1,10,000
0.2* 1,20,000 = ₹ 1,09,000

Salvage Value
0.3* 20,000
0.5* 50,000
0.2* 60,000 = ₹ 43,000

Present Value Factor @ 10% Cost of Capital

PVF_A (5,10%)	= 3.7908* 1,09,000	= ₹ 4,13,195.76
PVF (5,10%)	= 0.6209* 43,000	= ₹ 26,699.61
	Total	= **₹ 4,39,895.37**

i) Net Present Value = PV of Cash Inflows – PV of Cash Outflows
= 4,39,895.37– 4,00,000
= **₹ 39,895.37**

NPV in best and worst cases

	Best Case	Worst Case
Per year Cash flow (a)	1,20,000	1,00,000
PVF_A (b)	3.7908	3.7908
A= (a)* (b)	**4,54,896**	**3,79,080**
Salvage Value (c)	60,000	20,000
PVF (Year 5) (d)	0.6209	0.6209
B= (c)* (d)	**37,254**	**12,418**
A+B	4,92,150	3,91,498
Less: Investment	4,00,000	4,00,000
NPV	**92,150**	**(8,502)**

ii) Cash flows perfectly Dependent Overtime

First year cash flows determine cash flow of all subsequent years, of which probability is provided 0.3.

Cash flows are Independent Overtime.

Probability of worst case in all 5 years,

= 0.3*0.3*0.3*0.3*0.3 = **0.00243**

iii) Computation of Most Likely NPV

= -4,00,000+1,10,000*3.7908 + 50,000*0.6209

= 48,033

σ^2 = 0.3(-8,502-39,895)^2 + 0.5(48,033-39,895)^2 + 0.2(92,150-39,895)^2

= 1,28,19,11,409.4

σ = 35,803.79

Coefficient of Variation = $\frac{\sigma}{ENPV}$

= $\frac{35803.79}{39895}$ = 0.897

iv) Because Coefficient of Variation is 0.897, which is less than 0.95, the cost of capital will be 100 bps lower than 10% i.e it will 9%. 9% is RADR.

ENPV of project at 9% Cost of Capital:

Year	PVF@ 9%	
Year 0	1	4,00,000 Investment
Year 1-5	3.889	1,09,000 Inflow
Year 5	0.6499	43,000 Salvage
	NPV	**₹ 51,919.03**

21. Illustration

XY Ltd. has under its consideration a project with an initial investment of ₹ 1,00,000. Three probable cash inflow scenarios with their probabilities of occurrence have been estimated as below:

Annual cash inflow (₹)	**20,000**	**30,000**	**40,000**
Probability	**0.1**	**0.7**	**0.2**

The project life is 5 years and the desired rate of return is 20%. The estimated terminal values for the project assets under the three probability alternatives, respectively, are ₹ 0, 20,000 and 30,000.

You are required to:

(i) Find the probable NPV;

(ii) Find the worst-case NPV and the best-case NPV; and

(iii) State the probability occurrence of the worst case, if the cash flows are perfectly positively correlated over time.

(ICAI SM, MTP Nov'21, Old PM)

Solution:

i)

		S1		S2		S3		EV
Year	PVF @20%	Pi	CF	Pi	CF	Pi	CF	CF
0	1	0.1	(1)	0.7	(1)	0.2	(1)	(1)
1	0.833	0.1	0.2	0.7	0.3	0.2	0.4	0.31
2	0.694	0.1	0.2	0.7	0.3	0.2	0.4	0.31
3	0.579	0.1	0.2	0.7	0.3	0.2	0.4	0.31
4	0.482	0.1	0.2	0.7	0.3	0.2	0.4	0.31
5	0.402	0.1	0.2	0.7	0.3	0.2	0.4	0.31
5 (TV)	0.402	0.1	0	0.7	0.2	0.2	0.3	0.2
1-5	2.9906							

Expected Value for years 1-5

= 2.9906* 0.31 lakhs = ₹ 0.927086 lakhs

Expected Value for Terminal Value

= 0.402 * 0.2 lakhs = ₹ 0.080375 lakhs

NPV = **₹ 746.151**

ii) Worst Case NPV = Annual Cash Inflow* PVFA (20%, 5 years) + TV* PVF – Investment

= 20,000*2.9906 + 0 – 1,00,000

= – **₹ 40,188**

Best Case NPV = 40,000*2.9906 + 30,000*0.4018 – 1,00,000

= **₹ 31,680.33**

iii) If cash flows are perfectly positively correlated over time, first year cash flow will determine the subsequent year cash flows. Probability of worst case in first year is 0.1 or **10%.** That will be the probability of worst-case scenario throughout.

22. Illustration

XYZ Ltd. is considering a project for which the following estimates are available:

	₹
Initial Cost of the project	10,00,000
Sales price/unit	60
Cost/unit	40
Sales volumes	
Year 1	20000 units
Year 2	30000 units
Year 3	30000 units

Discount rate is 10% p.a.

You are required to measure the sensitivity of the project in relation to each of the following parameters:

(a) Sales Price/unit
(b) Unit cost
(c) Sales volume
(d) Initial outlay and
(e) Project lifetime Taxation may be ignored.

(ICAI SM, Old PM)

Solution:

a) Project NPV for the given data

Year	0	1	2	3
Cash Flow (WN1)	- 10,00,000	4,00,000	6,00,000	6,00,000
PVF @10%	1	0.909	0.8264	0.7513
PVCIF	-10,00,000	3,63,636.36	4,95,867.76	4,50,788.8
NPV	-10,00,000			13,10,293
NPV	**3,10,293**			

Method 1: Sensitivity, when Selling Price reduces by 10%

Year	0	1	2	3
Cash Flow (WN1)	- 10,00,000	2,80,000	4,20,000	4,20,000
PVF @10%	1	0.909	0.8264	0.7513
PVCIF	-10,00,000	2,54,545.45	3,47,107.44	3,15,552.22
NPV	-10,00,000			9,17,205.11
NPV	**-82,794.89**			

From a 10% reduction in selling price, the NPV fell by ₹ 3,93,087.89 to ₹ -82,794 from 3,10,293 i.e., a reduction of **126.68%**

Method 2: At what Standard Deviation Selling Price will NPV be zero.
Let S be the sale price,

$$\left\{ \frac{(S-40)*20,000}{1.1} + \frac{(S-40)*30,000}{1.1^2} + \frac{(S-40)*30,000}{1.1^3} \right\} - 10,00,000 = 0$$

-36,20,586 +65,514.65 S = 0

S = 55.26

At price of ₹ 55.26, i.e., a reduction in Selling Price by ₹ 4.74, i.e., a **reduction of 7.89%** the NPV reduces by 100% to zero.

Change in Unit Cost:

Method 1: Unit Cost Increases by 10% from 40 to 44

Year	0	1	2	3
Cash Flow (WN 3)	-10,00,000	3,20,000	4,80,000	4,80,000
PV @10%	-10,00,000			10,48,234.41
NPV				**₹ 48,234.41**

Reduction in NPV = 3,10,293 - 48,234

= **₹ 2,62,058** = **84.45%**

For 10% increase in cost, the NPV reduces by 84.45%

Method 2: At what cost, NPV will be zero

Year	0	1	2	3
Cash Flow (WN 3)	-10,00,000	(60-c)*20,000	(60-c)*30,000	(60-c)*20,000
		1.1	1.21	1.31

29,30,879 – 65,514.6 *C* = 0
C = 44.736
If Cost increases by **11.84%** i.e., ₹ 4.736 from ₹ 40 to ₹ 44.736, the NPV will reduce by 100% to zero.

Method 1: Reduction in Sales Volume by 10%

Year	0	1	2	3
C.F (WN 4)	= (10,00,000)	=20* 18,000 = 3,60,000	=20* 27,000 = 5,40,000	=20*27,000 = 5,40,000

PV @ 10% = ₹ 1,79,263
Reduction from original NPV ₹ 3,10,293
= ₹ 1,31,029
= **42.22%**
10% reduction in volume, reduces NPV by 42.22%

Method 2: At what volume will NPV be zero.
Since volume reduction is common across all 3 yrs, the reduced NPV equation for NPV @ 0
- Outflow+ current Inflow (1-x) = D
- 10,00,000+ 13,10,293(1-x) = 3,10,293- 3,10,293
3,10,293 = 13,10,293 x
x = $\frac{310293}{1310293}$
x = **23.68%**

b) Method 1: 10% increase in Outlay (₹ 1,00,000)
₹ 1,00,000 increase in outlay, reduces NPV by ₹ 1,00,000 i.e., 10% for ₹ 3,10,293 to ₹ 2,10,293 i.e., a reduction of **32.2276%**

Method 2: Increase in Outflow to be zero
Current Outflow = ₹ 10,00,000
Current Inflow = ₹ 13,10,293
Increase in outflow for NPV to be zero, is increase of ₹ 3,10,293 i.e., **31.03%** for NPV to reduce by 100%.

c) Method 1: If project life time reduces by 10%, i.e., from 3years to 2.7 years, last year reduction from 1 year to 0.7 years or 8.4 months or 255.5 days

Year	0	1	2	3
CF	(10,00,000)	4,00,000	6,00,000	6,00,000
Period	1	1	1	0.7
PV @ 10%	(10,00,000)	11,75,056		
NPV	**1,75,056**			

Reduction for 3,10,293 to 1,75,056 = **43.58%**

Method 2: At what period will NPV be zero.
Amount received in years 1 and 2
= 4,00,000/1.1 + 6,00,000/1.21 = ₹ 8,59,504
Balance to be recovered in PV terms = 10,00,000 – 8,59,504 = 1,40,496
Amount to be recovered in FV terms = 1,40,496 X 1.331 = 1,87,000
Total units manf per day = 30000 / 365 = 82.19 units | Profit per unit = ₹20 | Per day profit = 82.19 x 20 = ₹1643.84 | No of days of profit required = ₹1,87,000 / ₹1643.84 = 113.76 days. Unit has to operate for 114 days in Year 3. i.e a reduction of 251 days in Yr 3. So total period reduction = 251/1095 x 100 = 22.92 %

	Impact on NPV of a 10% Reduction	**Change required for NPV= 0**
Selling Price	126.68	7.89
Cost	84.45	11.84
Volume	42.22	23.68
Outlay	32.23	31.03
Time	43.58	22.92

Working Notes

1. Selling Price = ₹ 60/ unit Cost = ₹ 40/ unit Contribution = ₹ 20/ unit

Year 1 = 20,000 units, Cash flow = ₹ 4,00,000
Year 2 = 30,000 units, Cash flow = ₹ 6,00,000
Year 3 = 30,000 units, Cash flow = ₹ 6,00,000

2. Selling Price reduces by 10%
Selling Price = ₹ 54/ unit Cost = ₹ 40/ unit Contribution = ₹ 14/ unit

Year 1 = 20,000 units, Cash flow = ₹ 2,80,000
Year 2 = 30,000 units, Cash flow = ₹ 4,20,000
Year 3 = 30,000 units, Cash flow = ₹ 4,20,000

3. Unit Cost increase by 10%
Selling Price = ₹ 60 Cost = ₹ 40+ 10% = ₹ 44` Profit = ₹ 16

Year 1 = 20,000 units*16 Cash flow = ₹ 3,20,000
Year 2 = 30,000 units*16 Cash flow = ₹ 4,80,000
Year 3 = 30,000 units*16 Cash flow = ₹ 4,80,000

4. Reduction in Sales Volume

Year	Volume	%	Revised Volume
Year 1	20,000 units	10%	18,000
Year 2	30,000 units	10%	27,000
Year 3	30,000 units	10%	27,000

23. Illustration

From the following details relating to a project, analyze the sensitivity of the project to changes in initial project cost, annual cash inflow and cost of capital:

Initial Project Cost (₹)	**1,20,000**
Annual Cash Inflow (₹)	**45,000**
Project Life (Years)	**4**
Cost of Capital	**10%**

To which of the three factors, the project is most sensitive? (Use annuity factors: for 10% 3.169 and 11% 3.103).

(ICAI SM, Old PM, RTP May '18, MTP May'20 Old)

Solution:

Initial Cash Flow = (1,20,000)

Annual Cash Flow = 45,000

k_c = 10%

No. of Years = 4

NPV of the project at 10% k_c = PVFAC (10%, 4years) * 45,000 – 1,20,000

= 3.169*45,000 -1,20,000

= 1,42,605 – 1,20,000

NPV = ₹ **22,605**

Sensitivity computation for a 10% adverse change

	Current	Initial COF	ACF	k_c
Initial Cash Outflow (A)	(1,20,000)	(1,32,000)	(1,20,000)	(1,20,000)
CFA (B)	45,000	45,000	40,500	45,000
Disc. Factor (C)	10%	10%	10%	11%
PVA	3.169	3.169	3.169	3.103
PVIF (D) = (B)*(C)	1,42,606	1,42,606	1,28,344.5	1,42,606
NPV (A)+(D)	22,605	10,605	8,344.5	19,635
% change in NPV	**0%**	**– 53.08%**	**– 63.08%**	**– 13.14%**

Based on the above, **cash inflow is the most sensitive factor** as 10% change in it leads to 63.08% change in the NPV.

24. Illustration

Red Ltd. is considering a project with the following Cash flows:

₹

Years	Cost of Plant	Recurring Cost	Savings
0	10,000		
1		4,000	12,000
2		5,000	14,000

The cost of capital is 9%. Measure the sensitivity of the project to changes in the levels of plant value, running cost and savings (considering each factor at a time) such that the NPV becomes zero. The P.V. factor at 9% are as under:

Year	Factor
0	1.000
1	0.917
2	0.842

Which factor is the most sensitive to affect the acceptability of the project?

[ICAI SM, MTP Apr'24, Nov'17 QP (Old), Old PM]

Solution:

Year	Outflow	Cost	Saving	PVF	PV
0	(10,000)			1	(10,000)
1		(4,000)	12,000	0.917	7,336
2		(5,000)	14,000	0.842	7,578
					4,914

If initial cash outflow/ investment increases to 14,914 from 10,000, i.e., an increase of 49.14%, NPV becomes zero.

Change in recurring cost such that total recurring cost increases by 4,914

Current PV of recurring cost = 4,000* 0.917 + 5,000*0.842

= **7,878**

4,914+ 7,8,78= 12,792

Let x be the change %

4000(1+x)*0.917+ 5000(1+x)*0.842

= 12,792

x = 12,792/7878 -1

= **62.37%**

i.e., **increase by 62.37%**

Reduction in saving by ₹ 4,914 to make NPV as zero

(12,000*0.917) (1-x) + (14,000*0.842) (1-x) = 22,792- 4,914

22,792 (1-x%) = 17,878

22,792 – 17,878 = 22,792 x

x = 4914/22,792

x = **21.56%**

Since the lowest required change for the NPV to become zero is in **savings**, that is the most Sensitive factor.

25. Illustration

The Easygoing Company Limited is considering a new project with initial investment, for a product "Survival". It is estimated that IRR of the project is 16% having an estimated life of 5 years.
The Financial Manager has studied that project with sensitivity analysis and informed that annual fixed cost sensitivity is 7.8416%, whereas cost of capital (discount rate) sensitivity is 60%.
Other information available are:
Profit Volume Ratio (P/V) is 70%, Variable cost ₹ 60/- per unit Annual Cash Flow ₹ 57,500/-
Ignore Depreciation on initial investment and impact of taxation.
Calculate

(i) Initial Investment of the Project
(ii) Net Present Value of the Project
(iii) Annual Fixed Cost
(iv) Estimated annual unit of sales
(v) Break Even Units

Cumulative Discounting Factor for 5 years

8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%
3.993	3.890	3.791	3.696	3.605	3.517	3.433	3.352	3.274	3.199	3.127

(ICAI SM, Old PM)

Solution:

Given, IRR = -16%
Y = 5 years
FC Sensitivity = 7.8416%
k_c sensitivity = 60%
PV Ratio = 70%
Variable Cost = ₹ 60/ unit
Annual Cash Flow = ₹ 57,500

i) Initial Investment
Annual Cash Flow = ₹ 57,500
PVFA (5 yrs, 16%) = 3.274
PV of all Cash Inflows
(A) = PV of all Cash Outflows (B)
(A) = 57,500* 3.274
= ₹ 1,88,255
So, Initial Investment (B) = **₹ 1,88,255**

ii) k_e is <16% and sensitivity of k_c is 60%
At k_c of x, NPV = Y
At k_c of x(1+60%), NPV = 0 where IRR= 16%

So, x(1+60%) = 16%
x = 10%
So, k_c = 10%

At k_c of 10%, PVFA = 3.791
PVIF = 3.791* 57,500
= ₹ 2,17,982.5 (C)
PVOF = ₹ 1,88,255 (D)
NPV = **₹ 29,727.50** (C) -(D)

iii) Annual Fixed Cost
For a 7.8416% in Fixed Cost, the NPV becomes zero.
Let annual fixed cost be x
Then, PV of Annual Fixed Cost is 3.791x
If 3.791x increases by 7.8416%, then NPV of ₹ 29,727.50 becomes zero.
3.791x* 7.8416% = 29,727.50

Solving for x,

x *7.8416% $= \frac{29727.5}{3.791}$

x = ₹ 99,999.98 = ₹ **1,00,000 approx.**

iv) Estimated sale units

Contribution (PV Ratio)= 70%

Variable Cost (%) = 1- Contribution%

= 1 -70% = 30%

Given, Variable Cost = ₹ 60

$\frac{60}{S.P}$ = 30%

Selling Price = ₹ 200

Annual Cash Flow = Contribution* Units -Fixed Cost

₹ 57,500 = 140*Units -1,00,000

Units Sold $= 57{,}500 + \frac{1{,}00{,}000}{140}$ =**1,125 units**

v) Break Even Units

Break Even point at which,

Contribution = Fixed Cost

$\frac{Fixed Cost}{Contribution\ per\ unit}$

= Break Even Units

Contribution/unit*x-FC = 0

140 x = 1,00,000

x $= \frac{1{,}00{,}000}{140}$

x = 714.2857 units

Break Even Units = **715 units**

26. Illustration

Unnat Ltd. is considering investing ₹ 50,00,000 in a new machine. The expected life of machine is five years and has no scrap value. It is expected that 2,00,000 units will be produced and sold each year at a selling price of ₹ 30.00 per unit. It is expected that the variable costs to be ₹ 16.50 per unit and fixed costs to be ₹ 10,00,000 per year. The cost of capital of Unnat Ltd. is 12% and acceptable level of risk is 20%.

You are required to measure the sensitivity of the project's net present value to a change in the following project variables:

(a) sale price;

(b) sales volume;

(c) variable cost;

(d) On further investigation it is found that there is a significant chance that the expected sales volume of 2,00,000 units per year will not be achieved. The sales manager of Unnat Ltd. suggests that sales volumes could depend on expected economic states which could be assigned the following probabilities:

State of Economy	Annual Sales (in Units)	Prob.
Poor	1,75000	0·30
Normal	2,00,000	0·60
Good	2,25,000	0·10

Calculate expected net present value of the project and give your decision whether company should accept the project or not.

(ICAI SM, Old PM)

Solution:

i) Project NPV @ 2,00,000 units sales volume

Annual Cash Inflow (WN) = ₹ 17,00,000

No. of Years = 5

k_c = 12%

PVFA (5 years, 12%) = 3.6047

NPV = PVCIF -PVCOF

= 17,00,000* 3.6047 - 50,00,000

= **₹ 11,28,119.54**

ii) Sensitivity of NPV for a 10% reduction in Selling Price

Annual Cash Flow when Selling Price reduces by 10%

(WN 2) = ₹ 11,00,000

PVFA (5 years, 12%) = 3.6047

NPV = PVCIF -PVCOF

= 11,00,000* 3.6047 - 50,00,000

= (₹ 10,34,830)

% change in NPV = $\frac{-10,34,830-11,28,120}{11,28,120}$

= **-191.73%**

iii) Sensitivity of NPV for a 10% reduction in Sales Volume

Annual Cash Flow when Selling Price reduces by 10%

(WN 2) = ₹ 14,30,000

PVFA (5 years, 12%) = 3.6047

NPV = PVCIF -PVCOF

= 14,30,000* 3.6047 - 50,00,000

= ₹ 1,54,721

% change in NPV = $\frac{11,28,120-1,54,721}{11,28,120}$

= **86.29%**

iv) Sensitivity of NPV for a 10% reduction in Variable Cost

Annual Cash Flow when Selling Price reduces by 10%

(WN 3) = ₹ 13,70,000

PVFA (5 years, 12%) = 3.6047

NPV = PVCIF -PVCOF

= 13,70,000* 3.6047 - 50,00,000

= (₹ 61,561)

% change in NPV = $\frac{-11,28,120-61,561}{11,28,120}$

= - **105.46%**

v) Expected NPV of the Project

Selling Price = ₹ 30

Variable Cost = ₹ 16.5

Contribution = ₹ 13.5

Value = (1,75,000*0.3 + 2,00,000*0.6 + 2,25,000*0.1)

= 1,95,000

Total Contribution = 13.5* 1,95,000

= ₹ 26,32,500

Less: Fixed Cost = ₹ 10,00,000

Annual Profit = ₹ 16,32,500

PVAF (5yr, 12%)	= 3.6047
PVCIF	= ₹ 58,84,673
Less: PVCOF	= ₹ 50,00,000
NPV	= **₹ 8,84,673**

vi) There is 30% probability that the economy is in poor state, when NPV will be ₹ 88,596 (WN 5) i.e., negative value and the company's risk tolerance level is 20% and therefore the company should not undertake this project.

Working Notes:

1.

Selling Price	= ₹ 30
Less: Variable Cost	= ₹ 16.5
Contribution	= ₹ 13.5
Units sold	= ₹ 2,00,000
Total Contribution	= 2,00,000* 13.5
	= ₹ 27,00,0000
Less: Fixed Cost	= ₹ 10,00,000
Profit	= ₹ 17,00,000

Since tax details are not give, no tax or tax benefit on depreciation are considered

2.

Selling Price	= ₹ 27
Less: Variable Cost	= ₹ 16.5
Contribution	= ₹ 10.5
Units sold	= ₹ 2,00,000
Total Contribution	= 2,00,000* 10.5
	= ₹ 21,00,0000
Less: Fixed Cost	= ₹ 10,00,000
Profit	= ₹ 11,00,000

3.

Selling Price	= ₹ 30
Less: Variable Cost	= ₹ 16.5
Contribution	= ₹ 13.5
Units sold	= ₹ 1,80,000
Total Contribution	= 1,80,000* 13.5
	= ₹ 24,30,0000
Less: Fixed Cost	= ₹ 10,00,000
Profit	= ₹ 14,30,000

4.

Selling Price	= ₹ 30
Less: Variable Cost	= ₹ 18.15
Contribution	= ₹ 11.85
Units sold	= ₹ 2,00,000
Total Contribution	= 2,00,000* 11.85
	= ₹ 23,70,0000
Less: Fixed Cost	= ₹ 10,00,000
Profit	= ₹ 13,70,000

5. NPV in poor and good state

	Poor	Good
Contribution Per Unit	₹ 13.5	₹ 13.5
Units	1,75,000	2,25,000
Total Contribution	₹ 23,62,500	₹ 30,37,500
Less: Fixed Cost	₹ 10,00,000	₹ 10,00,000
Profit	₹ 13,62,500	₹ 20,37,500
PVFA (5y, 12%)	3.6047	3.6047
PVCIF	₹ 49,11,403.75	₹ 73,44,576
Less: PVCOF	₹ 50,00,000	₹ 50,00,000
NPV	(₹ 88,596)	₹ 23,44,576

27. Illustration

The Textile Manufacturing Company Ltd. is considering one of two mutually exclusive proposals, Projects M and N, which require cash outlays of ₹ 8,50,000 and ₹ 8,25,000 respectively. The certainty-equivalent (C.E) approach is used in incorporating risk in capital budgeting decisions. The current yield on government bonds is 6% and this is used as the risk-free rate. The expected net cash flows and their certainty equivalents are as follows:

	Project M		Project N	
Year-end	Cash Flow ₹	C.E.	Cash Flow ₹	C.E.
1	4,50,000	0.8	4,50,000	0.9
2	5,00,000	0.7	4,50,000	0.8
3	5,00,000	0.5	5,00,000	0.7

Present value factors of ₹ 1 discounted at 6% at the end of year 1, 2 and 3 are 0.943, 0.890 and 0.840 respectively. Required:

(i) Which project should be accepted?

(ii) If risk adjusted discount rate method is used, which project would be appraised with a higher rate and why?

(ICAI SM, RTP Nov'20, MTP Mar'24, MTP Mar'21, Old PM)

Solution:

NPV of Project M

	Year	Cash Flow	C.E	CECF	PVF @6%	PVCF
Instalment	0	-8,50,000	1	-8,50,000	1	- 8,50,000
Annual CF	1	4,50,000	0.8	3,60,000	0.943	3,39,480
	2	5,00,000	0.7	3,50,000	0.890	3,11,500
	3	5,00,000	0.5	2,50,000	0.840	2,10,000
			2.0			**10,980**

NPV of Project N

	Year	Cash Flow	C.E	CECF	PVF @6%	PVCF
Instalment	0	-8,25,000	1	-8,25,000	1	-8,25,000
Annual CF	1	4,50,000	0.7	4,05,000	0.943	3,81,915
	2	4,50,000	0.8	3,60,000	0.890	3,20,400
	3	5,00,000	0.7	3,50,000	0.840	2,94,000
			2.4			**1,71,315**

NPV on C.E basis = $\sum PV\ of\ Cfi * CEi$

i) Project N should be accepted as it has higher NPV.

ii) Project M has total C.E of 2.0/3 and Project N has C.E of 2.4/3. So, **Project M is more** uncertain as its C.E quotient is lower and hence is **riskier**, and will have higher discount rate/ Risk adjusted Discount Rate.

28. Illustration

Determine the risk adjusted net present value of the following projects:

	X	Y	Z
Net cash outlays (₹)	2,10,000	1,20,000	1,00,000
Project life	5 years	5 years	5 years
Annual Cash inflow (₹)	70,000	42,000	30,000
Coefficient of variation	1.2	0.8	0.4

The Company selects the risk-adjusted rate of discount on the basis of the coefficient of variation:

Coefficient of Variation	Risk-Adjusted Rate of Return	P.V. Factor 1 to 5 years @ Risk adjusted rate of discount
0.0	10%	3.791
0.4	12%	3.605
0.8	14%	3.433
1.2	16%	3.274
1.6	18%	3.127
2.0	22%	2.864
More than 2.0	25%	2.689

(ICAI SM, Old PM)

Solution:

	Cash Outflow	Annual Cash Inflow	Coeff. Of Var	Disc. Rate	PVFA	PVCIF	Risk Adjusted NPV
X	(2,10,000)	70,000	1.2	16%	3.274	2,29,180	19,180
Y	(1,20,000)	42,000	0.8	14%	3.433	1,44,186	24,186
Z	(1,00,000)	30,000	0.4	12%	3.605	1,08,150	8,150

29. Illustration

New Projects Ltd. is evaluating 3 projects, P-I, P-II, P-III. Following information is available in respect of these projects:

	P-I	P-II	P-III
Cost	₹ 15,00,000	₹ 11,00,000	₹ 19,00,000
Inflows-Year 1	6,00,000	6,00,000	4,00,000
Year 2	6,00,000	4,00,000	6,00,000
Year 3	6,00,000	5,00,000	8,00,000
Year 4	6,00,000	2,00,000	12,00,000
Risk Index	1.80	1.00	0.60

The minimum required rate of return of the firm is 15% and applicable tax rate is 40%. The risk-free interest rate is 10%.

Required:

(i) Find out the risk-adjusted discount rate (RADR) for these projects.

(ii) Which project is the best?

(ICAI SM, Old PM)

Solution:

Assumption: Cash flows and Discount Rates are Post Tax and tax need not be adjusted.

i) PART 1: RADR

RADR= Risk free rate + Risk Index * (Expected Return – Risk free rate)

	P1	P2	P3
Rf	10	10	10
Risk Index	1.8	1.0	0.6
Expected Return	15	15	15
Risk Premium	(15 -10) =5	(15 -10) =5	(15 -10) =5
RADR	10+ (1.8*5) = 19%	10+(1*5) = 15%	10+(0.6*5) = 13%

ii) PART 2

NPV of Project 1

PVCOF = -15,00,000

PVCIF = PVF (19%,4y) * 6,00,000
= 2.6385* 6,00,000
= 15,83,151

NPV = 15,83,151 -15,00,000
= **₹ 83,151**

NPV of Project 2

Year	Cash Flow	PVF @ 15%	PVCF
0	-11,00,000	1	- 11,00,000
1	6,00,000	0.8695	5,21,739
2	4,00,000	0.7561	3,02,457
3	5,00,000	0.6575	3,28,758
4	2,00,000	0.57175	1,14,350
			₹ 1,67,305

NPV of Project 3

Year	Cash Flow	PVF @ 13%	
0	-19,00,000	1	- 19,00,000
1	4,00,000	0.8849	3,53,982
2	6,00,000	0.7831	4,69,888
3	8,00,000	0.6930	5,54,440
4	12,00,000	0.6133	7,35,982
			₹ 2,14,292.91

P3 has higher NPV and hence it should be chosen.

30. Illustration

Jumble Consultancy Group has determined relative utilities of cash flows of two forthcoming projects of its client company as follows:

Cash Flow in ₹	-15000	-10000	-4000	0	15000	10000	5000	1000
Utilities	-100	-60	-3	0	40	30	20	10

The distribution of cash flows of project A and Project B are as follows:

Project A					
Cash Flow (₹)	-15000	- 10000	15000	10000	5000
Probability	0.10	0.20	0.40	0.20	0.10
Project B					
Cash Flow (₹)	- 10000	-4000	15000	5000	10000
Probability	0.10	0.15	0.40	0.25	0.10

Which project should be selected and why ?

(ICAI SM, RTP Nov'24, MTP Nov'21 Old, Old PM)

Solution:

A				B			
Cash Flow	*Pi*	Utility	*Pi** U	Cash Flow	*Pi*	Utility	*Pi** U
-15,000	0.1	-100	-10	-10,000	0.1	-60	-6
-10,000	0.2	-60	-12	-4,000	0.15	-3	-0.45
15,000	0.4	40	16	15,000	0.4	40	16
10,000	0.2	30	6	5,000	0.25	20	5
5,000	0.1	20	2	10,000	0.1	30	3
	1.0	-70	2		1.0	27	17.55

PW of Project A = -1,500 -2,000 +6,000 +2,000 +500
= 5,000

PW of Project B = -1,000 -600 +6,000 +1,250 +1,000
= 6,650

Evaluation

	A	B
Expected Cash Flow	5,000	6,650
Probability Weighted Utility Value	2	17.55

The company **should undertake Project B** given that its utility value and expected cash flow are higher than Project A.

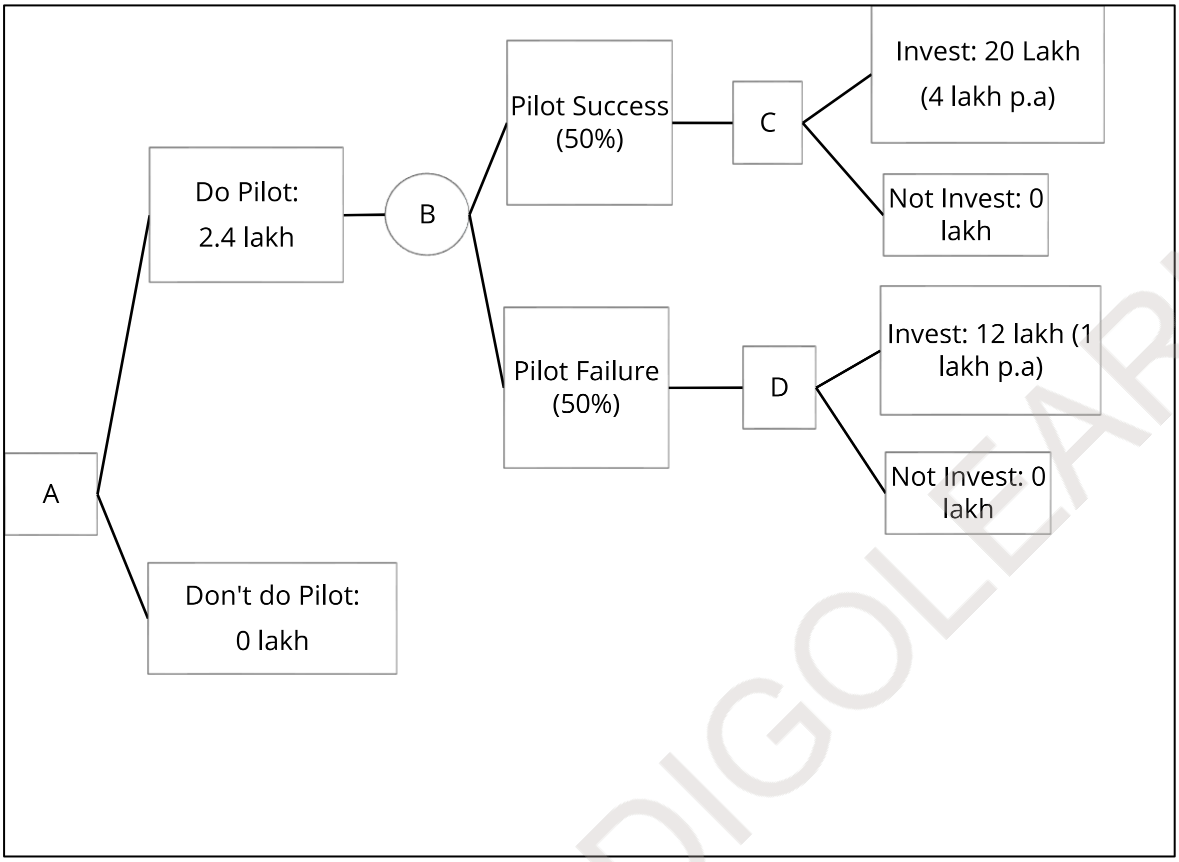
31. Illustration

L & R Limited wishes to develop new virus-cleaner software. The cost of the pilot project would be ₹ 2,40,000. Presently, the chances of the product being successfully launched on a commercial scale are rated at 50%. In case it does succeed. L&R can invest a sum of ₹ 20 lacs to market the product. Such an effort can generate perpetually, an annual net after tax cash income of ₹ 4 lacs. Even if the commercial launch fails, they can make an investment of a

smaller amount of ₹ 12 lacs with the hope of gaining perpetually a sum of ₹ 1 lac. Evaluate the proposal, adopting decision tree approach. The discount rate is 10%.

(ICAI SM)

Solution:



Value at Node C = ₹ 4,00,000 p.a till perpetuity

Value of ₹ 4,00,000 p.a till perpetuity (@10% discount rate) = $\frac{400000}{0.1}$

= ₹ 40,00,000

Less: Investment = ₹ 20,00,000

NPV = ₹ 20,00,000

Value of Node D = ₹ 1,00,000 p.a till perpetuity

Value of ₹ 1,00,000 p.a till perpetuity (@10% discount rate) = $\frac{100000}{0.1}$

= ₹ 10,00,000

Less: Investment = ₹ 12,00,000

NPV = -₹ 2,00,000

Value at Node B = Probability A* NPV (A) + Probability B* NPV (B)

= 0.5*20,00,000 + 0.5*0

= ₹ 10,00,000

Value at Node A = Invest ₹ 2,40,000 and get ₹ 10,00,000

NPV (When Pilot is done) = ₹ 7,60,000

NPV (No Pilot) = ₹ 0

The company **should go ahead with pilot as investment** of ₹ 2,40,000 will generate ₹ 10,00,000 that is NPV of ₹ 7,60,000

32. Illustration

A firm has an investment proposal, requiring an outlay of ₹ 80,000. The investment proposal is expected to have two years economic life with no salvage value. In year 1, there is a 0.4 probability that cash inflow after tax will be ₹ 50,000 and 0.6 probability that cash inflow after tax will be ₹ 60,000. The probability assigned to cash inflow after tax for the year 2 is as follows:

The cash inflow year 1	₹ 50,000		₹ 60,000	
The cash inflow year 2	Probability		Probability	
	₹ 24,000	0.2	₹ 40,000	0.4
	₹ 32,000	0.3	₹ 50,000	0.5
	₹ 44,000	0.5	₹ 60,000	0.1

The firm uses a 10% discount rate for this type of investment. Required:

(i) Construct a decision tree for the proposed investment project and calculate the expected net present value (NPV).

(ii) What net present value will the project yield, if worst outcome is realized? What is the probability of occurrence of this NPV?

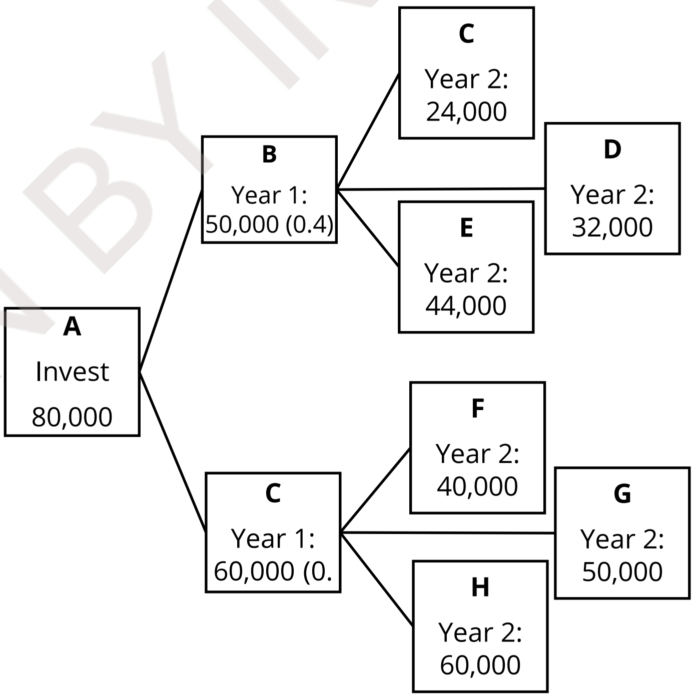
(iii) What will be the best outcome and the probability of that occurrence?

(iv) Will the project be accepted?

(Note: 10% discount factor 1 year 0.909; 2 year 0.826)

(ICAI SM, Old PM)

Solution:



i) Expected Value

Part	Year 1 Cf* PV	Year 2 Cf*PV	- Invest	NPV (1)	Joint Probability (2)	(1)* (2)
1	= 50,000*0.909 = 45,450	= 24,000*0.826 = 19,824	- 80,000	-14,726	= 0.4*0.2 = 0.08	-1,178.08
2	45,450	= 32,000*0.826 = 26,432	- 80,000	-8,118	= 0.4*0.3 = 0.12	-974.16
3	45,450	= 44,000*0.826 = 36,344	- 80,000	1,794	= 0.4*0.5 = 0.2	358.8
4	= 60,000*0.909 = 54,540	= 40,000*0.826 = 33,040	- 80,000	7,580	= 0.6*0.4 =0.24	1,819.2
5	54,540	= 50,000*0.826 = 41,300	- 80,000	15,840	= 0.6*0.5 = 0.3	4,752
6	54,540	= 60,000*0.826 = 49,560	- 80,000	24,100	= 0.6*0.1 = 0.06	1,446
						6,223.76

Expected Value = ₹ **6,223.76**

ii) If worst outcome is realised, the NPV will be –₹ **14,726** and probability will be 0.08
iii) If best outcome is realised, the NPV will be -₹ **24,100** and probability will be 0.06
iv) Yes, the project is accepted as the expected value of NPV is positive at ₹ **6,223.76.**

33. Illustration

A Company named Roby's cube decided to replace the existing Computer system of their organization. The original cost of the old system was ₹ 25,000 and it was installed 5 years ago. Current market value of old system is ₹ 5,000. Depreciation of the old system was charged with life of 10 years with Estimated Salvage value as Nil. Depreciation of the new system will be charged with life over 5 years. Present cost of the new system is ₹ 50,000. Estimated Salvage value of the new system is ₹ 1,000. Estimated cost savings with the new system is ₹ 5,000 per year. Increase in sales with new system is assumed at 10% per year based on original total sales of ₹ 1,00,000. Company follows straight line method of depreciation. The cost of capital of the company is 10% whereas tax rate is 30%. Evaluate the replacement decision.

[ICAI SM, similar Dec'21 QP (Old)]

Solution:

	Old System	New System
Original Cost	25,000	50,000
Original Life	10 years	5 years
Life Expected	5 years	0
Market Value (Today)	5,000	
Salvage Value (10 yrs)	0	1,000
Savings		5,000
Sales	1,00,000	
Increase In Sales		10,000 p.a
Depreciation	SLM	SLM
Kc	10%	10%
Tax	30%	30%

STEP 1:

Cash Outflow of New System + Cash Inflow of Old System

= -50,000 + 7,250 = **-42,750**

Calculation:

Book Value of Old System

Depreciation = $\frac{25,000-0}{10}$ = ₹ 2,500

Life Exhausted = 5 years

Depreciation for 5 years = 5* 2500

= 12,500

Current Book Value = 25,000- 12,500 = ₹ 12,500

Cash Inflow from Old System

i) Market Value = ₹ 5,000

ii) Tax Benefit on Loss = (Book Value - Market Value) * Tax Rate

= (12,500-5,000)*30% = ₹ 2,250

Cash Inflow = 5,000+ 2,250 = 7,250

Cash Outflow from New System

= - 50,000

STEP 2:

Changes in Annual Cash Flows

= (Change in Sales+ Change in Savings) (1-t) + (Change in Depreciation)*t

= (15,000) *(1-30%) + 7,300*30%

= 10,500+ 2,190 = **₹ 12,690**

Calculation:

i) Increase in Sales = ₹ 1,00,000* 10% = ₹ 10,000

ii) Savings = ₹ 5,000

iii) Change in Depreciation

= New depreciation - Old Depreciation

= 9,800 - 2,500 = 7,300

New depreciation = (Original cost - Salvage Value)/Life

= 50000-1000/5 = 9,800

STEP 3:

PV of Annual Cash Flow @10% for 5 Years

= (12,690* 3.790786) + (1000*0.62092)

= 48,105.08 + 620.92 = **₹ 48,726**

STEP 4:

PV of Cash Inflow+ PV of Cash Outflow

= 48,726- 42,750 = **₹ 5,976**

STEP 5:

Since NPV>0, the replacement **decision is correct.**

34. Illustration

X Ltd. is a taxi operator. Each taxi cost to company ₹ 4,00,000 and has a useful life of 3 years. The taxi's operating cost for each of 3 years and salvage value at the end of year is as follow s:

	Year 1	Year 2	Year 3
Operating Cost	₹ 1,80,000	₹ 2,10,000	₹ 2,38,000
Resale Value	₹ 2,80,000	₹ 2,30,000	₹ 1,68,000

You are required to determine the optimal replacement period of taxi if cost of capital of X Ltd. is 10%.

(ICAI SM)

Solution:

EAC at the end of Year 1

Period	0	1
Cash Flow	(4,00,000)	(1,80,000)+ 2,80,000 = 1,00,000
PVF	1	0.909
PVCF	(4,00,000)	90,909.09
PVCF		(3,09,090.9)
EAC	$\frac{3,09,090.9}{0.909}$	**= 3,40,034**

EAC at the end of Year 2

Period	0	1	2
Cash Flow	(4,00,000)	(1,80,000)	(2,10,000)+ 2,30,000 = 20,000
PVF	1	0.909	0.826
PVCF	(4,00,000)	1,63,636	16,528.92
PVCF		(5,47,107.08)	
EAC	$\frac{5,47,107.08}{1.7353}$	**= 3,15,237.89**	

EAC at the end of Year 3

Period	0	1	2	3
Cash Flow	(4,00,000)	(1,80,000)	(2,10,000)	(70,000)
PVF	1	0.909	0.826	0.7513
PVCF	(4,00,000)	(1,63,636.36)	(1,73,553.72)	(52,592.04)
PVCF		(7,89,782.12)		
EAC	$\frac{7,89,782.12}{2.48685}$	**= 3,17,583**		

The optimum replacement period is at the end of two years because EAC is less at the end of year 2

35. Illustration

Company X is forced to choose between two machines A and B. The two machines are designed differently but have identical capacity and do exactly the same job. Machine A costs ₹ 1,50,000 and will last for 3 years. It costs ₹ 40,000 per year to run. Machine B is an 'economy' model costing only ₹ 1,00,000, but will last only for 2 years, and costs ₹ 60,000 per year to run. These are real cash flows. The costs are forecasted in rupees of constant purchasing power. Ignore tax. Opportunity cost of capital is 10 per cent. Which machine company X should buy?

(ICAI SM, Old PM)

Solution:

Machine	A	B
Cost	1,50,000	1,00,000
Annual Maintenance (p.y)	40,000	60,000
Life	3	2
Cost of Capital	10%	10%

Machine	A	B
Cost	1,50,000	1,00,000
PVAF	2.48685	1.7355
PV of Maintenance Cost	2.4865* 40,000 = (99,474.08)	1.7355* 60,000 = (1,04,132.23)
Total PV of all Cash Flows (PVCF)	(2,49,474.08)	(2,04,132.23)
EAC= $\frac{PVCF}{PVAF}$	1,00,331.42	1,17,621.56

Since **EAC of Machine A is lower**, X Ltd. Is advised to go with it.

36. Illustration

Company Y is operating an elderly machine that is expected to produce a net cash inflow of ₹ 40,000 in the coming year and ₹ 40,000 next year. The current salvage value is ₹ 80,000 and next year's value is ₹ 70,000. The machine can be replaced now with a new machine, which costs ₹ 1,50,000, but is much more efficient and will provide a cash inflow of ₹ 80,000 a year for 3 years. Company Y wants to know whether it should replace the equipment now or wait a year with the clear understanding that the new machine is the best of the available alternatives and that it in turn be replaced at the optimal point. Ignore tax. Take opportunity cost of capital as 10 per cent. Advise with reasons.

(ICAI SM, Old PM)

Solution:

	Year	Old Machine	New Machine
Cash Inflow	Year 1	40,000	80,000
	Year 2	40,000	80,000
	Year 3		80,000
Salvage Value	Year 0	80,000	
	Year 1	70,000	
Cash Outflow	Year 0		(1,50,000)

Alternative 1: Replace Now

Salvage Value of Old Machine	= 80,000
Cost of New Machine	= (1,50,000)
Cash Flow of Year 0	= (70,000)
Cash Flows Year (1-3)	= 80,000 * 2.48685 (PVAF- 3,10%)
	= 1,98,948
PVCF	= (70,000) + 1,98,948
	= **1,28,948**

Alternative 2: Replace after a Year

	Year	Cash Flow	PVF	PVCF
Cash Inflow Old	1	40,000	0.909	36,364
Salvage Old	1	70,000	0.909	63,636
Expense New	1	(1,50,000)	0.909	(1,36,364)
			(A)	**(36,364)**
Year 2-4 Cash flows from New Machine	2-4	80,000	2.260	**(B)**1,80,862

PV at Year 0 for New Machine (cash Inflows)	= 1,80,862 **(B)**
NPV as on day 0 for Replacement After Year 1	= **(A) + (B)**
	= 1,80,862- 36,364 = **₹ 1,44,498**

Hence it is better to **replace after Year 1** as NPV is higher.

37. Illustration

Trouble Free Solutions (TFS) is an authorized service center of a reputed domestic air conditioner manufacturing company. All complaints/service-related matters of Air conditioner are attended by this service center. The service center employs many mechanics, each of whom is provided with a motorbike to attend to the complaints. Each mechanic travels approximately 40000 kms per annum. TFS decides to continue its present policy of always buying a new bike for its mechanics but wonders whether the present policy of replacing the bike every three years is optimal or not. It is believed that as new models are entering into the market on a yearly basis, it wishes to consider whether a replacement of either one year or two years would be a better option than present three-year period. The fleet of bikes is due for replacement shortly soon.The purchase price of the latest model bike is ₹ 55,000. Resale value of used bike at current prices in market is as follows:

Period	₹
1 Year old	35,000
2-Year-old	21,000
3-Year-old	9,000

Running and Maintenance expenses (excluding depreciation) are as follows:

Year	Road Taxes Insurance etc. (₹)	Petrol Repair Maintenance etc. (₹)
1	3,000	30,000
2	3,000	35,000
3	3,000	43,000

Using opportunity cost of capital as 10% you are required to determine optimal replacement period of bike.

(ICAI SM, Old PM)

Solution:

Cost of running a bike for years 1,2,3 and salvage value.

Year	Total Expenses	Salvage Value	PVF @ 10%	Cash Flow	PVCF
1	33,000	35,000	0.909	2,000	1,818
2	38,000	21,000	0.8264	(17,000)	(14,049)
3	46,000	9,000	0.7513	(37,000)	(27,798.1)

NPV of Cumulative Cash Flows for the bike

Year	PV of Previous Year Expenses	PV of CY	Cumulative NPV
1	0	1,818	(1,818)
2	(33,000) * 0.909	(14,049)	(44,046)
3	(33,000) * 0.909 (38,000)* 0.8264	(27,798)	(89,198)

Expenses for a new Bike + Cumulative Costs

Year	PV of Previous Year Expenses	Cumulative Cash Flows NPV	Cumulative Cost PV
1	-55,000	(1,818)	53,182
2	-55,000	(44,046)	99,046
3	-55,000	(89,198)	1,44,198

EAC of Cumulative Cost

Year	Cumulative Cost PV	Cumulative Cash Flows NPV	EAC
1	53,182	0.909	58,506
2	99,046	1.7354	57,074
3	1,44,198	2.4867	57,987

As the EAC is lowest for replacement after year 2, it is advised to **replace after year 2.**

38. Illustration

A machine used on a production line must be replaced at least every four years. Costs incurred to run the machine according to its age are:

Age of the Machine (years)

	0	1	2	3	4
Purchase price (in ₹)	60,000				
Maintenance (in ₹)		16,000	18,000	20,000	20,000
Repair (in ₹)		0	4,000	8,000	16,000
Scrap Value (in ₹)		32,000	24,000	16,000	8,000

Future replacement will be with identical machine with same cost. Revenue is unaffected by the age of the machine. Ignoring inflation and tax, determine the optimum replacement cycle. PV factors of the cost of capital of 15% for the respective four years are 0.8696, 0.7561, 0.6575 and 0.5718.

(ICAI SM, May'24 QP, Old PM)

Solution:

Replacement Cycle:

Repl. Cycle Years		1		2	
Year	PVF 15%	CF	PVCF	CF	PVCF
0	1	-60,000	-60,000	-60,000	-60,000
1	0.896	16,000	13,913.6	-16,000	-13,913.6
2	0.7561			2,000	1,512.2
3	0.6575				
4	0.5718				
			-46,086.4		**-72,401**

Repl. Cycle Years		3		4	
Year	PVF 15%	CF	PVCF	CF	PVCF
0	1	-60,000	-60,000	-60,000	-60,000
1	0.896	-16,000	-13,913.6	-16,000	-13,913.6
2	0.7561	-22,000	-16,634.2	-22,000	-16,634.2
3	0.6575	-12,000	-7,890	-28,000	-18,410
4	0.5718			-28,000	-16,010.4
			-98,437.8		**-1,24,968.2**

Optimum Replacement Cycle:

$$EAC = \frac{CumCF}{PVAF}$$

Replacement Period	Cum. PV of CF (1)	PVAF (2)	(1)/ (2)
1	-46,086.4	0.896	52,997
2	-72,401	1.6257	44,535
3	-98,437.8	2.2832	43,113.96
4	-1,24,968.2	2.855	43,771.7

The optimum replacement cycle is **after 3 years.** When EAC is lowest at ₹ 43,113.96.

Working Notes:

1. Replacement at the end of Year 1:

Year	0	1
Cash Flows:		
Purchase	-60,000	
Maintenance		-16,000
Repairs		0
Scrap		32,000
	-60,000	16,000

2. Replacement at the end of Year 2:

Year	0	1	2
Cash Flows:			
Purchase	-60,000		
Maintenance		-16,000	-18,000
Repairs			-4,000
Scrap			24,000
	-60,000	-16,000	2,000

3. Replacement at the end of Year 3:

Year	0	1	2	3
Cash Flows:				
Purchase	-60,000			
Maintenance		-16,000	-18,000	-20,000
Repairs			-4,000	-8,000
Scrap				16,000
	-60,000	16,000	-22,000	-12,000

4. Replacement at the end of Year 4:

Year	0	1	2	3	4
Cash Flows:					
Purchase	-60,000				
Maintenance		-16,000	-18,000	-20,000	-20,000
Repairs			-4,000	-8,000	-16,000
Scrap					8,000
	-60,000	**16,000**	**-22,000**	**-28,000**	**-28,000**

39. Illustration

A company has an old machine having book value zero – which can be sold for ₹ 50,000. The company is thinking to choose one from following two alternatives:

(i)To incur additional cost of ₹ 10,00,000 to upgrade the old existing machine.

(ii)To replace old machine with a new machine costing ₹ 20,00,000 plus installation cost ₹ 50,000.

Both above proposals envisage useful life to be five years with salvage value to be nil. The expected after-tax profits for the above three alternatives are as under:

Year	Old existing Machine (₹)	Upgraded Machine (₹)	New Machine (₹)
1	5,00,000	5,50,000	6,00,000
2	5,40,000	5,90,000	6,40,000
3	5,80,000	6,10,000	6,90,000
4	6,20,000	6,50,000	7,40,000
5	6,60,000	7,00,000	8,00,000

The tax rate is 40 per cent. The company follows straight line method of depreciation. Assume cost of capital to be 15 per cent.
P.V.F. of 15%, 5 = 0.870, 0.756, 0.658, 0.572 and 0.497. You are required to advise the company as to which alternative is to be adopted.

(ICAI SM, Old PM)

Solution:

i) Upgraded Machine

Year	0	1	2	3	4	5
PAT		5,50,000	5,90,000	6,10,000	6,50,000	7,00,000
Add: Depreciation		2,00,000	2,00,000	2,00,000	2,00,000	2,00,000
CFAT		7,50,000	7,90,000	8,10,000	8,50,000	9,00,000
Investment	-10,00,000					

PV Factor	Cash Flows	PV of Cash Flows
1	-10,00,000	-10,00,000
0.87	7,50,000	6,52,500
0.756	7,90,000	5,97,240
0.658	8,10,000	5,32,980
0.572	8,50,000	4,86,200
0.497	9,00,000	4,47,300
	NPV	**17,16,220**

Depreciation:
No Salvage of Old Machine
Upgradation Cost = 10,00,000
Depreciation = $\frac{10,00,000}{5}$ = 2,00,000

ii) Replacement:

Year	0	1	2	3	4	5
PAT		6,00,000	6,40,000	6,90,000	7,40,000	8,00,000
Scrap (After Tax)	30,000					
Add: Depreciation		4,10,000	4,10,000	4,10,000	4,10,000	4,10,000
CFAT		10,10,000	10,50,000	11,00,000	11,50,000	12,10,000
Investment	-20,50,000					

PV Factor	Cash Flows	PV of Cash Flows
1	-20,20,000	-20,20,000
0.87	10,10,000	8,78,700
0.756	10,50,000	7,93,800
0.658	11,00,000	7,23,800
0.572	11,50,000	6,57,800
0.497	12,10,000	6,01,370
	NPV	**16,35,470**

Replacement Scrap Value = 50,000
Less: Tax = (20,000)
Net Scrap = 30,000

Cost of New = 20,50,000
Depreciable Life = 5 years
Depreciation = 4,10,000

iii) PAT of Old Machine:

Year	0	1	2	3	4	5
PAT	0	5,00,000	5,40,000	5,80,000	6,20,000	6,60,000
PVF	1	0.87	0.756	0.658	0.572	0.497
PVCF	0	4,35,000	4,08,240	3,81,640	3,54,640	3,28,020
NPV	**19,07,540**					

Continuing with old machine is most preferred as it gives the highest NPV of **₹19,07,540**.

40. Illustration

A & Co. is contemplating whether to replace an existing machine or to spend money on overhauling it. A & Co. currently pays no taxes. The replacement machine costs ₹ 90,000 now and requires maintenance of ₹ 10,000 at the end of every year for eight years. At the end of eight years it would have a salvage value of ₹ 20,000 and would be sold. The existing machine requires increasing amounts of maintenance each year and its salvage value falls each year as follows:

Year	Maintenance (₹)	Salvage (₹)
Present	0	40,000
1	10,000	25,000
2	20,000	15,000
3	30,000	10,000
4	40,000	0

The opportunity cost of capital for A & Co. is 15%. Required:
When should the company replace the machine?
(Notes: Present value of an annuity of Re. 1 per period for 8 years at interest rate of 15% : 4.4873; present value of Re. 1 to be received after 8 years at interest rate of 15% : 0.3269).

(ICAI SM, Old PM)

Solution:

Year	0	1-8	8
Cash Flow	-90,000 Investment	-10,000 Maintenance	20,000 Salvage Value
PV Factor	1	4.4873	0.3269
Present Value	-90,000	-44,873	6538

Net Present Value = -1,28,335 (excluding old machine scrap)

Equivalent Annual Cost $= -\frac{1,28,335}{4.4873}$ = - 28,599.6 = -28,600

Case: Machine Replaced in Year 0		PVF	PVCF
Cash Flow	40,000	1	40,000
EAC of New Machine	(28,600)	1	(28,600)
			11,400

Case: Machine Replaced in Year 1		PVF	PVCF
Cash Flow	25,000	0.8695	
	(10,000)	0.8695	
	(28,600)	0.8695	
	(13,600)	0.8695	**(11,825.2)**

Case: Machine Replaced in Year 2

Year	Cash Flow	PVF	PVCF
1	(10,000)	0.8695	(8,695)
2	(20,000)	0.7561	(25,405)
2	15,000	0.7561	
2	(28,600)	0.7561	
			(34,100)

Case: Machine Replaced in Year 4

Year	Cash Flow	PVF	PVCF
1	(10,000)	0.8695	(8,695)
2	(20,000)	0.7561	(15,122)
3	(30,000)	0.6575	(19,725)
4	(40,000)	0.5717	(22,868)
4	(28,600)	0.5717	(16,350.62)
			(82,760.62)

Case: Machine Replaced in Year 3

Year	Cash Flow	PVF	PVCF
1	(10,000)	0.8695	(8,695)
2	(20,000)	0.7561	(15,122)
3	(30,000)	0.6575	(19,725)
3	10,000	0.6575	6,575
3	(28,600)	0.6575	(18,804.5)
			(55,771.5)

It is better to **replace in year 0** itself as NPV of replacement cost is the least

41. Illustration

PQ Ltd. expects sales of ₹100 lakhs in the year 1. The same will increase by 20 lakhs per year over the next four years. At the end of 5 years the project would be wound up. Depreciation would be charged at 20% p.a on SLM Basis

The Expenses excluding depreciation, will be 40% of the sales. There will be no salvage value of the plant. PQ Ltd. proposes to invest in the plant an amount where the Net Present Value will be zero.

Corporate Tax rate is 30%.

You are required to calculate the investment which can be made in the plant.

(Nov'24 QP 8 marks)

Solution:

Assume IRR is 10% because, without IRR no computation can be made. IRR can be assumed as 15% or 20% also in which case the answer would be different

Assume Total Investment made is ₹ X Lacs

Year	1	2	3	4	5
Sales	100	120	140	160	180
Dep	X/5	X/5	X/5	X/5	X/5
Exp	-40	-48	-56	-64	-72
PBT	60-X/5	72-X/5	84-X/5	96-X/5	108-X/5
Tax	30% *(60-X/5)	30% *(72-X/5)	30% *(84-X/5)	30% *(96-X/5)	30% *(108-X/5)
PAT	42-(0.7X/5)	50.4- (0.7X/5)	58.8- (0.7X/5)	67.2- (0.7X/5)	75.6- (0.7X/5)
Add Dep	X/5	X/5	X/5	X/5	X/5
CFAT	42+(0.3X/5)	50.4+(0.3X/5)	58.8+(0.3X/5)	67.2+(0.3X/5)	75.6+(0.3X/5)
PVF	0.9090	0.8264	0.7513	0.6380	0.6209
PVCIF	38.18 + 0.9090 (0.3X/5)	41.65 + 0.8264 (0.3X/5)	44.18 + 0.7513 (0.3X/5)	45.90 + 0.6380 (0.3X/5)	46.94+ 0.6209 (0.3X/5)

PVCIF = 216.852 + 3.7907 (0.3X/5)
PVCOF = X
216.852+ 0.2274 X = X
216.852 = X-0.2274X
0.7726X =216.852
X = 216.85/0.773
X = 280.53
Original Investment Value = ₹280.53 Lacs

42. Illustration

BC Ltd. is contemplating on buying a new machine at ₹ 70,00,000 with an additional working capital requirement of ₹ 10,00,000. The machine is expected to have an economic useful life of 5 years, with no salvage value. The company follows straight line method of depreciation and same is accepted for tax purposes. The machine is expected to generate an incremental increase in the before tax cash operating income of ₹ 25,00,000 (in real terms) per year for a period of 5 years. The relevant tax rate is 35%. Inflation is expected to be 6% per year and the firm's cost of capital in real term is 10% per year. Assuming that the working capital requirement will remain unchanged throughout the period, in spite of inflation.

Advise the company whether the machine should be purchased or not. Show your NPV calculation in real term.
Calculation upto two decimal place.
PV Factor at 10% & 6% are as under –

PV factor	1	2	3	4	5
At 10%	0.909	0.826	0.751	0.683	0.621
At 6%	0.943	0.890	0.840	0.792	0.747

(May'25 3(a) – 6 Marks)

Solution:

Method 1

Amount in ₹

Year	0	1	2	3	4	5
Investment (A)	- 7,000,000					
Working Capital Investment (B)	- 1,000,000					
OP CF (Pre Tax) i.e EBITDA (Real) (X)		2,500,000.00	2,500,000.00	2,500,000.00	2,500,000.00	2,500,000.00
Inflation Adjustment(Y)		0.943	0.890	0.840	0.792	0.747
OP CF (Pre Tax) i.e EBITDA (Nominal) (X/Y)		2,651,113.47	2,808,988.76	2,976,190.48	3,156,565.66	3,346,720.21
Depreciation		-1,400,000.00	-1,400,000.00	-1,400,000.00	-1,400,000.00	-1,400,000.00
EBIT (Nominal)		1,251,113.47	1,408,988.76	1,576,190.48	1,756,565.66	1,946,720.21
Tax @35%		-437,889.71	-493,146.07	-551,666.67	-614,797.98	-681,352.07
PAT (Nominal)		813,223.75	915,842.70	1,024,523.81	1,141,767.68	1,265,368.14
Add: Depreciation		1,400,000.00	1,400,000.00	1,400,000.00	1,400,000.00	1,400,000.00
CFAT (Nominal)		2,213,223.75	2,315,842.70	2,424,523.81	2,541,767.68	2,665,368.14
Deflator @ 6%		0.943	0.890	0.84	0.792	0.747
PVF @ 10%		0.909	0.826	0.751	0.683	0.621
PV CFAT (Real)		1,897,146.63	1,702,468.60	1,529,486.60	1,374,933.64	1,236,429.63
PV CFAT (Real) (C)	7,740,465.10					
NPV (excluding Terminal CF) (D = A + B + C)	-2,59,534.90					

Terminal CF – WC Release

WC Amount of ₹10,00,000 is Nominal, then,
WC amount in PV Real terms = ₹10,00,000 x 0.621 x 0.747 = 4,63,887 – (E)
NPV of Project (F) = (D + E) = -₹2,59,534.90 + ₹4,63,887 = ₹2,04,352.10

The project has +ve NPV and hence it should be undertaken

Method 2

Annual CFAT (Excl Dep) + Annual Tax Savings on Depreciation

Details	Amount in ₹
OP CF (Pre Tax) i.e EBITDA (Real)	2,500,000.00
Tax @35%	- 8,75,000.00
CFAT(Excl Dep) (Real) PA	16,25,0000

PV of all CF for period 1-5 of CFAT Excl Dep = 16,25,000 x PVIFA (10%, 5 years = 3.790) = ₹61,58,750 - A

Depreciation PA (Nominal Equivalent) - ₹14,00,000

Tax benefit on Depreciation @ 35% = 14,0,000 x 35% = ₹ 4,90,000

PV of Inflation adjusted Depreciation

Year	Dep Tax benefit (Nominal) A	Conversion factor (Adjusting for Inflation & PV Factor) B	PV in Real terms A*B
1	4,90,000	0.909 x 0.943	4,20,021.63
2	4,90,000	0.826 x 0.890	3,60,218.60
3	4,90,000	0.751 x 0.840	3,09,111.60
4	4,90,000	0.683 x 0.792	2,65,058.64
5	4,90,000	0.621 x 0.747	2,27,304.63
	Total - B		**15,81,715.10**

PV of Outflow – real	-80,00,000.00
PV of CF Inflow – real A	61,58,750.00
PV of tax benefit on Dep B	15,81,715.10
PV of WC release (10,00,000 x 0.621 x 0.747)	4,63,887.00
NPV	2,04,352.10

The project has +ve NPV and hence it should be undertaken

SECURITY ANALYSIS (6Q)

1. Illustration

Closing values of BSE Sensex from 6th to 17th day of the month of January of the year 2020 were as follows.

Days	Date	Day	Sensex
1	6	Thursday	14522
2	7	Friday	14925
3	8	Saturday	No trading
4	9	Sunday	No Trading
5	10	Monday	15222
6	11	Tuesday	16000
7	12	Wednesday	16400
8	13	Thursday	17000
9	14	Friday	No Trading
10	15	Saturday	No Trading
11	16	Sunday	No Trading
12	17	Monday	18000

Calculate EMA of Nifty during the above period. The previous day EMA of Sensex can be assumed as 15,000. The Value of exponent for 31 days EMA is 0.062

(ICAI SM, May'24 QP, Nov '19 QP, May'18 QP, MTP March 18,4 Old PM)

Solution :

Day	Sensex	Pt	EMA of Previous Day	Exponent (2/n+1)	EMA for the Day
6	14522	14522	15000	0.062	(14522*0.062) + (1- 0.062) × 15000 = **14,970**
7	14925	14925	14970	0.062	(14925*0.062) + (1- 0.062) × 14970 = **14,968**
10	15222	15222	14968	0.062	(15222*0.062) + (1- 0.062) × 14968 = **14,983**
11	16000	16000	14,983	0.062	(16000*0.062) + (1- 0.062) × 14,983 = **15,046**
12	16400	16400	15,046	0.062	(16,400*0.062) + (1- 0.062) × 15,046 = **15,130**
13	17000	17000	15,130	0.062	(17000*0.062) + (1- 0.062) × 15,130 = **15,246**
17	18000	18000	15,246	0.062	(18000*0.062) + (1- 0.062) × 15,246 = **15,417**

The SENSEX Values are consistently above the EMA. This indicates a bullish outlook

2. Illustration

Closing values of TCS for the month of January and August 2021 were as follows.

Date	Close Price	Date	Close Price
30-Aug-21	3,701.40	29-Jan-21	3,112.90
27-Aug-21	3,720.15	28-Jan-21	3,199.45
26-Aug-21	3,671.05	27-Jan-21	3,261.05
25-Aug-21	3,659.50	25-Jan-21	3,290.20
24-Aug-21	3,612.20	22-Jan-21	3,303.40
23-Aug-21	3,635.85	21-Jan-21	3,274.80
20-Aug-21	3,557.45	20-Jan-21	3,308.20
18-Aug-21	3,558.50	19-Jan-21	3,265.15
17-Aug-21	3,552.40	18-Jan-21	3,220.80
16-Aug-21	3,471.75	15-Jan-21	3,233.60
13-Aug-21	3,461.90	14-Jan-21	3,250.15
12-Aug-21	3,353.95	13-Jan-21	3,158.80
11-Aug-21	3,340.55	12-Jan-21	3,172.85
10-Aug-21	3,333.25	11-Jan-21	3,175.05
09-Aug-21	3,322.40	08-Jan-21	3,120.35
06-Aug-21	3,310.15	07-Jan-21	3,032.70
05-Aug-21	3,285.15	06-Jan-21	3,052.95
04-Aug-21	3,273.45	05-Jan-21	3,092.30
03-Aug-21	3,284.15	04-Jan-21	3,039.25
02-Aug-21	3,217.90	01-Jan-21	2,928.20

You are required:

To test the Weak Form of Market Efficiency using Auto-Correlation test, assuming the time lag is 211 days.

Solution :

Date	Close Price	X (Change from Previous Day)	X mean	X - X Mean	(X-X Mean)^2
30-Aug-21	3,701.40	(18.75)	25.45	=(18.75)-25.45 = (44.20)	1,953.41
27-Aug-21	3,720.15	49.10	25.45	23.65	559.45
26-Aug-21	3,671.05	11.55	25.45	(13.90)	193.14
25-Aug-21	3,659.50	47.30	25.45	21.85	477.54
24-Aug-21	3,612.20	(23.65)	25.45	(49.10)	2,410.55
23-Aug-21	3,635.85	78.40	25.45	52.95	2,803.98
20-Aug-21	3,557.45	(1.05)	25.45	(26.50)	702.11
18-Aug-21	3,558.50	6.10	25.45	(19.35)	374.32
17-Aug-21	3,552.40	80.65	25.45	55.20	3,047.33
16-Aug-21	3,471.75	9.85	25.45	(15.60)	243.28
13-Aug-21	3,461.90	107.95	25.45	82.50	6,806.68
12-Aug-21	3,353.95	13.40	25.45	(12.05)	145.14
11-Aug-21	3,340.55	7.30	25.45	(18.15)	329.33
10-Aug-21	3,333.25	10.85	25.45	(14.60)	213.08
09-Aug-21	3,322.40	12.25	25.45	(13.20)	174.17
06-Aug-21	3,310.15	25	25.45	(0.45)	0.2
05-Aug-21	3,285.15	11.70	25.45	(13.75)	188.99
04-Aug-21	3,273.45	(10.70)	25.45	(36.15)	1,306.63
03-Aug-21	3,284.15	66.25	25.45	40.80	1,664.85
02-Aug-21	3,217.90				

N			19
Average/Mean	25.45		
Sum			23,594.18
Variance			=23,594.18/19 = 1,241.80
Standard Deviation			$= \sqrt{1241.80}$ = **35.24**

Date	Close Price	X (Change from Prev Day)	X mean	X - X Mean	(X-X Mean)^2	(x-X mean) (y-Y mean)
29-Jan-21	3,112.90	(86.55)	9.72	(96.27	9268.12	4254.93
28-Jan-21	3,199.45	(61.60)	9.72	(71.32)	5,086.69	(1686.93)
27-Jan-21	3,261.05	(29.15)	9.72	(38.87)	1510.96	540.21
25-Jan-21	3,290.20	(13.20)	9.72	(22.92)	525.37	(500.89)
22-Jan-21	3,303.40	28.60	9.72	18.88	356.41	(926.91)
21-Jan-21	3,274.80	(33.40)	9.72	(43.12)	1,859.43	(2283.37)
20-Jan-21	3,308.20	43.05	9.72	33.33	1,110.82	(883.13)
19-Jan-21	3,265.15	44.35	9.72	34.63	1,199.16	(669.98)
18-Jan-21	3,220.80	(12.80)	9.72	(22.52)	507.2	(1243.22)
15-Jan-21	3,233.60	(16.55)	9.72	(26.27)	690.17	409.76
14-Jan-21	3,250.15	91.35	9.72	81.63	6,663.29	6,734.60
13-Jan-21	3,158.80	(14.05)	9.72	(23.77)	565.06	286.38
12-Jan-21	3,172.85	(2.20)	9.72	(11.92)	142.11	216.34
11-Jan-21	3,175.05	54.70	9.72	44.98	2,023.11	(656.57)
08-Jan-21	3,120.35	87.65	9.72	77.93	6,072.92	(1028.46)
07-Jan-21	3,032.70	(20.25)	9.72	(29.97)	898.26	13.41
06-Jan-21	3,052.95	(39.35)	9.72	(49.07)	2,407.97	674.60
05-Jan-21	3,092.30	53.05	9.72	43.33	1,877.40	(1566.23)
04-Jan-21	3,039.25	111.05	9.72	101.33	10,267.56	4,134.49
01-Jan-21	2,928.20					
N		19				
Average / Mean		9.72				
Sum					53,032.00	
Variance					=53,032/19 = 2,791.16	
Standard Deviation					$=\sqrt{2791.16}$ **= 52.83**	

Sum of (x - X Mean) (y-Y Mean)						=5,819.018
Covariance						= 5,819.018/19 Sum of (x-X Mean) (y-Y Mean)/N = 306.26
Correlation						= Cov (X,Y)/ $SD_x \times SD_y$ =306.26/(35.24 × 52.83) = 16.45%

Alternatively, students can take percentage change instead of absolute price changes.

3. Illustration

Mr. X is of the opinion that market has recently shown the Weak Form of Market Efficiency. In order to test the validity of his impression he has collected the following data relating to the movement of the SENSEX for the last 20 days.

Days	Open	High	Low	Close
1	33470.94	33513.79	33438.03	33453.99
2	33453.64	33478.11	33427.82	33434.83
3	33414.06	33440.29	33397.65	33431.99
4	33434.94	33446.18	33377.78	33383.41
5	33372.92	33380.27	33352.12	33370.93
6	33375.85	33389.49	33331.42	33340.75
7	33340.89	33340.89	33310.95	33330.93
8	33326.84	33340.91	33306.17	33335.08
9	33307.16	33328.22	33296.43	33301.97
10	33298.64	33318.60	33254.28	33259.03
11	33260.04	33228.85	33241.66	33251.53
12	33255.92	33289.46	33249.46	33285.89
13	33288.86	33535.67	33255.98	33329.28
14	33335.00	33346.21	33276.72	33284.17
15	33293.83	33310.86	33278.54	33298.78
16	33300.02	33337.79	33300.02	33325.38
17	33323.36	33356.34	33322.44	33329.95
18	33322.81	33345.98	33317.44	33319.67
19	33317.51	33321.18	33294.19	33302.32

20	33290.86	33324.96	33279.62	33319.61

You are required to test the Weak Form of Market Efficiency using Auto-Correlation test, taking time lag of 10 days

(Jan'21 QP)

Solution :

Day	Sensex	Price Change from Previous Day (X)
1	33453.99	
2	33434.83	(19.16)
3	33431.99	(2.84)
4	33383.41	(48.58)
5	33370.93	(12.48)
6	33340.75	(30.18)
7	33330.93	(9.82)
8	33335.08	4.15
9	33301.97	(33.11)
10	33259.03	(42.94)
11	33251.53	(21.66)
12	33285.89	34.36
13	33329.28	43.39
14	33284.17	(45.11)
15	33298.78	14.61
16	33325.38	26.60
17	33329.95	4.57
18	33319.67	(10.28)
19	33302.32	(17.35)
20	33319.61	17.29

Day	Sensex	Price change	Day	Sensex	Price change
		X			Y
1	33,453.99		11	33251.53	
2	33,434.83	(19.16)	12	33285.89	34.36
3	33,431.99	(2.84)	13	33329.28	43.39
4	33,383.41	(48.58)	14	33284.17	(45.11)
5	33,370.93	(12.48)	15	33298.78	14.61
6	33,340.75	(30.18)	16	33325.38	26.60
7	33,330.93	(9.82)	17	33329.95	4.57
8	33,335.08	4.15	18	33319.67	(10.28)
9	33,301.97	(33.11)	19	33302.32	(17.35)
10	33,259.03	(42.94)	20	33319.61	17.29
	X mean	(21.66)			7.56

Days	x - x mean	x - x mean	y - y mean	y - y mean
1	-19.16 - -21.66	2.50	34.36 - 7.56	26.80
2	-2.84 - -21.66	18.82	43.39 - 7.56	35.83
3	-48.58 - -21.66	(26.92)	-45.11 - 7.56	- 52.67
4	-12.48 - -21.66	9.18	14.61 - 7.56	7.05
5	-30.18 - -21.66	(8.52)	26.6 - 7.56	19.04
6	-9.82 - -21.66	11.84	4.57 - 7.56	- 2.99
7	4.15 - -21.66	25.81	-10.28 - 7.56	- 17.84
8	-33.11 - -21.66	(11.45)	-17.35 - 7.56	- 24.91
9	-42.94 - -21.66	(21.28)	17.29 - 7.56	9.73

	X - X mean	$(X - X\ mean)^2$	Y - Y Mean	$(Y - Y\ Mean)^2$	(X - X Mean) (Y - Y Mean)
1	2.5	6.26	26.80	718	67.05
2	18.82	354.28	35.83	1,283.47	674.32
3	(26.92)	724.57	(52.67)	2,774.60	1417.88
4	9.18	84.31	7.05	49.64	64.69
5	(8.52)	72.55	(19.04)	362.35	(162.14)
6	11.84	140.24	(2.99)	8.97	(35.46)
7	25.81	666.27	(17.84)	318.42	(460.60)
8	(11.45)	131.05	(24.91)	620.73	285.22
9	(21.28)	452.74	9.73	94.59	(206.94)
Sum		2,632.27		6,230.77	1644.01
Variance		=2632.27/9 =292.47		=6230.77/9 =692.31	
SD		$=\sqrt{292.47}$ =17.10		$=\sqrt{692.31}$ =26.31	

Co- Variance = Sum of (X - X Mean) × (Y - Y Mean)/n

= 1,644.01/9

= 182.67

Correlation = Cov (X,Y)/ SD_x × SD_y

=182.67/(17.10 × 26.31)

= 40.59%

Given that correlation is 40.59%, the market does not exhibit even weak form of efficiency, it is not at all efficient.

4. Illustration

The closing value of a Stock Market Index for the month of October 2007 is given below

Date	Index
Closing	
1.10.07	2800
3.10.07	2780

4.10.07	2795
5.10.07	2830
8.10.07	2760
9.10.07	2790
10.10.07	2880
11.10.07	2960
12.10.07	2990
15.10.07	3200
16.10.07	3300
17.10.07	3450
19.10.07	3360
22.10.07	3290
23.10.07	3360
24.10.07	3340
25.10.07	3290
29.10.07	3240
30.10.07	3140
31.10.07	3260

You are required to test the weak form of efficient market hypothesis by applying the run test at 5% and 10% level of significance.

Value of t at 5% is 2.101 at 18 degrees of freedom

Value of t at 10% is 1.734 at 18 degrees of freedom.

(ICAI SM, RTP Nov '23, MTP April '21, MTP Oct'18, Old PM)

Solution :

Date	Index Value		Run
1-Oct-07	2800		
03-Oct-07	2780	-	1
04-Oct-07	2795	+	2
05-Oct-07	2830	+	
08-Oct-07	2760	-	3
09-Oct-07	2790	+	4
10-Oct-07	2880	+	
11-Oct-07	2960	+	
12-Oct-07	2990	+	
15-Oct-07	3200	+	
16-Oct-07	3300	+	
17-Oct-07	3450	+	
19-Oct-07	3360	-	5
22-Oct-07	3290	-	
23-Oct-07	3360	+	6
24-Oct-07	3340	-	7
25-Oct-07	3290	-	
29-Oct-07	3240	-	
30-Oct-07	3140	-	
31-Oct-07	3260	+	8

Number of Sign Changes	19
+	11
-	8
Runs	8

	t	Degree of Freedom = Number of Sign Changes – 1	
5%	2.101	18	
10%	1.734	18	

Mean	= 2n1n2/(n1+n2) + 1 = (2 × 11 × 8)/(11+8) + 1 = 10.2632
Standard Deviation	$\sigma = \frac{\sqrt{2n1n2(2n1n2 - n1 - n2)}}{\sqrt{n1 + n2)\char`^2(n1 + n2 - 1))}}$ $\sigma = \frac{\sqrt{2 \times 11 \times 8\,(2 \times 11 \times 8 - 11 - 8)}}{\sqrt{11 + 8)\char`^2(11 + 8 - 1))}}$ $= \frac{\sqrt{(176)\,(157)}}{\sqrt{361)(18))}}$ = 2.06213

Particulars	**5%**	**10%**
t	2.10	1.734
Mean	10.26	10.26
SD	2.06	2.06
R (Number of runs)	8	8
Mean + t × SD	=10.26+ (2.10 × 2.06) = 14.60	=10.26+ (1.734 × 2.06) = 13.84
Mean – t × SD	=10.26 - (2.10 × 2.06) = 5.93	=10.26 - (1.734 × 2.06) = 6.69
R	8	8

As the number of runs lies between the 1 SD Value from mean at t Levels of 5% & 10% , the market exhibits weak form of efficiency.

5. Illustration

The closing value of TCS stock for the month of August 2021 is given below

Date	Close Price
02-Aug-21	3,217.90
03-Aug-21	3,284.15
04-Aug-21	3,273.45
05-Aug-21	3,285.15
06-Aug-21	3,310.15
09-Aug-21	3,322.40
10-Aug-21	3,333.25
11-Aug-21	3,340.55
12-Aug-21	3,353.95
13-Aug-21	3,461.90
16-Aug-21	3,471.75
17-Aug-21	3,552.40
18-Aug-21	3,558.50

20-Aug-21	3,557.45
23-Aug-21	3,635.85
24-Aug-21	3,612.20
25-Aug-21	3,659.50
26-Aug-21	3,671.05
27-Aug-21	3,720.15
30-Aug-21	3,701.40

You are required to test the weak form of efficient market hypothesis by applying the run test at 5% and 10% level of significance.

(Faculty created)

Solution :

Date	Close Price	Sign Changes	Runs
02-Aug-21	3,217.90		
03-Aug-21	3,284.15	+	1
04-Aug-21	3,273.45	-	2
05-Aug-21	3,285.15	+	3
06-Aug-21	3,310.15	+	
09-Aug-21	3,322.40	+	
10-Aug-21	3,333.25	+	
11-Aug-21	3,340.55	+	
12-Aug-21	3,353.95	+	
13-Aug-21	3,461.90	+	
16-Aug-21	3,471.75	+	
17-Aug-21	3,552.40	+	
18-Aug-21	3,558.50	+	
20-Aug-21	3,557.45	-	4
23-Aug-21	3,635.85	+	5
24-Aug-21	3,612.20	-	6
25-Aug-21	3,659.50	+	7
26-Aug-21	3,671.05	+	
27-Aug-21	3,720.15	+	
30-Aug-21	3,701.40	-	8

Number of Sign Changes	19
+	15
-	4
Runs	8

	T	Degree of Freedom = Number of Sign Changes - 1	
5%	2.101	18	
10%	1.734	18	

Mean = 2n1n2/(n1+n2) + 1

= (2 × 15 × 4)/(15+4) + 1

= 7.32

Standard Deviation	$\sigma = \frac{\sqrt{2n1n2(2n1n2 - n1 - n2)}}{\sqrt{n1 + n2)^\wedge 2(n1 + n2 - 1))}}$ $\sigma = \frac{\sqrt{2 \times 15 \times 4\,(2 \times 15 \times 4 - 15 - 4)}}{\sqrt{15 + 4)^\wedge 2(15 + 4 - 1))}}$ $= \frac{\sqrt{120\,(101)}}{\sqrt{361)(18))}}$ = 1.37

Particulars	**5%**	**10%**
t	2.10	1.734
Mean	7.32	7.32
SD	1.37	1.37
R (Number of runs)	8	8
Mean + t × SD	**=7.32+ (2.10 × 1.37) = 10.1851**	**=7.32+ (1.734 × 1.37) = 9.6840**
Mean – t × SD	**=7.32 - (2.10 × 1.37) = 4.44652**	**=7.32+ (1.734 × 1.37) = 4.94754**

Under both 5% and 10% levels of Significance the number of runs I.e., 8 lies within the confidence interval , Hence, the market exhibits atleast a weak form of efficiency.

6. Illustration

You are a financial analyst at a prominent investment firm and have been tasked with empirically verifying the weak form of Efficient Market Hypothesis (EMH) Theory for the XYZ Stock Index, a collection of diverse stocks. You decided to conduct three different tests to assess whether the stock market follows the principles of the weak form of EMH.

Test 1

For the past five years, you collected daily price changes of the stocks in the XYZ Stock Index. You calculated correlation coefficients for different lag periods and analyzed whether past price changes exhibit any significant correlation with future price changes. You considered price changes to be serially independent. The results indicated that most auto correlation coefficients are close to zero and statistically insignificant, suggesting those past price changes do not predict future price changes.

Test 2

You further investigated the randomness of price changes in the XYZ Stock Index. Analyzing the sequence of daily price changes, you count the number of runs where price changes are consistently positive or negative. Upon comparing the observed number of runs with the expected number based on randomness, you find that they align closely, supporting the idea that price changes follow a random pattern.

Test 3

To examine the efficacy of trading strategies based on historical price trends, you implemented a simple trading rule for the XYZ Stock Index. The rule involves buying when the price crosses

a moving average of 5% threshold and selling when it crosses another 7% threshold. Over a period of testing, you computed the returns generated by the trading strategy. The results revealed that the returns are not consistently better than random chance, implying that past price trends do not reliably predict future price movements.

Conclusion:
After conducting the three tests the evidence supports the weak form of Efficient Market Theory for the XYZ Stock Index you concluded that past price trends do not reliably predict future price movements.

(RTP May'24)

Based on the above information answer the following questions:

I. Test 1 is ....

(a) **Serial Correlation test**
(b) Filter Rules test
(c) Run test
(d) Variance Ratio test

II. Test 2 is ....

(a) Serial Correlation test
(b) Filter Rules test
(c) **Run test**
(d) Variance Ratio test

III. Test 3 is ....
(a) Serial Correlation test
(b) **Filter Rules test**
(c) Run test
(d) Variance Ratio test.

IV. The Filter Rule Test should not be applied for buy and hold strategy if...............

(a) the behavior of stock price changes is predictable.
(b) the behavior of stock price changes is dependent on past trends.
(c) the behavior of stock price changes is correlated.
(d) **the behavior of stock price changes is random.**
If Stock prices are random then filter rules will not work.

V. Results of your studies support the..............

(a) Semi-strong EMH Theory
(b) Strong EMH Theory
(c) **Random Walk Theory**
(d) Markowitz Theory

1. **Illustration**

A company has a book value per share of Rs. 137.80. Its return on equity is 15% and it follows a policy of retaining 60% of its earnings. If the Opportunity Cost of Capital is 18%, compute the price of the share today using both Dividend Growth Model and Walter's Model.

(ICAI SM, Old PM)

Solution 1:

Given:

Book value per share = 137.80

ROE = 15%

Retention Ratio = 60%

Payout Ratio (p) = 1-60% = 40%

Opportunity Cost (K_e) = 18%

Dividend Growth Model:

$P_0 = \frac{D_1}{K_e - g}$

g = b x r = Retention x Return on equity

D = *EPS (1-b) = EPS x p*

Where, EPS = $\frac{PAT}{Share\ Count}$ & ROE = $\frac{PAT}{Equity} = \frac{EPS}{BVPS}$

i.e; 15% = $\frac{EPS}{137.80}$ => EPS = ₹137.80 x 15% = **₹20.67 per share**

Assumed that Data of ROE & BVPS is past data and hence EPS is for the year 0.

$E_0 = 20.67$

P = 40%

$D_0 = 20.67 * 40\% = 8.268$

G = b x r = 60% x 15% = 9%

$D_1 = D_0(1+g)$ = 8.268(1 + 9%) = 9.01212

$$\boldsymbol{P_0 = \frac{9.01212}{18\% - 9\%} = 100.135}$$

Walter's Model:

$P_0 = \frac{D + (E-D) * \frac{r}{K_e}}{K_e}$

$= \frac{8.268 + (20.67 - 8.268) * \frac{15\%}{18\%}}{18\%}$ = **₹ 103.35**

Where,

P_0 = Price per share

D_0 = Dividend per Share for the just concluded year

E = Earnings per share

r = Return on equity

K_e – Cost of Equity

2. Illustration

ABC Limited's shares are currently selling at Rs. 13 per share. There are 10,00,000 shares outstanding. The firm is planning to raise Rs. 20 lakhs to Finance a new project.

Required:

- **What are the ex-right price of shares and the value of a right, if**
  - **The firm offers one right share for every two shares held.**
  - **The firm offers one right share for every four shares held.**
- **How does the shareholders' wealth (holding 100 shares) change from (i) to (ii)? How does right issue increase shareholders' wealth?**

(ICAI SM, RTP Nov'18, Old PM)

Solution 2

Given

Company Name : ABC Ltd.
Current Market price of equity Share is ₹13
Existing share count 10,00,000
New Project = ₹20,00,000

(i) 1 Right share for 2 shares held

∴ No. of rights shares issued = 10,00,000/2 = 5,00,000 shares
Hence, Project of ₹20,00,000 will be financed by issuing 5,00,000 shares

Price per right share = $\frac{Money\ raised}{No.of\ shares\ issued}$ = $\frac{₹20,00,000}{5,00,000}$ = ₹4 per share

So, Ex Right Price of share = $\frac{N_0P_0 + S}{N_0 + N_1}$ = $\frac{(10,00,000*13)+20,00,000}{10,00,000+5,00,000}$ = ₹10 per share

N = Number of existing equity shares
P_0 = Price of Share Pre-Right Issue
S = Subscription amount raised from Right Issue
N_1 = No. of new shares offered

Value of Right = ₹10 per share - ₹4 per share = ₹6 per share

Value of Right on per share basis = $\frac{₹10-₹4}{2}$ = ₹3

(ii) 1 Right share for 4 shares held

Total Existing share count 10,00,000
∴ No. of rights shares issued = 10,00,000/4 = 2,50,000 shares

Price per right share = $\frac{Money\ raised}{No.of\ shares\ issued}$ = $\frac{₹20,00,000}{2,50,000}$ = ₹8 per share

Ex Right Price of share = $\frac{N_0P_0 + S}{N_0 + N_1}$ = $\frac{(10,00,000*13)+20,00,000}{10,00,000+2,50,000}$ = ₹12 per share

Value of Right = ₹12 per share - ₹8 per share = ₹4per share

Value of Right on per share basis = $\frac{₹12-₹8}{4}$ = **₹1**

Shareholder's wealth pre issue =

Shares held	100
CMP	₹13
Total shareholder's wealth	₹1300

Shareholder's wealth in (i)

Shares held	100
New right shares	50
Total shares	150
Ex right price	₹10
Total Value of shareholding	₹1500
Less: Subscription amt paid (Price per right share * No. of shares subscribed)	₹200 i.e (50*4)
Shareholder's wealth	₹1300

Shareholder's wealth in (ii)

Shares held	100
New right shares	25
Total shares	125
Ex right price	₹12
Total Value of shareholding	₹1500
Less: Subscription amt paid (Price per right share * No. of shares subscribed)	₹200 (25*8)
Shareholder's wealth	₹1300

Conclusion: There is no change in shareholder's wealth due to rights issue in both the scenarios.

3. Illustration

MNP Ltd. has declared and paid annual dividend of Rs. 4 per share. It is expected to grow @ 20% for the next two years and 10% thereafter. The required rate of return of equity investors is 15%. Compute the current price at which equity shares should sell.

Note: Present Value Interest Factor (PVIF) @ 15%:

- For year 1 = 0.8696.
- For year 2 = 0.7561

(ICAI SM, Old PM)

Solution 3

Given,

D_0 = 4

$D_1 = D_0$ (1+g) = 4 x 1.2 = 4.8
$D_2 = D_1$ (1+g)= 4 x $(1.2)^2$ = 5.76
$D_3 = D_2$ (1+g) = 5.76 x 1.10 = 6.336
K_e = 15%

Yr	1	2	2
P_0	4.8	5.76	126.72
PVIF	0.8696	0.7561	0.7561

$\therefore P_0$ = 4.17408 + 100.168 = ₹104.34

4) Illustration

X Limited, just declared a dividend of Rs. 14.00 per share. Mr. B is planning to purchase the share of X Limited, anticipating increase in growth rate from 8% to 9%, which will continue for three years. He also expects the market price of this share to be Rs. 360.00 after three years.

You are required to determine:

i. The maximum amount Mr. B should pay for shares if he requires a rate of return of 13% per annum.

ii. The maximum price Mr. B will be willing to pay for share, if he is of the opinion that the 9% growth can be maintained indefinitely and require 13% rate of return per annum.

iii. The price of share at the end of three years, if 9% growth rate is achieved and assuming other conditions remaining same as in (ii) above.

Calculate rupee amount up to two decimal points.

	Year 1	Year 2	Year 3
FVIF @ 9%	1.090	1.188	1.295
FVIF @ 13%	1.130	1.277	1.443
PVIF @ 13%	0.885	0.783	0.693

(ICAI SM, MTP May'20, RTP Nov'18 Old, MTP Apr'18 Old, Old PM)

Solution 4

(i) Expected dividend for next 3 years.

Year 1 (D_1) 14.00 (1.09) = 15.26
Year 2 (D_2) 14.00 (1.09)2 = 16.63
Year 3 (D_3) 14.00 (1.09)3 = 18.13

Required rate of return = 13% (K_e) & Market price of share after 3 years = (P_3)= 360

The present value of share

$$P_0 = \frac{D_1}{1+K_e} + \frac{D_2}{(1+K_e)^2} + \frac{D_3}{(1+K_e)^3} + \frac{P_3}{(1+K_e)^3}$$

$$= \frac{15.26}{(1+0.13)} + \frac{16.63}{(1+0.13)^2} + \frac{18.13}{(1+0.13)^3} + \frac{360}{(1+0.13)^3}$$

P_0 = 15.26(0.885) + 16.63(0.783) +18.13(0.693)+360(0.693)

= 13.50 + 13.02 + 12.56 + 249.48

P_0 = 288.56

(ii) If growth rate 9% is achieved for indefinite period, then maximum price of share
should Mr.B willing be to pay is

$$P_0 = \frac{D_1}{K_e - g} = \frac{15.26}{0.13-0.09} = ₹381.50$$

(iii) Assuming that conditions mentioned above remain same, the price expected after 3
years will be:

$$P_3 = \frac{D_4}{K_e - g} = \frac{D_3(1.09)}{0.13-0.09} = \frac{18.13 \text{ x } 1.09}{0.04} = ₹494$$

5) Illustration

On the basis of the following information:

Current dividend (D_0)	Rs. 2.50
Discount Rate (k)	10.5%
Growth Rate	2%

i. Calculate the present value of stock of ABC Ltd.

ii. Is its stock overvalued if stock price is Rs.35, ROE= 9% and EPS = Rs. 2.25? Show detailed calculation. Using PE Multiple Approach and Earning Growth Model.

(ICAI SM, May-25 Similar 6M, MTP Mar'24, Old PM)

Solution 5

(i) Present Value of the stock of ABC Ltd.is:

$$P_0 = \frac{D_1}{K_e - g} = \frac{D_0(1+g)}{0.105-0.02} = \frac{2.5*1.02}{0.085} = ₹30/-$$

(ii) Value of stock under the PE Multiple Approach

Particulars	Value
Actual Stock Price	₹35.00
EPS	₹2.25
PE Multiple (1/ ROE)= 1/9%	11.11
Market Price per Share	₹25

Since, Actual Stock Price is higher, hence it is overvalued

(iii) Value of the Stock under the Earnings Growth Model

Actual Stock Price	₹35
Return on equity	9%
EPS	2.25
Growth Rate	2%
Market Price per Share [EPS ×(1+g)]/(ROE – g)] = 2.25 × 1.02/0.07	₹32.79

Since, Actual Stock Price is higher, hence it is overvalued. *(However, there is known rationale other than ICAI SM where PE = 1/ ROE & P_0 = EPS_1/ (ROE-g))*

6) Illustration

Piyush Loonker and Associates presently pay a dividend of Rs. 1.00 per share and has a share price of Rs. 20.00.

i. If this dividend were expected to grow at a rate of 12% per annum forever, what is the firm's expected or required return on equity using a dividend-discount model approach?

ii. Instead of this situation in part (i), suppose that the dividends were expected to grow at a rate of 20% per annum for 5 years and 10% per year thereafter. Now what is the firm's expected, or required, return on equity?

(ICAI SM, RTP Nov'18, Old PM)

Solution 6

(i) Firm's Expected or Required Return on Equity (Using a dividend discount model approach)

According to Dividend discount model approach the firm's expected or required return on equity is computed as follows:

$$P_0 = \frac{D_1}{K_e - g} \quad or \quad K_e = \frac{D_1}{P_0} + g$$

Where,

K_e= Cost of equity share capital or (Firm's expected or required return on equity share capital)

D_1= Expected dividend at the end of year 1

P_0= Current market price of the share.

g = Expected growth rate of dividend.

Now, D_1 = D_0 (1 + g) or 1 (1 + 0.12) or 1.12, P0 = 20 and g = 12% per annum

Therefore, $K_e = \frac{₹1.12}{₹20}$ + 12% = 17.6%

(ii) Firm's Expected or Required Return on Equity

(If dividends were expected to grow at a rate of 20% per annum for 5 years and 10% per year thereafter) i.e. $P_0 = 20 \quad g_1 = 20\% \quad g_2 = 10\%\ D_0 = 1\ \ K_e = ?$

Assume $K_e = 18\%$ and see if it leads us to $P_0 = 20$

Year	Growth	Dividend	Dividend(₹)	PVF@18%	PV
0	-	1			
1	20%	(1.2)	1.2	0.847	1.0164
2	20%	$(1.2)^2$	1.44	0.718	1.03392
3	20%	$(1.2)^3$	1.728	0.609	1.0505
4	20%	$(1.2)^4$	2.074	0.516	1.0701
5	20%	$(1.2)^5$	2.488	0.437	1.087256
					∑= 5.258
6	10%	$(1.2^5)(1.1)$	2.737		

$$P_0 = \frac{D_1}{1+K_e} + \frac{D_2}{(1+K_e)^2} + \frac{D_3}{(1+K_e)^3} + \frac{D_4}{(1+K_e)^4} + \frac{D_5}{(1+K_e)^5} + \frac{P_5}{(1+K_e)^5}$$

$P_5 = \frac{D_6}{K_e - g} = \frac{2.737}{18\% - 10\%}$ = **₹34.2125**

$P_0 = 5.258 + \text{PV of } 34.2125$ = 5.258 + 0.437(34.2125) = **₹20.209**

Computation using $K_e = 19\%$, g = 10%

Year	Dividend(₹)	PVF@19%	PV
1	1.2	0.840	1.008
2	1.44	0.706	1.01664
3	1.728	0.593	1.0247
4	2.074	0.499	1.0349
5	2.488	0.419	1.0424
			∑= 5.127
6	2.737		

P_0 = 5.127 + $\frac{P_5}{(1+K_e)^5}$

P_5= $\frac{D_6}{K_e - g}$ = $\frac{2.737}{19\% - 10\%}$ = ₹30.411

$P_0 = 5.127 + PV\ of\ 30.411$ = 5.127 + 0.419(30.411) = **₹17.869**

Since the present value of dividend stream is more than required it indicates that K_e is greater than 18%.

	A	B	C	(B-C)	(A-B)
P_0	20	20.209	17.869	2.34	0.209
K_e	?	18%	19%	1%	? (1%*0.209//2.34)= 0.089%

K_e = 18% + 0.089% = **18.089%**

Alternatively,

$K_e = \ LR + \frac{NPV\ at\ LR - NPV\ Required}{NPV\ at\ LR - NPV\ at\ HR}$ x Δr

Where,

LR = Lower Rate

NPV at LR = Present value of share at LR

NPV at HR = Present value of share at Higher Rate

Δr = Difference in rates

K_e= 18% + $\frac{(₹20.209 - ₹20)}{₹20.209 - ₹17.869}$ x 1% = **18.089%**

7) Illustration

ABC Ltd. has been maintaining a growth rate of 10 percent in dividends. The company has paid dividend @ Rs. 3 per share. The rate of return on market portfolio is 12 percent and the risk-free rate of return in the market has been observed as 8 percent. The Beta co-efficient of company's share is 1.5.

You are required to calculate the expected rate of return on company's shares as per CAPM model and equilibrium price per share by dividend growth model.

(ICAI SM, MTP Aug'18 Old, Old PM)

Solution 7

CAPM formula for calculation of Expected Rate of Return is :

ER = R_f+ β (R_m – R_f) = 8 + 1.5 (12 – 8) = 8 + 1.5 (4)

= 8 + 6

=14% or 0.14

Applying Dividend Growth Model for the calculation of per share equilibrium price:

ER = $K_e = \frac{D_1}{P_0} + g$

0.14 = $\frac{3(1.10)}{P_0}$ + 0.10

0.04P_0 = 3.30

P_0= **₹82.50**

∴ Per share equilibrium price will be 82.50

8) Illustration

A Company pays a dividend of Rs. 2.00 per share with a growth rate of 7%. The risk-free rate is 9% and the market rate of return is 13%. The Company has a beta factor of 1.50. However, due to a decision of the Finance Manager, beta is likely to increase to 1.75. Find out the present as well as the likely value of the share after the decision.

(Old PM)

Solution 8

In order to find out the value of a share with constant growth model, the value of K_e should be ascertained with the help of 'CAPM' model as follows:

$K_e = R_f + \beta\,(R_m - R_f)$

Where, K_e = Cost of equity R_f = Risk free rate of return β = Portfolio Beta i.e. market sensitivity index K_m = Expected return on market portfolio

By substituting the figures,
K_e = 0.09 + 1.5 (0.13 – 0.09) = 0.15 or 15% and the value of the share as per constant growth model is –

$P_0 = \frac{D_1}{K_e - g} = \frac{2(1.07)}{0.15-0.07}$ = **₹26.75**

However, if the decision of finance manager is implemented, the beta (β) factor is likely to increase to 1.75 therefore,

$K_e = R_f + \beta\,(R_m - R_f)$
= 0.09 + 1.75 (0.13 – 0.09) = 0.16 or 16%

The value of share is $P_0 = \frac{D_1}{K_e - g} = \frac{2(1.07)}{0.16-0.07}$ = **₹23.78**

∴ At β of 1.5, price is ₹26.75 & as β increases to 1.75 price falls to ₹23.77

9) Illustration

Shares of Voyage Ltd. are being quoted at a price-earnings ratio of 8 times. The company retains Rs. 5 per share which is 45% of its Earning Per Share.

You are required to compute

- **The cost of equity to the company if the market expects a growth rate of 15% p.a.**
    - **If the anticipated growth rate is 16% per annum, calculate cost of capital with same market price.**
    - **If the company's cost of capital is 20% p.a. & the anticipated growth rate is 19% p.a., calculate the market price per share.**
- **At anticipated growth rate in (ii), and cost of capital calculated above, compute indicative market price**

(Similar MTP Apr'24, Old PM)

Solution 9

Given,

PE = 8 times

Retained earnings = 5

45% of EPS = b

b x EPS = 5

45% x EPS = 5

∴ EPS = ₹11.11

(i) K_e =? if g= 15% where, $\frac{MPS}{EPS} = PE\ Ratio$

so, $\frac{MPS}{11.11}$ = 8 then, MPS = ₹88.88; MP = $\frac{D_1}{K_e - g}$

D_1 = EPS x P = EPS(1-b) = 11.11(1 - 45%) = **6.11**

88.88 = $\frac{6.11}{K_e - 15\%}$

K_e= 15% + 6.87% = 21.87%

(ii) if g = 16% then, $K_e = ?$

88.88 = $\frac{6.11}{K_e - 16\%}$ so, K_e = 22.87%

(iii) K_e = 20% & g=19% MP = $\frac{D_1}{K_e - g}$ = $\frac{6.11}{20\% - 19\%}$ = **₹611 per share**

(iv) MP @ 16%=g & K_e=22.87% MP = $\frac{D_1}{K_e - g}$

$\frac{6.11}{22.87\% - 16\%}$= **₹88.93 per share**

10) Illustration

Capital structure of Sun Ltd., as at 31.3.2003 was as under:

(Rs. In Lakhs)

Equity share capital (FV 100)	80
8% Preference share capital	40
12% Debentures	64
Reserves	32

Sun Ltd. earns a profit of Rs. 32 lakhs annually on an average before deduction of income-tax, which works out to 35%, and interest on debentures.

Normal return on equity shares of companies similarly placed is 9.6% provided:

a. Profit after tax covers fixed interest and fixed dividends at least 3 times.
b. Capital gearing ratio is 0.75
c. Yield on share is calculated at 50% of profits distributed and at 5% on undistributed profits

Sun Ltd. has been regularly paying equity dividend of 8%.

Compute the value per equity share of the company assuming:

i. 1% for every one time of difference for Interest and Fixed Dividend Coverage.

ii. 2% for every one time of difference for Capital Gearing Ratio.

(ICAI SM, MTP Oct'18, Old PM)

Solution 10

Working note 1: Calculation of Profit after tax (PAT)

Profit before interest and tax (PBIT)	32,00,000
Less: Debenture interest (64,00,000 × 12/100)	7,68,000
Profit before tax (PBT)	24,32,000
Less: Tax @ 35%	8,51,200
PAT Profit after tax (PAT)	**15,80,800**
Less: Preference Dividend(40,00,000 × 8/100)	3,20,000
Equity Dividend (80,00,000 × 8/100)	6,40,000
Retained Earnings (Undistributed Profit)	**6,20,800**

(i) Calculation of Interest and Fixed Dividend Coverage

$$\frac{\text{PAT}}{\text{Debenture interest+ Preference dividend}} = \frac{15{,}80{,}800}{7{,}68{,}000 + 3{,}20{,}000} = 1.453$$

(iii) Capital Gearing Ratio

$$= \frac{\textit{Fixed interest bearing funds}}{\textit{Equity shareholders funds}}$$

$$= \frac{\textit{Preference Share Capital + Debentures}}{\textit{Equity Share Capital + Reserves}}$$

$$= \frac{40{,}00{,}000+64{,}00{,}000}{80{,}00{,}000 + 32{,}00{,}000} = \mathbf{0.93}$$

(iv) Calculation of Yield on Equity Shares: Yield on equity shares is calculated at 50% of profits distributed and 5% on undistributed profits:

50% on distributed profits (6,40,000 × 50/100)	3,20,000
5% on undistributed profits (6,20,800 × 5/100)	31,040
Yield on equity shares	3,51,040

Yield on equity shares % = $\frac{\text{Yield on shares}}{\text{Equity share capital}}$ *100 = $\frac{3{,}51.040}{80{,}00{,}000}$ *100 = **4.388%**

(v) Calculation of Expected Yield on Equity shares

(a) Interest and fixed dividend coverage of Sun Ltd. is 2.16 times but the industry average is 3 times. Therefore, risk premium is added to Sun Ltd. Shares @ 1% for every 1 time of difference.

Risk Premium = 3.00 – 1.453 (1%) = (1%) = **1.547%**

(b) Capital Gearing ratio of Sun Ltd. is 0.93 but the industry average is 0.75 times. Therefore, risk premium is added to Sun Ltd. shares @ 2% for every 1 time of difference.
Risk Premium = (0.75 – 0.93) (2%) = 0.18 (2%) = **0.36%**

Particulars	%
Normal return expected	9.60
Add: Risk premium for low interest and fixed dividend coverage	1.547
Add: Risk premium for high interest gearing ratio	0.36
	11.507%

Value of Equity Share

Actual yield Expected yield × Paid-up value of share = 4.39/11.507 × 100 = **₹38.15**

Note: Alternate solution as per ICAI

Consider PAT as computed above

(i) Calculation of Interest and Fixed Dividend Coverage

$$\frac{\text{PAT+ Debenture interest}}{\text{Debenture interest+ Preference dividend}} = \frac{15,80,800+7,68,000}{7,68,000+3,20,000} = \mathbf{2.16}$$

(ii) Capital Gearing Ratio

$$= \frac{\textit{Fixed interest bearing funds}}{\textit{Equity shareholders funds}}$$

$$= \frac{\textit{Preference Share Capital + Debentures}}{\textit{Equity Share Capital + Reserves}}$$

$$= \frac{40,00,000+64,00,000}{80,00,000+32,00,000} = \mathbf{0.93}$$

(iii) Calculation of Yield on Equity Shares: Yield on equity shares is calculated at 50% of profits distributed and 5% on undistributed profits:

Particulars	Amount(₹)
50% on distributed profits (6,40,000 × 50/100)	3,20,000
5% on undistributed profits (6,20,800 × 5/100)	(+)31,040
Yield on equity shares	3,51,040

Yield on equity shares % = $\frac{\text{Yield on shares}}{\text{Equity share capital}}$ *100 = $\frac{3,51.040}{80,00,000}$ *100 = **4.388% or 4.39%**

(iv) Calculation of Expected Yield on Equity shares (a) Interest and fixed dividend coverage of Sun Ltd. is 2.16 times but the industry average is 3 `s. Therefore, risk premium is added to Sun Ltd. Shares @ 1% for every 1 time of difference.

Risk Premium = 3.00 – 2.16 (1%) = (1%) = **0.84%**

Capital Gearing ratio of Sun Ltd. is 0.93 but the industry average is 0.75 times. Therefore, risk premium is added to Sun Ltd. shares @ 2% for every 1 time of difference.

Risk Premium = (0.75 – 0.93) (2%) = 0.18 (2%) = **0.36%**

Particulars	%
Normal return expected	9.60
Add: Risk premium for low interest and fixed dividend coverage	0.84
Add: Risk premium for high interest gearing ratio	0.36
	10.80%

Value of Equity Share

Actual yield Expected yield × Paid-up value of share = 4.39/10.80 × 100 = **₹40.65**

11) Illustration

Following Financial data are available for PQR Ltd. for the year 2008:

	(Rs. In Lakhs)
8% debentures	125
10% bonds (2007)	50
Equity shares (Rs. 10 each)	100
Reserves and Surplus	300
Total Assets	600
Assets Turnovers ratio	1.1
Effective interest rate	8%
Effective tax rate	40%
Operating margin	10%
Dividend payout ratio	16.67%
Current market Price of Share	Rs. 14
Required rate of return of investors	15%

You are required to:

i. Draw income statement for the year

ii. Calculate its sustainable growth rate of earnings

iii. Calculate the fair price of the Company's share using dividend discount model, and

iv. What is your opinion on investment in the company's share at current price?

(ICAI SM, July'21 QP, May'19 QP 8 marks, RTP May'20 Old, Old PM)

Solution 11

Working Note 1:

Asset turnover ratio	1.1
Total Assets	600 lakhs
Turnover (600 lakhs × 1.1)	660 lakhs
Effective interest rate	8%
Liabilities (125 lakhs + 50 lakhs)	175 lakhs
Interest	= 175 lakhs × 0.08 = 14 lakh

Working Note 2:

Operating Margin = 10% i.e. $\frac{Operating\ Profit}{Turnover} = 10\%$

$$\frac{Operating\ Profit}{660\ lakhs} = 10\%$$

So, EBIT/Operating profit = 660 lakhs x 10% = 66 lakhs

(i) **Income statement**

Particulars	(₹ Lakhs)
Sale	660
EBIT	66
(-) Interest	14
= EBT	52
(-) Tax @ 40%	20.80
= EAT	31.20
Dividend @ 16.67%	5.20
Retained Earnings	**26**

(ii) Computation of Growth rate: **g = b x r**

$r = \frac{PAT}{Net\ Worth} = \frac{31.20}{ESC+Reserves} = \frac{31.20}{100+300} = 7.8\%$

b = 1 – payout ratio = 1 – 16.67%= 83.35%

g = 83.33% x 7.8% = **6.5%**

(iii) Calculation of fair price of share using dividend discount model

$P_0 = \frac{D_1}{K_e - g} = \frac{D_0(1+g)}{0.15-0.065} = \frac{0.52*1.065}{0.085} =$ **₹6.52/-**

Where, g= 6.5% , k = 15%

No. of shares = 100 lakhs/₹10 = 10 lakhs & $D_0 = \frac{₹5.2\ lakhs}{10\ lakhs}$ = **₹0.52**

(iv) DDM Price = ₹6.52 and CMP = ₹14

Since the current market price of share is 14, the share is **overvalued**. Hence the investor should not invest in the company.

12) Illustration

M/s X Ltd. has paid a dividend of Rs. 2.50 per share on a face value of Rs. 10 in the financial year ending on 31st March 2009. The details are as follows:

Current market price of share	Rs. 60
Growth rate of earnings and dividends	10%
Beta of share	0.75
Average market return	15%
Risk free rate of return	9%

Calculate the intrinsic value of the share . *(Old PM)*

Solution 12

Intrinsic value $D_0 = \frac{D_1}{K_e - g}$ & ER = R_f+ β (R_m - R_f)

Where,

R_f= Risk Free Rate   β = Beta of Security R_m = Market Return

ER (K_e) = 9% + 0.75 (15% - 9%) = **13.5%**

$\therefore P_0 = \frac{2.5*1.1}{0.135-0.10}$ = **₹78.57**

13) Illustration

Mr. A is thinking of buying shares at Rs. 500 each having face value of Rs. 100. He is expecting a bonus at the ratio of 1: 5 during the fourth year. Annual expected dividend is 20% and the same rate is expected to be maintained on the expanded capital base.

He intends to sell the shares at the end of seventh year at an expected price of Rs. 900 each. Incidental expenses for purchase and sale of shares are estimated to be 5% of the market price. He expects a minimum return of 12% per annum.

Should Mr. A buy the share. If so, what maximum price should he pay for each share? Assume no tax on dividend income and capital gain.

(ICAI SM, RTP Nov'19, Similar MTP Sept'22, MTP Apr'19, Old PM)

Solution 13

Year	Bonus	Dividend/Cash flow	PVF (12%)	Present Value
0	1	-525 (500 + 5%)	1	-525
		Total Outflow		**Σ = -525**
1	1	₹20	0.893	17.86
2	1	₹20	0.797	15.94
3	1	₹20	0.712	14.24
4	1.2	₹24	0.636	15.264
5	1.2	₹24	0.567	13.608
6	1.2	₹24	0.507	12.168
7	1.2	₹24	0.452	10.848
7	1.2	900 x 1.2 x 95% = 1026	0.452	463.752
		Total Inflow		Σ =563.68
		Net gain (Outflow – Inflow)		**38.68**

Since Mr. A is gaining 38.68 per share, he should buy the share.

Maximum price Mr. A should be ready to pay is 563.68 which will include incidental expenses. So the maximum price should be 563.68 x 100/105 = **536.84 (purchase price before expense)**

***Assumptions :**

1. Mr.A purchases only 1 share
2 Bonus of 1:5 is pre-dividend i.e. dividend is paid on bonus shares also
3 Bonus share 1:5 for every share will get 0.2 shares as bonus. So, revised shares is 1.2
4. Revised dividend in Yr 4 is Rev shares x FV x Dividend % = 1.2 x 100 x 20% =₹24

14) Illustration

SAM Ltd. has just paid a dividend of Rs. 2 per share and it is expected to grow @ 6% p.a. After paying dividend, the Board declared to take up a project by retaining the next three annual dividends. It is expected that this project is of same risk as the existing projects. The results of this project will start coming from the 4th year onward from now. The dividends will then be Rs. 2.50 per share and will grow @ 7% p.a.

An investor has 1,000 shares in SAM Ltd. and wants a receipt of at least Rs. 2,000 p.a. from this investment.

Show that the market value of the share is affected by the decision of the Board. Also show as to how the investor can maintain his target receipt from the investment for first 3 years and improved income, thereafter, given that the cost of capital of the firm is 8%.

(ICAI SM, RTP May'18 New & Old, MTP Oct'22, MTP Mar'22, MTP Oct'20, MTP Mar'18, Old PM)

Solution 14

Given, D_0 = 2 $D_1 - D_3$= 0 D_4 = 2.5

g_1 = 6% g_2 = 7% from D_4 K_e= 8%

(i) Market Value of share is affected by the board's decision Yes or No?

Scenario 1 : Project not undertaken

$P_0 = \frac{D_1}{K_e - g} = \frac{2(1+6\%)}{8\%-6\%}$ = **₹106**

Scenario 2 : Price at end of $P_3 = \frac{D_4}{K_e - g_2} = \frac{2.5}{8\%-7\%}$ = **₹250**

$P_0 = \frac{P_3}{(1.08)^3} = \frac{250}{1.257}$ = **₹198.46**

Yes, because of the project the share price will go up.

(ii) Investor holds 1000 shares - Year wise price of the share

Year	FVF@8%	Price(₹)
0	1	198.46
1	1.08	214.33 (1.08 x 198.46)
2	1.166	231.48
3	1.259	250.00

No. Of shares to be sold to get ₹2000 each year

Year	***Cash required***	**Share price per share**	**Shares he can sell**
1	₹2000	214.33	2000/214.33 = 9.33 i.e. 10 shares
2	₹2000	231.48	2000/231.48 = 8.64 i.e. 9 shares
3	₹2000	250	8 shares
	Total shares sold		**∑ = 27 shares**

Shares left = 1000 - 27 = 973 shares

So, the dividend on 973 shares in Yr 4 = 973 x 2.5 = ₹2432.5

The share holder needs to sell 27 shares in 3 yrs to get minimum of ₹2000 p.a and In those 3 years balance shares will be earning dividend greater than ₹2000 thereafter.

15) Illustration

XYZ Ltd. paid a dividend of Rs. 2 for the current year. The dividend is expected to grow at 40% for the next 5 years and at 15% per annum thereafter. The return on 182 days T-bills is 11% per annum and the market return is expected to be around 18% with a variance of 24%.

The co-variance of XYZ's return with that of the market is 30%. You are required to calculate the required rate of return and intrinsic value of the stock.

(Old PM)

Solution 15

Given,

D_0 = 2 $g_{1-}g_5$ = 40% thereafter g =15% R_f = 11% R_m = 18% Covariance = 30% Variance =24%

β =? K_e = ? P_0 = ?

β = $\frac{\text{Covariance of Market Return and Security Return}}{\text{Variance of Market Return}} = \frac{30\%}{24\%}$ = 1.25 &

K_e= R_f+ β (R_m – R_f) = 11% + 1.25 (18% - 11%) = **19.75%**

Intrinsic value of the stock = $P_0 = \frac{D_1}{1+K_e} + \frac{D_2}{(1+K_e)^2} + \frac{D_3}{(1+K_e)^3} + \frac{D_4}{(1+K_e)^4} + \frac{D_5}{(1+K_e)^5} + \frac{P_5}{(1+K_e)^5}$

Year	D(1+g)	Dividend	PVIF@19.75%	PFCF
1	2(1.4)	2.8	0.835	2.338
2	$2(1.4)^2$	3.92	0.697	2.732
3	$2(1.4)^3$	5.488	0.582	3.194
4	$2(1.4)^4$	7.638	0.486	3.712
5	$2(1.4)^5$	10.76	0.406	4.368
5		*260.505	0.406	105.76
				₹122.129

$* P_5 = \frac{D_6}{K_e - g} = \frac{10.76(1+0.15)}{19.75\% - 15\%}$ = ₹260.505

16) Illustration

Rahul Ltd. has surplus cash of Rs. 100 lakhs and wants to distribute 27% of it to the shareholders. The company decides to buy back shares. The Finance Manager of the company estimates that its share price after re-purchase is likely to be 10% above the buyback price-if the buyback route is taken. The number of shares outstanding at present is 10 lakhs and the current EPS is Rs. 3.

You are required to determine:

- **The price at which the shares can be re-purchased if the market capitalization of the company should be Rs. 210 lakhs after buyback,**
- **The number of shares that can be re-purchased, and**
- **The impact of share re-purchase on the EPS, assuming that net income is the same.**

(ICAI SM, RTP Nov'22, MTP Sept'22, Old PM)

Solution 16

(i) Given,

Surplus Cash = 100 lakhs

Buy back value ₹27 lakhs

Let Buy back price = P

Post buy back price = 1.1P

Current shares = 10 lakhs

Shares re-purchased = $\frac{₹27 lakhs}{P}$

Outstanding shares post buy back = 10 lakhs - $\frac{₹27 lakhs}{P}$

Market capitalisation = 210 lakhs

Market capitalisation = O/S share x price per share

₹210 lakhs = (10 lakhs - $\frac{₹27 lakhs}{P}$) x 1.1P

₹210 lakhs = $\frac{(₹10\ lakhsP - ₹27\ lakhs)}{P}$ x 1.1P

On solving the above equation we get,

P = ₹21.79

(ii) No. of shares re-purchased = $\frac{₹27\ lakhs}{₹21.79}$ = 123910 shares

Current EPS = ₹3

Current shares 10 lakhs ∴Net profit = ₹3 x 10 lakhs = ₹30 lakhs

Share count after buy back = 10,00,000 - 1,23,910 = **8,76,090**

Revised EPS = ₹30,00,000/8,76,090 = **₹3.424**

If Net Income is same EPS will go up from ₹3 to ₹3.424.

17) Illustration

Goldi locks Ltd was started a year back with equity capital of Rs. 40 lakhs. The other details are as under:

Earnings of the company	Rs. 4,00,000
Price Earnings ratio	12.5
Dividend paid	Rs. 3,20,000
Number of Shares	40,000

Find the current market price of the share. Use Walter's Model.
Find whether the company's D/ P ratio is optimal, use Walter's formula

(MTP Nov'21 Old, Old PM)

Solution 17

Given

PAT = ₹4,00,000

PE Ratio = 12.5

Dividend = ₹3,20,000

No. Of shares = 40,000

Equity Capital = ₹40,00,000

Walter's model $P_0 = \frac{D+(E-D)*\frac{r}{Ke}}{Ke}$

DPS = Dividend/Shares = ₹3,20,000/40,000 = ₹8

EPS = PAT/No. of shares = ₹4,00,000/40,000 = ₹10 per share

For investment of ₹40,00,000 PAT = ₹4,00,000. ∴ Return(r) = 10%

PE = $\frac{MPS}{EPS}$ or $\frac{EPS}{MPS} = \frac{1}{PE}$ i.e $K_e = \frac{1}{PE} = \frac{1}{12.5}$ = 8%

Price as per Walter's model = $\frac{₹8+(10-8)*\frac{10\%}{8\%}}{8\%}$ = **₹131.5**

r Vs. Ke	Company Phase	Correlation (DPS, MPS)	Optimum Dividend Payout Ratio
r > Ke	Growth	Negative	Zero
r = Ke	Constant / Stable	Zero	Any Ratio
r < Ke	Decline	Positive	100%

Given that r @ 10% is > Ke @ 8%, it is advisable that Goldi locks Ltd. **NOT PAY** any dividend at all &instead invest all the money / profits in the business, as the business is generating a return higher than the expected shareholder return

18) Illustration

X Ltd. earns Rs. 6 per share having capitalization rate of 10% and return on investment of 20%. According to Walter's model, what should be the price of the share at 25 percent dividend pay-out.
(Similar Old PM)

Solution 18

Given,

EPS = ₹6 Capitalisation rate(K_e) = 10% ROI(r) = 20%

Pay out ratio = 25% ∴ 6 x 25% = ₹1.5

As per Walter's Model = P_0 = $\frac{D+(E-D)*\frac{r}{Ke}}{Ke}$ = $\frac{₹1.5+(6-1.5)*\frac{20\%}{10\%}}{10\%}$ = **₹105**

19) Illustration

The following information is collected from the annual reports of J Ltd:

Profit before tax	Rs. 2.5 crore
Tax rate	40%
Retention ratio	40%
Number of outstanding shares	50,00,000
Equity capitalization rate	12%
Rate of return on investment	15%

What should be the market price per share according to Gordon's model of dividend policy? *(Old PM)*

Solution 19

From annual return of J Ltd given,

PBT = ₹2.5 crore PAT = ₹2.5(1 - 40%) = ₹1.5 crore

T = 40% b = 40% No. of shares O/S = ₹50 lakhs K_e = 12%

r =15%

Market price as per Gordon's growth model

EPS = PAT/No. of shares = ₹1.5cr/50lakh = **₹3 per share**

EPS_1 = EPS(1+g) = 3(1+6%) = **₹3.18**

g =br = 40% x 15% = **6%**

Therefore, P_0= $\frac{D_1}{Ke-g}$ = $\frac{EPS_1\,(1-b)}{Ke-g}$ = $\frac{₹3.18(1-40\%)}{12\%-6\%}$ = **₹31.8**

20) Illustration

The following information is given for QB Ltd.

Earnings per Share	Rs. 12
Dividend per share	Rs. 3
Cost of Capital	18%
IRR on investment	22%

Calculate market price per share using

i. Gordon's Formula

ii. Walter's Formula

(RTP Nov'20 Old, Old PM)

Solution 20

Earnings per Share	₹ 12
Dividend per share	₹ 3
Cost of Capital(K_e)	18%
IRR on investment	22%
g = br where b = $\frac{(12-3)}{12}$ = 75% and r = 22% So, 75% x 22% = 16.5%	16.5%

(i)Gordon's formula = $P_0 = \frac{D_1}{Ke-g} = \frac{D_0(1+g)}{K_e-g} = \frac{3(1+16.5\%)}{18\%-16.5\%}$ = **₹233**

(ii) Walter's formula : $P_0 = \frac{D+(E-D)*\frac{r}{Ke}}{Ke} = \frac{3+(12-3)*\frac{22\%}{18\%}}{18\%}$ = ***₹77.77***

21) Illustration

A share of Tension free Economy Ltd. is currently quoted at a price earnings ratio of 7.5 times. The retained earning being 37.5% is Rs. 3 per share.
Calculate

- The company's cost of equity if investor's expected rate of return is 12%
- Market price of share, if anticipated growth rate is 13% per annum with same cost of capital.
- Market price per share, if company's cost of capital is 18% and anticipated growth rate is 15% per annum, assuming other conditions remaining the same.

(RTP Nov'23, Old PM)

Solution 21

Particulars	Computation	Value
PE Ratio	given	7.5
Retention ratio (b)	given	37.5%
r	given	12%
EPS	3/37.5% * 100% EPS RE 100% 37.5% ? ₹3	8
Payout ratio(p)	(1-37.5%)	62.5%
g = br	37.5% x 12%	4.5%
PE	given	7.5
Price(MPS)	PE X EPS = 7.5 X 8	₹60

$P_0 = \frac{D_1}{Ke-g} = \frac{D_0(1+g)}{K_e-g}$

₹60 = $\frac{5(1+4.5\%)}{K_e-4.5\%}$

K_e - 4.5% = $\frac{5.225}{60}$

K_e = **13.21%**

(ii) P_0 = ? if, g = 13% K_e = 13.21%

$P_0 = \frac{D_1}{Ke-g} = \frac{D_0(1+g)}{K_e-g} = \frac{5(1+13\%)}{13.21\%-13\%}$ = **₹2690.5**

(iii) g = 15% K_e = 18% D_0 = ₹5

$P_0 = \frac{D_1}{Ke-g} = \frac{5(1+15\%)}{18\%-15\%}$ = **₹191.7**

22) Illustration

A firm had been paid dividend at Rs. 2 per share last year. The estimated growth of the dividends from the company is estimated to be 5% p.a. Determine the estimated market price of the equity share if the estimated growth rate of dividends

- Rises to 8%, and
- Falls to 3%.

Also find out the present market price of the share, given that the required rate of return of the equity investors is 15.5%.

(Old PM)

Solution 22

Given,

D_0 = 2	g = 5%	g_A =8%	g_B =3%	K_e=15.5%

Case 1 Market price = ? when g= 5%,

$P_0 = \frac{D_1}{K_e - g} = \frac{D_0(1+g)}{K_e - g} = \frac{2(1+5\%)}{15.5\% - 5\%}$ = ₹20

Case 2 Market price = ? when g= 8%

$P_0 = \frac{D_1}{K_e - g} = \frac{D_0(1+g)}{K_e - g} = \frac{2(1+8\%)}{15.5\% - 8\%}$ = ₹28.8

Case 2 Market price = ? when g= 3%

$P_0 = \frac{D_1}{K_e - g} = \frac{D_0(1+g)}{K_e - g} = \frac{2(1+3\%)}{15.5\% - 3\%}$ = ₹16.48

23) Illustration

Shares of Voyage Ltd. are being quoted at a price-earnings ratio of 8 times. The company retains 50% of its earnings which are Rs. 10 per share.

You are required to compute

- The cost of equity to the company if the market expects a growth rate of 15% per annum.
- If the anticipated growth rate is 16% per annum, calculate the indicative market price with the same cost of capital.
- If the company's cost of capital is 20% p.a. & the anticipated growth rate is 18% p.a., calculate the market price per share.

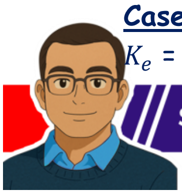
(ICAI SM, Nov'18 QP 8 marks)

Solution 23

Given,	
P/E Ratio	8 times
EPS	₹10
∴MPS	₹80 (EPS x PE)(8 x 10)
Retention ratio(b)	50%
Payout ratio(p)	50%

Case 1

K_e = ? when g = 15% and D_0 = 10 x 50% = ₹5



$P_0 = \frac{D_1}{K_e - g} = \frac{D_0(1+g)}{K_e - g}$

₹80 = $\frac{5(1+15\%)}{K_e - 15\%}$

$\therefore K_e = \frac{5.75}{80}$ + 15% = **22.1875%**

Case 2

P_0 = ? K_e = 22.1875% g = 16%

$P_0 = \frac{D_1}{K_e - g} = \frac{D_0(1+g)}{K_e - g} = \frac{5(1+16\%)}{22.1875\% - 16\%}$ = **₹93.74**

Case 3

P_0 = ? K_e = 20% g = 18%

$P_0 = \frac{D_1}{K_e - g} = \frac{D_0(1+g)}{K_e - g} = \frac{5(1+18\%)}{20\% - 18\%}$ = **₹295**

24) Illustration

In December 2011 AB Co's share was sold for Rs. 146 per share. A long-term earnings growth rate of 7.5% is anticipated. AB Co. is expected to pay dividend of Rs. 3.36 per share.
What rate of return an investor can expect to earn assuming that dividends are expected to grow along with earnings at 7.5% per year in perpetuity?
It is expected that AB Co. will earn 10% on book Equity and shall retain 60% of earnings. In this case, whether there would be any change in growth rate and cost of Equity?

(RTP Nov'18 Old, RTP Nov'20 Old, Old PM)

Solution 24

Given,

Method 1

(i) P_0 = 146 g = 7.5% K_e = ? D_1 = 3.36

$P_0 = \frac{D_1}{K_e - g}$

146 = $\frac{3.36}{K_e - 7.5\%}$

$K_e = \frac{3.36}{146}$ + 7.5% = 2.30% + 7.5% = **9.8%**

(ii) r = 10% b = retention ratio = 60% so, g = br = 60% x 10% = **6%**

Hence the growth rate will change to 6% from 7.5%.

Now for old data , $D_1 = D_0(1+g)$

So, 3.36 = $D_0(1 + 7.5\%)$

$\boldsymbol{D_0}$ = $\frac{3.36}{1.075}$ = **3.126**

$P_0 = \frac{D_1}{K_e - g}$

146 = $\frac{3.126(1+6\%)}{K_e - 6\%}$

$K_e = \frac{3.314}{146}$ + 6% = 2.27% + 6% = **8.27%**

Method 2

g = 7.5% means, br = 7.5%

So, b x 10% = 7.5% hence b = 75% then, p =25%
It is assumed that ROE remains same and retention ratio changes
Then, dividend being ₹3.36 is 25% so, EPS = $\frac{3.36}{25\%}$ = **₹13.44**

(ii) EPS = 13.44 & b = 60% so, Payout = 40% then, D_1 = 13.44 x 40% = ₹5.376

$P_0 = \frac{D_1}{K_e - g}$

$146 = \frac{5.376}{K_e - 6\%}$

$K_e = \frac{5.376}{146} + 6\% = 3.68\% + 6\% = \mathbf{9.68\%}$

25) Illustration

A company has an EPS of Rs. 2.5 for the last year and the DPS of Rs. 1. The earnings is expected to grow at 2% a year in long run. Currently it is trading at 7 times its earnings. If the required rate of return is 14%, compute the following.
An estimate of the PE Ratio using Gordon growth model.
The long-term growth rate implied by the current PE Ratio.

[MTP May'20, Nov'18 QP (Old), MTP Mar'21 Old]

Solution 25

EPS=2.5	DPS_0=1	g = 2%	PE = 7 times	K_e=14%

MPS = PE x EPS = 7 x 2.5 = ₹17.5

(i) PE = ? by using **Gordon Growth Model** to find P_0

MPS(P_0)= $\frac{D_1}{K_e - g} = \frac{1(1+2\%)}{14\% - 2\%}$ = ₹8.5

So, PE = $\frac{\text{MPS}}{\text{EPS}} = \frac{₹8.5}{₹2.5}$ = 3.4

(ii) Current PE = 7

Current price = PE x EPS = 7 x 2.5 = 17.5

$P_0 = \frac{D_0(1+g)}{K_e - g}$

$17.5 = \frac{1(1+g)}{14\% - g}$

So, 17.5(14 - g) = 1(1 + g)
17.5 x 14 – 17g = 1+g
2.45 – 1 = 17.5g +g

g = $\frac{1.45}{18.5}$ = **7.84% is the long term growth rate for the current PE ratio**

26) Illustration

Rahim Enterprises is a manufacturer and exporter of woollen garments to European countries. Their business is expanding day by day and in the previous financial year the company has registered a 25% growth in export business. The company is in the process of considering a new investment project. It is an all equity financed company with 10,00,000 equity shares of face value of Rs. 50 per share. The current issue price of this share is Rs. 125 ex-divided.

Annual earning is Rs. 25 per share and in the absence of new investments will remain constant in perpetuity. All earnings are distributed at present. A new investment is available which will cost Rs. 1,75,00,000 in one year's time and will produce annual cash inflows thereafter of Rs. 50,00,000. Analyse the effect of the new project on dividend payments and the share price.

[Nov'17 QP (Old), MTP Oct'21 Old, Old PM]

Solution 26

Rahim Enterprises

Equity share count = 10 lakh share
FV = ₹50
MPS = ₹125 ex-dividend
EPS = 25

Since all earnings have been paid, DPS = EPS =₹25

New investment = ₹1.75 Cr

Cash flow from investment = ₹50 lakh

If new project is undertaken then total amount invested is 1.75 Cr
PAT = Dividend = 25 per share x 10 lakh shares = ₹2.5 Cr

Next year out of ₹2.5 Cr, ₹1.75 Cr will be invested. So, dividend for Yr 1 will be 2.5Cr – 1.75Cr = 0.75 Cr
i.e. ₹7.5/- per share which will be compared to 25 per share in Yr 0

From Yr 2 dividend will increase to ₹3 Cr i.e. ₹2.5 Cr from existing business and ₹50 lakhs from new investments.

So, Dividend will increase to ₹3 Cr/10 lakh = ₹30 per share.

Analysis (for classroom purpose)

Year	Yr 0	Yr 1	Yr 2	Yr 3	Yr 4
Earlier	25	25	25	25	25
Now	25	↓ 7.5	↑ 30	↑ 30	↑ 30

So if the company goes through with the project, next year dividend will fall to 7.5 but will increase to ₹30 from the year after.

(ii) Impact on share price

$P_0 = \frac{D_1}{K_e - g}$ where K_e can be determined using existing data considering there is no growth (g= 0)

$125 = \frac{25}{K_e - 0}$ So, $K_e = \frac{25}{125}$ = 20%

Now using two stage dividend discount model

$P_0 = \frac{D_1}{1+K_e} + \frac{P_1}{1+K_e}$

Where, $P_1 = \frac{D_2}{K_e - g} = \frac{30}{20\%}$ = ₹150

So, $P_0 = \frac{7.5}{1+20\%} + \frac{150}{1+20\%} = \frac{157.5}{1.2}$ = **₹131.25**

27) Illustration

A company had paid a dividend of Rs. 2.50 per share last year and its required rate of return for equity investors is 20%. What will be the market price of the share at the end of the year, if? There is no growth in dividend?

Dividend grows at constant rate of 5% per annum in perpetuity.
Constant dividend for first five years and then grows at constant rate of 5% per annum in perpetuity?
Constant dividend for first five years and then shares is sold at the price of Rs. 20.

Solution 27

Given,

D_0 = 2.5 K_e = 20%

Part 1: Where there is no growth in dividend

$P_0 = \frac{D_1}{K_e - g}$ since g=0 $P_0 = \frac{D}{K_e} = \frac{2.5}{20\%}$ = **₹12.5**

Part 2 : g=5% then,

$P_0 = \frac{D_1}{K_e - g}$ where, $P_0 = \frac{D_0(1+g)}{K_e - g}$

then, $\frac{2.5(1+5\%)}{20\% - 5\%} = \frac{2.625}{15\%}$ = **₹17.5**

Part 3 : $D_1 - D_5$ = constant $D_6 = D_5(1+5\%)$ g_6 = 5%
g_6 - g_∞ = 5%

That can be tabulated as,

Yr	0	1	2	3	4	5	6
Dividend	2.5	2.5	2.5	2.5	2.5	2.5	2.625
growth	-	-	-	-	-	-	5%
K_e = 20% so, PV factor is	1	0.833	0.694	0.578	0.482	0.402	

So, the PVAF for 5 yrs @20% = 2.989 (0.833 + 0.694 + 0.578 + 0.482 + 0.402)

$D_6 = D_5(1+5\%)$ = 2.5(1+ 5%) = **2.625** so, $P_5 = \frac{D_6}{K_e - g} = \frac{2.625}{20\% - 5\%}$ = ₹17.5

P_0 = PVAF (5,20%) + $\frac{P_5}{(1+20\%)^5}$

= 2.989 x (2.5) + 17.5 x 0.402

= 7.473 + 7.035 = **14.508**

Part 4 : when, $D_1 - D_5$ = constant and P_5 = 20

P_0 = PVAF(5,20%) + $\frac{P_5}{(1+20\%)^5}$

= 2.989 x (2.5) + 20 x 0.402 = **₹15.513**

28) Illustration

An investor is considering purchasing the equity shares of LX Ltd., whose current market price is Rs. 150. The company is proposing a dividend of Rs. 6 for the next year. LX is expected to grow at 18% per annum for the next four years. The growth will decline linearly to 14% per annum after first four years. Thereafter, it will stabilise at 14% per annum infinitely. The required rate of return is 18% per annum.

You are required to determine:

i. The intrinsic value of one share

ii. Whether it is worth to purchase the share at this price

t	1	2	3	4	5	6	7	8
PVIF (18,t)	0.847	0.718	0.609	0.516	0.437	0.370	0.314	0.266

[MTP Oct'21 New & Old, May'19 QP (Old)]

Solution 28

MPS =₹150	D_1 = ₹6	$g_1 - g_4$ = 18%	$g_5 - g_8$ =17%/16%/15%/14%	g_9 - g_∞= 14%	K_e = 18%

Year	Growth rate	Computation	Dividend	PVIF @ 18%	PV
1	18%	-	6	0.847	5.082
2	18%	6(1.18)	7.08	0.718	5.084
3	18%	$6(1.8)^2$	8.35	0.609	5.085
4	18%	$6(1.8)^3$	9.86	0.516	5.088
5	17%	9.86(1.17)	11.53	0.437	5.005
6	16%	11.53(1.16)	13.38	0.370	4.95
7	15%	13.38(1.15)	15.39	0.314	4.83
8	14%	15.39(1.14)	17.54	0.266	4.66
Present value of explicit factor					**Σ = 39.79**
9	14%	17.54(1.14)	20		

$P_8 = \frac{D_9}{K_e - g} = \frac{20}{18\% - 14\%}$ = ₹500

PV of P_8 = ₹500 x 0.266 = **₹133**

So, the total value of cash inflow = ₹39.79 + ₹133 = ₹172.79

Intrinsic Value = 172.79 and investor should buy shares of LX Ltd as MPS (₹150) is less than the intrinsic value.

Alternatively,

Explicit factor (discounting for 7 years) = 35.13 (5.082 + 5.084 + 5.085 + 5.088 + 5.005 + 4.95 + 4.83)

Terminal value of $P_7 = \frac{D_8}{K_e - g} = \frac{17.54}{18\% - 14\%}$ = ₹438.5

PV(7[th] yr @18%) of ₹438.5 is 0.314 x 438.5 = ₹137.69

So, the total value of cash inflow = Explicit factor + Terminal value of P_7
= ₹35.13 + ₹137.69
= **₹172.81**

Intrinsic Value = 172.81 and investor should buy shares of LX Ltd as MPS (₹150) is less than the intrinsic value.

29) Illustration

X Ltd. is a Shoes manufacturing company. It is all equity financed and has a paid up Capital of Rs. 10,00,000 (Rs. 10 per share) X Ltd. has hired Swastika consultants to analyse the future earnings. The report of Swastika consultants states as follows:

i. **The earnings and dividend will grow at 25% for the next two years.**
ii. **Earnings are likely to grow at the rate of 10% from 3rd year and onwards.**
iii. **Further, if there is reduction in earnings growth, dividend pay-out ratio will increase to 50%.**

The other data related to the company are as follows:

Year	EPS (Rs.)	Net Dividend per share (Rs.)	Share Price (Rs.)
2010	6.30	2.52	63.00
2011	7.00	2.80	46.00
2012	7.70	3.08	63.75
2013	8.40	3.36	68.75
2014	9.60	3.84	93.00

You may assume that the tax rate is 30% (not expected to change in future) and post-tax cost of capital is 15%. By using the Dividend Valuation Model, calculate

i. **Expected Market Price per share**
ii. **P/E Ratio.**

(MTP Oct'19 Old, RTP May'19 Old, RTP May'20 Old, Old PM)

Solution 29

(i) Assuming we're on 1/1/2015 & EPS AD DPS AND Growth rate is from this date.

Year	g	EPS Computation	EPS	DPS computation	DPS
2014			9.6		
2015	25%	9.6(1.25)	12	3.84 x 1.25	4.8
2016	25%	12(1.25)	15	4.8 x 1.25	6
2017	10%	15(1.10)	16.5	16.5 x 50%	8.25
2018	10%	16.5(1.10)	18.15	18.15 x 50%	9.075

$$P_0 = \frac{D_1}{1+K_e} + \frac{D_2}{(1+K_e)^2} + \frac{D_3}{(1+K_e)^3} + \frac{P_3}{(1+K_e)^3} \quad \& \; P_3 = \frac{D_4}{K_e - g}$$

Year	Dividend	PVF @ 15%	PV of dividend
1	4.8	0.8696	4.17408
2	6	0.7561	4.5366
3	8.25	0.6575	5.4243
Present value of explicit factor			**Σ = 14.13498**
4	9.075	0.5717	

$P_3 = \frac{D_4}{K_e - g} = \frac{9.075}{15\% - 10\%}$ = **₹181.5**

P_0 = **14.13498 + PV of** P_3 = **14.13498 + 0.6575 x 181.5 = ₹133.47**

(ii) PE = $\frac{MPS}{EPS}$

	Forward	Trailing
Market price(MPS₹)	133.47	133.47
EPS(₹)	12	9.6
PE Ratio	11.12	13.90

30) Illustration

AMKO Limited has issued 75,000 equity shares of Rs. 10 each. The current market price per share is Rs. 36. The company has a plan to make a rights issue of one new equity share at a price of Rs. 24 for every four shares held.
You are required to:
Calculate the theoretical post-rights price per share.
Calculate the theoretical value of the right alone

[Nov'18 QP (Old)]

Solution 30

Share capital = 75000 units of ₹10 each	CMP/MPS = ₹36	Rights issue price = ₹24	Rights issue = 1:4

<u>Theoretical Ex- Right Price (TERP)</u>

Issue size is 1 for 4 held. So, total shares issued = $\frac{75000}{4}$ * 1 = 18750

TERP = $\frac{nP_0 + S}{n + n_1} = \frac{75000 * 36 + 18750 * 24}{75000 + 18750}$ = **₹33.6**

Where,

n = No. of existing equity shares
P_0 = Price of Share Pre-Right Issue
S = Subscription amount raised from Right Issue
n_1 = No. of new shares offered

Now, Theoretical value of the right = ₹33.6 - ₹24 = ₹9.6 (is for 4 right shares)
Value of the right per existing share = ₹9.6/4 = **₹2.4**

31) Illustration

AB Limited's shares are currently selling at Rs. 130 per share. There are 10,00,000 shares outstanding. The firm is planning to raise Rs. 2 crores to finance new project.
Required

- **What is the ex-right price of shares and value of a right, if.**
  - **The firm offers one right share for every two shares held.**
  - **The firm offers one right share for every four shares held.**

Solution 31

Share capital = 10 lakhs	CMP/MPS = ₹130	New project = ₹2 Crores

Scenario	1	2
Rights issue	1:2	1:4
Project Value	2 Cr	2 Cr
Existing share count	10 lakhs	10 lakhs
No. of rights shares	10lakhs x 1/2= 5lakhs	10 lakhs x 1/4 = 2.5 lakhs
Price per share	₹2Cr/5lakhs = ₹40	₹2 Cr/2.5lakhs = ₹80
CMP	₹130	₹130
TERP	$\frac{10lacs * 130 + 5lacs * 40}{10lacs + 5lacs}$ = ₹100	$\frac{10lacs * 130 + 2.5lacs * 40}{10lacs + 2.5lacs}$ = ₹120
Theoretical value of the right	₹100 – ₹40 = ₹60	₹120 – ₹80 = ₹40
Shares held	2	4
Value per share	₹60/2 = 30	₹40/4 = ₹10

32) Illustration

PragyaLtd has issued 75000 equity shares of Rs. 10 each. Current market price is Rs. 24. Company has a plan to make rights issue of one equity share at a price of Rs. 16 for every 4 shares held. You are required to calculate

- **Theoretical post right price per share**
- **Calculate value of the right**
- **Show the effect of rights on the wealth of the shareholder who has 1000 shares assuming he sells the entire rights and show the effect of same holder if same shareholder does not take any action and ignores the issue.**

(RTP Nov'18 Old)

Solution 32

Given,

Share count 75000	FV = ₹10	CMP = ₹24	Rights issue = 1:4	Rights price = ₹16

Total rights shares issued = 75000 x $\frac{1}{4}$ = 18750

(i) Theoretical Ex Right Price = $\frac{75000 * 24 + 18750 * 16}{75000 + 18750} = \frac{21,00,000}{93750}$ = ₹22.40

(ii) Value of right = ₹22.4 - ₹16 = ₹6.4
Value per share held by existing shareholder = 6.4/4 = ₹1.6

(iii) Impact on shareholder

Value before rights issue = 24 x 1000 = 24000

Value after rights issue = 1000 x 22.4 = 22400

For 1000 shares he gets 250 rights.
Value realised on sale of rights shares = 250 x 6.4 = ₹1600

Post rights wealth = 22400 + 1600 = 24000
Pre rights wealth = 24000

Hence there is no change

(iv) If shareholder does not sell, he loses ₹1600 is the form of loss on rights unsold

33) Illustration

Telbel Ltd. is considering undertaking a major expansion an immediate cash outlay of Rs. 150 crores. The Board of Director of company are expecting to generate an additional profit of Rs. 15.30 crore after a period of one year. Further, it is expected that this additional profit shall grow at the rate of 4% for indefinite period in future.

Presently, Telbel Ltd. is completely equity financed and has 50 crore shares of Rs. 10 each. The current market price of each share is Rs. 22.60 (cum dividend). The company has paid a dividend of Rs. 1.40 per share in last year. For the last few years dividend is increasing at a compound rate of 6% p.a. and it is expected to be continued in future also. This growth rate shall not be affected by expansion project in any way.

Board of Directors are considering following ways of financing the possible expansion:
(1) A right issue on ratio of 1:5 at price of Rs. 15 per share.
(2) A public issue of shares.

In both cases the dividend shall become payable after one year.
You as a Financial Consultant required to:
Determine whether it is worthwhile to undertake the project or not.
Calculate ex-dividend market price of share if complete expansion is financed from the right issue.
Calculate the number of new equity shares to be issued and at what price assuming that new shareholders do not suffer any loss after subscribing new shares.
Calculate the total benefit from expansion to existing shareholders under each of two financing option.

Solution 33

Given,

Project outflow = 150 Cr	g = 4% till ∞	$Profit_1$(inflow)=15.3 Cr	Current share capital is 50 Cr shares @₹10 each	CMP = 22.6 cum dividend	g rate of dividend= 6% & D =1.4

To find out project value till perpetuity and thereafter to find NPV K_e is required.

$P_0 = \frac{D_1}{K_e - g}$ where, P_0 (Ex-dividend) = 22.6 – 1.4 = ₹21.2

21.2 = $\frac{1.4(1.06)}{K_e - 6\%}$

$K_e - 6\% = \frac{1.484}{21.2}$

K_e = 13%

Using K_e = 13% , g=4%and Cash inflow = 15.3Cr PV of future cashflows from the new project are computed as

PV of inflow = $\frac{15.3}{13\%-4\%}$ = ₹170 Cr

PV of outflow = ₹150 Cr

So, NPV = ₹170Cr - ₹150 Cr = ₹20 Cr

(ii)

Rights issue size = 150Cr	Price per share = 15	Issue terms = 1:5	New shares issued = ₹50Cr/5 = 10Cr

TERP = $\frac{nP_0 + S}{n + n_1}$ + $\frac{NPV\ from\ the\ project}{60}$ = $\frac{50Cr * 21.2 + 10Cr * 15}{50Cr + 10Cr}$ + $\frac{20Cr}{60Cr}$ = ₹20.5

The NPV of the project should also be included in the value of the share.

(iii) Let the number of new equity share = m

New issue price = p

M x p = ₹150 So, P = $\frac{150}{M}$

Post issue price = $\frac{nP_0 + S}{n + n_1}$ = $\frac{(m * p) + 20Cr + (21.2 * 50)}{50+m}$

P = $\frac{(m * p) + 20Cr + (21.2 * 50)}{50+m}$

$\frac{150}{M}$ = $\frac{(150Cr + 20Cr + 1060)}{50+m}$

$150(50 + m) = (150Cr + 20Cr + 1060)m$

7500 = 1230m - 150m

M = 7500/1080 = 6.944 Put M=6.944 in P = $\frac{150}{M}$

Then, P = $\frac{150}{6.944}$ = ₹21.60

Existing shareholders = 21.2 - 21.6 = 0.4

Value = 0.4 x 50 = ₹20Cr

(iv) (a) Rights issue

(Amt. in Cr)

	Pre Rights Issue	**Post Rights Issue**
Share Count	50	60 (50 + 10)
Price(Ex-dividend)	21.2	20.5
Value	1060	1230
Add : Subscription Amt	150	
	1210	**1230**

Net gain in case of rights issue is ₹20Cr

(b) In case of Public Issue

	Pre	**Post**
Shares	50	50
Price	21.2	21.6
	1060	**1080**

Net gain in case of public issue is ₹20Cr

34) Illustration

X is evaluating PQR Co., which has been growing at the annual rate of 15% for the last 5 years. Here is her forecast of the company's future dividend growth trend.

2022	13.5%
2023	12%
2024	10.5%
2025	9.0%
2026	7.5%
2027	6%
Till perpetuity	6%

After considerable study, she has decided to use the following information in her valuation (as of the end of 2021):

- Beta: 1.2.
- 2021 dividend: ₹5.
- The risk-free rate: 4%.
- Expected equity premium: 6%.

What is the value of the PQR stock?

Solution 34

Beta: 1.2	D_0: ₹5	$R_f = 4\%$	$R_p = 6\%$

H Model formula

$P_0 = \frac{D_0(1+g_c)}{K_e - g_c} + \frac{D_0\left(\frac{1}{2}*2H\right)(g_h - g_c)}{K_e - g_c}$ where,

g_h= Highest growth g_c = constant growth 2H = 6yrs (2021 to 2027)

$K_e = R_f + \beta\,(R_m - R_f)$ = 4% + 1.2(6%) = 11.2%

So, $P_0 = \frac{5(1+6\%)}{11.2\%-6\%} + \frac{5\left(\frac{1}{2}*6\right)(15\%-6\%)}{11.2\%-6\%} = \frac{5.3+1.35}{5.2\%}$ = ₹127.88

35) Illustration

The management of T Ltd, an internationally diversified conglomerate, believes that the recent strong performance of its wholly owned medical supply subsidiary, S Ltd, has gone unnoticed. To realize S Ltd's full value, T Ltd has announced that it will List S Ltd.

Data for 2021 & 2022 is given below

Income Statement Rs.cr	2021	2022
Revenue	474	598
Depreciation	20	23
Other Operating costs	368	460
Income before taxes	86	115
Taxes	26	35
Net Income	60	80
Dividends	18	24
EPS	Rs. 0.714	Rs. 0.952
Dividends per share	Rs. 0.214	Rs. 0.286
Common Shares Outstanding	84	84

Balance Sheet – Rs. Cr	2021	2022
Current Assets (Includes Rs. 5 Cr. cash in 2021 and 2022)	201	326
Net property, plant, and equipment	474	489
Total Assets	675	815
Current Liabilities (all non-interest-bearing)	57	141
Long Term Debt	0	0
Shareholder's Equity	618	674
Total Liabilities and equity	675	815
Capital Expenditures	34	38

A. Calculate the amount of FCFE per share for 2022 by using the data from Exhibit.
B. Calculate the current value of a share of S Ltd. based on the two - stage FCFE model.
For the next three years, growth rate is 10% and after 3 years at 5% in perpetuity.
Cost of Equity is 11%.

Solution 35

FCFF = EBIT (1 – Tax) + Depreciation -/+ Capital Expenditure / Assets Disposed off +/- Change in Working capital
FCFE = FCFF - Interest (1 – Tax rate) - Debt / Preference Shares repaid / redeemed + New Debt / Preference shares Issued - Preference Dividend

	2021	2022
PBIT	86	115
Int	-	-
PBT	86	115
Tax	26	35
PAT	60	80

EBIT(1 – t) = PAT
Int = 0 & Not accrued taxes & No adj.
Now, FCFF

	2021	2022
PAT	60	80
(+) DEP	20	23
(-) CAPEX*	(34)	(38)
(-) Inc in WC	-	(41)
FCFF	6	24

*WN :1 Increase in working capital

	2021	2022
Assets excluding cash	201	326
(-) Cash	(5)	(5)
	196	**321**
Current Liabilities	57	141
	139	180
Increase in working capital	**41 (180-139)**	

FCFE = FCFF - Interest (1 – Tax rate) - Debt /Preference Shares repaid / redeemed + New Debt / Preference shares Issued - Preference Dividend

FCFE = FCFF – 0

FCFE = 24Cr – 0 = 24Cr

O/S shares = 84Cr
FCFE/Share = 0.2857/share

(ii)

$g_1 - g_3$ = 10%	g_4 - g_∞= 5%	K_e = 11%

Two stage FCFE Model

Stage 1

FCFE 0.2857

Year	Growth	Computation	FCFE Value
2022			0.2857
2023	10%	0.2857(1.10)	0.3143
2024	10%	0.3143(1.10)	0.3457
2025	10%	0.3457(1.10)	0.3803
2026	5%	0.3803 (1.05)	0.3993

P_0 = Explicit period value + perpetuity value

Year	Cashflow	PVF@11%	PVCF
1	0.3143	0.9009	0.2831
2	0.3457	0.8116	0.2806
3	0.3803	0.7312	0.2780
			∑ = 0.8417
4	0.3803(1.05) = 0.3993		

Stage 2

Perpetuity Value : $\frac{P_3}{(1+K_e)^3}$

P_3 = $\frac{FCFE_4}{K_e - g}$ = $\frac{0.3993}{11\% - 5\%}$ = ₹6.6547

PV of P_3 = 6.6547 x 0.7312 = ₹4.8659

Now, P_0 = **Explicit period value + perpetuity value**

P_0 = 0.8417 + 4.8659 = **₹5.7076 per share**

36) Illustration

Mr. AF is analysing the financial statements of SD Limited. He has a 2007 income statement and balance sheet, as well as 2008 income statement, balance sheet, and cash flow from operations forecasts (as shown in the tables below). Assume there will be no sales of long-term assets in 2008. Calculate forecasted free cash flow to the firm (FCFF) and free cash flow to equity (FCFE) for 2008

Income Statement — In Crores

	2008 (Forecast)	2007 (Actual)
Sales	300	250
Cost of Goods Sold	120	100
Gross Profit	180	150
SG & A	35	30
Depreciation	50	40
EBIT	95	80
Interest Expense	15	10
Pre-Tax Earnings	80	70
Taxes (at 30%)	24	21
Net income	56	49

Balance Sheet — In Crores

	2008 (Forecast)	2007 (Actual)
Cash	10	5
Accounts Receivable	30	15
Inventory	40	30
Current Assets	80	50
Gross property, plant, and equipment	400	300
Accumulated Depreciation	(190)	(140)
Total Assets	290	210
Accounts payable	20	20
Short term Debt	20	10
Current Liabilities	40	30
Long term Debt	114	100
Common stock	50	50
Retained Earnings	86	30
Total Liabilities and Owner's Equity	290	210

SD Ltd Cash Flow from Operations Forecast for 2008

Net Income	56
Add: Depreciation	50
Less: Investment in working capital	25
Cash Flow from operations	81

Solution 36

FCFF = EBIT (1 – Tax) + Depreciation -/+ Capital Expenditure / Assets Disposed off +/- Change in Working capital

= 95(1-30%)+50-100*-25**

= 66.5 – 75

FCFF = (8.5) – negative cashflow

FCFE = FCFF – Int(1 – tax) – Debt Paid + Debt raised

FCFE = -8.5 – 15(1 – 0.3) + Net borrowing***

= -8.5 -10.5 +24 = **5**

FCFE = 5

*WN 1

Capex = GFA (2008) – GFA (2007) = 400-300 = **100Cr**

**WN 2

Working capital investment

	2008	2007
Current Asset	80	50
(-) Cash	10	5
Current asset excluding cash	**70**	**45**
Current Liabilities	40	30
(-) Short term debt	20	10
Current liabilities excluding short term debt	**20**	**20**
Net working capital	**50**	**25**

Increase in working capital 25

*****WN 3 : Net borrowing**

	2008	**2007**
Short term loan	20	10
Long term Loan	114	100
	134	110

Increase in net borrowing 24

EBITDA

FCFF = EBITDA(1-tax) + Dep x tax – WC – Capex

= 145(1-0.3) + 50(0.30) – 25 – 100

= 101.5 – 110

FCFF = (8.5)

CFO

CFO = NI + Dep- Inc in WC +Int Exp

CFO = 56 +50 -25+15= 96

FCFF = CFO - Int(tax) – Capex

= 96- 15(0.30) - 100

FCFF = (8.5)

PAT based approach

FCFF = PAT + Dep - Inc in WC – Capex + Int(1 – tax)

= 56 + 50 -25 – 100 + 15(0.7)

= -19 + 10.5

FCFF = -8.5

37) Illustration

Calculate the value of share from the following information:

Profit after tax of the company	Rs. 290 crores
Equity capital of company	Rs. 1,300 crores
Par value of share	Rs. 40 each
Debt ratio of company (Debt/ Debt + Equity)	27%
Long run growth rate of the company	8%
Beta 0.1; risk free interest rate	8.7%
Market returns	10.3%
Capital expenditure per share	Rs. 47
Depreciation per share	Rs. 39
Change in Working capital	Rs. 3.45 per share

(ICAI SM, Similar May'22 QP 8 marks, MTP Aug'18, RTP May'24, RTP May'20, RTP Nov'19 Old, Old PM)

Solution 37

Given,

PAT	₹290Cr
Equity Capital	₹1300Cr
FV per share	40
Debt : Equity	27%
g	8%
β	0.1
R_f	8.7%
R_m	10.3%
Capital expenditure per share	₹47
Depreciation per share	₹39
Change in WC per share	₹3.45

K_e = R_f + β (R_m – R_f)

= 8.7% + 0.1(10.3% - 8.7%)

K_e = 8.86%

FCFE = FCFF – Int(1 – tax) + Net borrowing + Net pref raised – Pref dividend

FCFF = PAT + Dep - Inc in WC – Capex + Int(1 – tax)

So, FCFE = PAT – WC – Capex + Dep + Net Borrowing

= PAT – WC (1 - Debt rate) – (1 – Debt rate)(Capex – Dep)

=*8.92 – (1 - 0.27)(3.45) – (1 - 0.27)(47 - 39) = 8.92 – 8.36 = 0.56

FCFE = ₹0.56

*[Instead of net borrowing multiply with 1-debt rate]

<u>*PAT per share</u>

Share count = 1300/40 = 32.5

PAT per share = 290/32.5 = **8.92**

Now, Value per share is

$FCFE_1 = FCFE_0$ (1+g) = 0.56 (1+8%) = 0.605

$$P_0 = \frac{FCFE_1}{K_e - g} = \frac{0.605}{8.86\% - 8\%} = ₹70.35$$

38) Illustration

Mr. SV has gathered the following data for CSI Ltd.

Recent Share Price	Rs. 31.25
Shares Outstanding	30 Lakhs
Market Value of Debt	115 Lakhs
Cash and Marketable Securities	47.6 Lakhs
Investments	247 Lakhs
Net Income	119.4 Lakhs
Interest Expense	5.8 Lakhs
Depreciation	6.9 lakhs
Amortization	2.3 Lakhs
Taxes	85.9 Lakhs

Based on these, compute EV, EV/EBITDA, Market Cap

Solution 38

Computation of EV, EV/EBITDA, Market Cap

Particulars	Computation	Value
Share Price		₹31.25
Shares O/S		30 Lakhs
Market Capitalisation	MP x shares O/S =31.25 x 30lakhs	₹9.375Cr
EV	Market cap + Debt - Cash -Invst = 9.375 + 1.15 - 0.476 - 2.47	₹7.579Cr
EBITDA	PAT +Tax + Dep +Int = 1.194Cr + 0.859Cr + 0.023Cr + 0.069Cr + 0.058Cr	₹2.203Cr
EV/EBITDA	₹7.579/₹2.203	**3.44 times**

39) Illustration

An investor is considering purchasing the equity shares of LX Ltd., whose current market price (CMP) is Rs. 112. The company is proposing a dividend of Rs. 4 for the next year. LX Ltd. is expected to grow @ 20 per cent per annum for the next four years. The growth will decline linearly to 16 per cent per annum after first four years. Thereafter, it will stabilize at 16 per cent per annum infinitely. The investor requires a return of 20 per cent per annum

You are required

To calculate the intrinsic value of the share of LX Ltd.

Whether it is worth to purchase the share at this price.

Period	1	2	3	4	5	6	7
PVIF(20%, n)	0.833	0.694	0.579	0.482	0.402	0.335	0.279

(Nov'20 QP 8 marks)

Solution 39

CMP = 112

Yr	Computation	Cash flow	PVF@20%	Present Value
D_0	$\frac{4}{1.2}$	3.33	1	
D_1	3.33(1.2)	4	0.833	3.332
D_2	4(1.2)	4.8	0.694	3.331
D_3	4.8(1.2)	5.76	0.579	3.335
D_4	5.76(1.2)	6.912	0.482	3.331
D_5	6.912(1.19)	8.225	0.402	3.306
D_6	8.225(1.18)	9.706	0.335	3.251
D_7	9.706(1.17)	11.356	0.279	3.168
Explicit period value				Σ = 23.056
D_8	11.356(1.16)	13.173		

P_0 = Explicit period value + perpetuity value

Explicit period value = Sum of PV of D_1 to D_7 = 23.056

Perpetuity value = $\frac{P_7}{(1+K_e)^7}$

$P_7 = \frac{D_8}{K_e - g} = \frac{13.173}{20\% - 16\%}$ = ₹329.325 ,

PV of 329.325 = 329.325 x 0.279 = ₹91.88

P_0 = 23.056 + 91.88 = ₹114.93

Hence the intrinsic value is 114.93. Since the CMP is ₹112, the investor may buy.

40) Illustration

SK Ltd. has a surplus cash of ₹150 lakhs and wants to distribute 30% of it to the shareholders. The company decided to buy-back shares.
The company estimates that its share price after the buy-back is likely to be 15% above the buy-back price. The number of shares outstanding at present is 15 lakhs and the current EPS is ₹4.

You are required to determine:
The price at which the shares can be bought-back, if the market capitalization of the company should be ₹400 lakhs after buy back.
The number of shares that can be bought-back, and
The impact of this buy-back on the EPS, assuming that the net income remains the same.

(July'21 QP 8 marks)

Solution 40

(i) Let P be the buyback price decided by SK Ltd.

Market Capitalisation after Buyback

1.15P (Original Shares – Shares Bought Back)

400 Lakhs = 1.15P[15lakhs - $\frac{30\% \text{ of } 150 \text{ lakhs}}{P}$

400 Lakhs = 17.25lakhs x P - 45 lakhs × 1.15 = 17.25 lakhs P – 51.75 lakhs

Again, 400 Lakhs = 17.25 lakhs P – 51.75 lakhs

or 17.25 lakhs P = 400 lakhs + 51.75 lakhs

or P = $\frac{451.75}{17.25}$ = 26.19

(ii) Number of Shares to be Bought Back : $\frac{45 \text{ lakh}}{26.19}$ = 1.718 lakhs (approx.) or 171821 share

(iii) Impact of Buy Back on the EPS:

No. of equity shares after buy back :- 15 lakhs – 1.718 lakhs = 13.282 lakhs or 15,00,000 – 1,71,821 = 13,28,179 shares

∴EPS = $\frac{4 \text{ x } 15 \text{ lakhs}}{13.282 \text{ lakhs}}$× = 4.52 or $\frac{4 \text{ x } 15 \text{ lakhs}}{13,28,179}$ = 4.52

Thus, EPS of SK Ltd., increases to 4.52 or increases by 0.52 (4.52 - 4.00)

41) Illustration

ABC Limited, just declared a dividend of ₹28.00 per share. Mr. A is planning to purchase the share of ABC Limited, anticipating increase in growth rate from 8% to 9%, which will continue for three years. He also expects the market price of this share to be ₹720.00 after three years.

You are required to determine:

- **the maximum amount Mr. A should pay for shares, if he requires a rate of return of 13% per annum.**
- **the maximum price Mr. A will be willing to pay for share, if he is of the opinion that the 9% growth can be maintained indefinitely and require 13% rate of return per annum.**
- **the price of share at the end of three years, if 9% growth rate is achieved and assuming other conditions remaining same as in (ii) above.**

Note : Calculate rupee amount up to two decimal points and use PVF upto 3 decimal points.

(RTP May'21 New & Old)

Solution 41

(i) Expected dividend for next 3 years.

Year 1 (D_1) 28.00 (1.09) = ₹30.52
Year 2 (D_2) 28.00 $(1.09)^2$ = ₹ 33.27
Year 3 (D_3) 28.00 $(1.09)^3$ = ₹36.26

Required rate of return = 13% (Ke)

Market price of share after 3 years = (P_3) = ₹720

The present value of share

$$P_0 = \frac{D_1}{1+K_e} + \frac{D_2}{(1+K_e)^2} + \frac{D_3}{(1+K_e)^3} + \frac{P_3}{(1+K_e)^3}$$

$$= \frac{30.52}{(1+0.13)} + \frac{33.27}{(1+0.13)^2} + \frac{36.26}{(1+0.13)^3} + \frac{720}{(1+0.13)^3}$$

P_0 = 30.52(0.885) + 33.27(0.783) +36.26(0.693) +720(0.693)

$\boldsymbol{P_0}$ **= ₹577.15**

(ii) If growth rate 9% is achieved for indefinite period, then maximum price of share should Mr. A willing be to pay is

$P_0 = \frac{D_1}{K_e - g} = \frac{30.52}{0.13-0.09}$ = **₹763**

(iii) Assuming that conditions mentioned above remain same, the price expected after 3 years will be:

$P_3 = \frac{D_4}{K_e - g} = \frac{D_3(1+9\%)}{0.13-0.09} = \frac{36.26(1.09)}{0.13-0.09}$ = **₹988**

42) Illustration

KLM Limited has issued 90,000 equity shares of ₹10 each. KLM Limited's shares are currently selling at ₹72. The company has a plan to make a rights issue of one new equity share at a price of ₹48 for every four shares held.

You are required to:

- **Calculate the theoretical post-rights price per share and analyse the change**
- **Calculate the theoretical value of the right alone.**

Suppose Mr. A who is holding 100 shares in KLM Ltd. is not interested in subscribing to the right issue, then advice what should he do.

(RTP May'21 New & Old)

Solution 42

(i) Calculation of theoretical Post-rights (ex-right) price per share

Ex Right Price of share = $\frac{N_0 P_0 + S}{N_0 + N_1} = \frac{72 \times 4 + 48 \times 1}{4+1}$ = **₹67.20**

Thus, post right issue the price of share has reduced by 4.80 per share

(ii) Calculation of theoretical value of the rights alone:

= Ex-right price – Cost of rights share
= 67.20 – 48 = 19.20
= **₹4.80**

(iii) If Mr. A is not interested in subscribing to the right issue, he can renounce his right eligibility @ 19.20 per right and can earn a gain of 480.

43) Illustration

M/s. B Ltd. has declared dividend of ₹2.50 per share on the EPS of ₹7. Earnings of the company are expected to grow at the rate of 10% for the next 3 years and to be stabilized at 3% thereafter. The pay-out ratio is expected to remain at the same level during 3 years and then will increase to 60%. If required rate of return is 16% calculate:

i. The current price of the share.

ii. The expected price of share of B Ltd. At the end of 3rd year.

Following table may be used for calculations.

Present Values	t_1	t_2	t_3	t_4	t_5
$PVIF_{0.16,\ t}$	0.862	0.743	0.641	0.553	0.477

[Jan'21 QP (Old)]

Solution 43

Working Notes:

Period	EPS	Dividend
1	7.70	2.750
2	8.47	3.025
3	9.317	3.327
4th onwards	9.60	5.76

(i) **Current price of the Share**

= PV of Dividends upto 3 Years + PV of Expected price of share of at the end of 3rd year

Expected price of share of B Ltd. at the end of 3rd year = $\frac{5.76}{0.16-0.03}$ = **₹44.31**

Accordingly, Current Market Price of Share shall be:

= 2.750 x 0.862 + 3.025 x 0.743 + 3.327 x 0.641 + 44.31 x 0.641
= 2.371 + 2.248 + 2.133 + 28.403 = **35.155** say **35.16**

(ii) **Expected price of share of B Ltd. at the end of 3rd year**

= $\frac{5.76}{0.16-0.03}$ = **₹44.31**

44) Illustration

Mr. A is holding 1000 shares of face value of ₹ 100 each of M/s. ABC Ltd. He wants to hold these shares for long term and has no intention to sell.

On 1st January 2020, M/s XYZ Ltd. has made short sales of M/s. ABC Ltd.'s shares and approached Mr. A to lend his shares under Stock Lending Scheme with following terms:

i. Shares to be borrowed for 3 months from 01-01-2020 to 31-03-2020
ii. Lending Charges/Fees of 1% to be paid every month on the closing price of the stock quoted in Stock Exchange and
iii. Bank Guarantee will be provided as collateral for the value as on 01-01-2020.

Other Information:

a. Cost of Bank Guarantee is 8% per annum,
b. On 29-02-2020, M/s. ABC Ltd., declared dividend of 25%
c. M/s ABC Ltd.'s share quoted in Stock Exchange on various dates are as follows:

Date	Share Price in Scenario -1 Bullish	Share Price in Scenario -2 Bearish
01-01-2020	1000	1000
31-01-2020	1020	980
29-02-2020	1040	960
31-03-2020	1050	940

You are required to find out:

i. Earning of Mr. A through Stock Lending Scheme in both the scenarios
ii. Total Earnings of Mr. A during 01-01-2020 to 31-03-2020 in both the scenarios
iii. What is the Profit or loss to M/s. XYZ by shorting the shares using through Stock Lending Scheme in both the scenarios?

[RTP May'22, RTP Nov'24, MTP Apr'23, Jan'21 QP (Old)]

Solution 44

Earnings of Mr. A through stock lending scheme

		Scenario 1	Scenario 2
(i)	Lending fee		
	31-01-20 1020 x 1% and 980 x 1%	10.20	9.80
	29-02-20 1040 x 1% and 960 x 1%	10.40	9.60
	31-03-20 1050 x 1% and 940 x 1%	10.50	9.40
	Earnings from lending per Share (A)	**31.10**	**28.80**
	Total No. of Shares	1000	1000
	Total Earning from Lending	**31,100**	**28,800**
(ii)	Dividend income per Share (B)	25.00	25.00
	Total earnings per share (A) + (B)	56.10	53.80
	Total No. of Shares	1000	1000

	Total Earning	**56,100**	**53,800**
(iii)	Gain on shorting the shares (1,050 - 1,000) and (1,000 - 940)	(50.00)	60.00
	Lending fees paid	(31.10)	(28.80)
	Bank guarantee charges @ 8%	(20.00)	(20.00)
	Dividend Exp (If paid before March 31st)	(25.00)	(25.00)
	Gain Per Share	**(126.10)**	**(13.80)**
	Total No. of Shares	1000	1000
	Total Gain on shorting the shares	(1,26,100)	(13,800)

45) Illustration

Mr. X has submitted the following data:

Particulars	(₹) in Lakhs
Total Assets	250
Total Liabilities	220
Net Income	12
Dividend Paid	4.5
Sales	100

Mr. X wants to know to what extent sales can be increased without going for additional borrowings by using the Sustainable Growth Rate (SGR) concept?

(Nov'22 QP)

Solution:

SGR = $ROE * Retention\ Rate$

= $r * b$

b = $1 - d$

d = Dividend payout rate

Given,

Total Assets = 250 lakhs

Total Liability = 220 Lakhs

Equity = 30 lakhs

Net Income = 12lakhs

ROE = $12/30$ = 40%

Dividend Paid = 4.5 lakhs

Retained Earnings = 12- 4.5 = 7.5 lakhs

Retained Earnings % = $7.5/12$ = 62.5%

SGR (g) = b* r

= 62.5%* 40% = **25%**

Sales (Original) = 100 lakhs

g = 25%

Revised Sales = 100(1+25%) = **125 Lakhs**

46) Illustration

An investor is considering purchasing equity shares of Alpha Ltd., whose current Market price in May 2023 is 172.45. The company is proposing a dividend of 6 for the year ending 31st March 2024. Alpha Ltd. is expected to grow @ 20 percent per annum for the next four years. Thereafter, the growth, over the next three years, will decline linearly by 100 basis points per annum. Thereafter, it will stabilize at a certain growth rate per annum infinitely. The required rate of return for the investor is 20%.

Dividend value is to be taken in 2 decimal points only. You are required:

(i) To calculate the stable growth rate of Alpha Ltd. after the end of 7 years.

(ii) To advise whether it is worth purchasing the share at this price if the investor has a stable target growth rate of 15% per annum.

Period	1	2	3	4	5	6	7
PVIF (20%, n)	0.8333	0.6944	0.5787	0.4823	0.4019	0.3349	0.2791

(May'23 QP 8 marks)

Solution:

Assuming growth rate of 20% from D_0 - D_4 i.e., 4 years and D_5 - D_7 @ 19% ,18% and 17%.

D_1 = 6

D_2 = 6*1.2 = 7.2

D_3 = 6* $(1.2)^2$ = 8.64

D_4 = 6* $(1.2)^3$ = 10.37

D_5 = 6* $(1.2)^3$ *1.19 = 12.34

D_6 = 6* $(1.2)^3$ *1.19 * 1.18 = 14.56

D_7 = 6* $(1.2)^3$ *1.19 * 1.18 * 1.17 = 17.03

Present Value of Dividends

	Dividend	PVF @ 20%	PV of Dividend
D_1	6	0.833	5
D_2	7.2	0.6944	5
D_3	8.64	0.5782	5
D_4	10.37	0.5823	5
D_5	12.34	0.4019	4.96
D_6	14.56	0.3349	4.88
D_7	17.03	0.2791	4.75
			34.59

P_0 = PV of Dividend for Explicit Period + PV of Dividend till Infinity

172.45 = 34.59 + PV of Dividend till Infinity

PV of D till Infinity

= 172.45 - 34.59

= 137.86

PV of D_8 - D_∞

D_7 = 17.03

k_e = 20%

D_8 = D_7 (1+g)

$P_7 = \frac{D_8}{ke-g} = \frac{D_7\,(1+g)}{ke-g} = \frac{17.03\,(1+g)}{0.2-g}$ **(1)**

$P_7 = \frac{PV\ of\ P_7}{PVF} = \frac{137.86}{0.2791}$ = 493.95 **(2)**

Equating **(1)** and **(2)** ;

$\frac{17.03\,(1+g)}{0.2-g} = 493.95$

$17.03\,(1+g) = 98.79 - 493.95g$

$17.03 - 98.79 = -17.03\,g - 493.95\,g$

$g = \frac{81.76}{510.98}$ = **16%**

i) Stable growth rate of Alpha Ltd. After 7 years is **16%.**

ii) If target Growth rate is 15% it is **advisable to purchase** this stock as it has a higher stable growth.

47) Illustration

Calculate the value of one equity share of X Ltd. from the following Information:

Profit of the company (Before tax)	₹ 8000 crores
Equity share capital of the Company	₹ 19000 crores
No. of Equity Shares	380 crores
Long run growth rate of the company	7%
Risk free Rate of Return	9.50%
Beta of the company	0.1
Market Risk Premium	3.10%
Total Capital expenditure	₹ 20140 crore
Chargeable Depreciation	₹ 17100 crore
Total Increase in working capital	₹ 1755.60 crore
New Debt to be issued for funding	₹ 2062.108 crore
Tax Rate	30%

Note: All calculation to rounded off upto 4 decimal points and final value of equity share to be rounded off upto 2 decimal points.

(MTP Oct'24)

Solution:

Profit After Tax (PAT) or Net Income = ₹ 8000 crores (1 – 0.30) = ₹ 5600 crores

FCFE = FCFF + New Debt Issued - Debt Repayments + Net issue of Preference Shares – Preference Share Dividends

FCFF = Net Income - Capital Expenditures + Depreciation -/+ Change in Net Working Capital

FCFF = ₹ 5600 crores - ₹ 20140 crore + ₹ 17100 crore - ₹ 1755.60 crore

FCFF = 804.4

FCFE = 804.4+ ₹ 2062.108 crore = ₹ 2866.508 crore

Cost of Equity = Rf + ß (Rm – Rf) or Rf + ß Market Risk Premium

= 9.50% + 0.1 x 3.10% = 9.81%

Value of Equity = FCFE (1+g) / (ke-g)

= 2866.508 Cr x (1+7%) / (9.81% - 7%)

= 3067.1636 Cr/ 2.81%

= ₹ 109151.7295 Cr

Value of one Equity Share = 109151.7295 crore / 380 Crore Shares = ₹ 287.24

48) Illustration (Nov'24) Case Based MCQ

Z Ltd. paid a dividend of ₹ 5 for the current year. The dividend is expected to grow at 25% for the next 6 years and at 10% per annum thereafter. The return of government bond is 13% per annum and market return is expected to be around 20%. The correlation between market return and Z Ltd. share return is 0.3733. The standard deviation of market return and Z Ltd. shares is 12% and 18% respectively. Round off to two decimal places.

From the information given above, choose the correct answer to the Question no. 1 to 5:

(Case Based MCQ Nov'24 QP 2 marks each)

Question no. 1 to 5:

What is the present value at the end of 4th year?

A. ₹ 23.71
B. ₹ 12.56
C. ₹ 6.53
D. ₹ 6.99

What is the intrinsic value of Z Ltd. shares?

A. ₹ 156.69
B. ₹ 303.14
C. ₹ 349.62
D. ₹ 341.30

What is the expected return of Z Ltd shares?

A. 15%
B. 23.92%
C. 16.92%
D. 16.5%

What is value in perpetuity at the start of the 6th year?

A. ₹ 156.69
B. ₹ 303.14
C. ₹ 349.62
D. ₹ 341.30

If current market price of the shares is ₹ 315 then stock is

A. Over valued
B. Under valued
C. Fairly valued
D. Cannot be determined

Solution:

Given:

1. Current Dividend (D0) = ₹5
2. Growth Rate of Dividend:
   - For the next 6 years = 25% per annum
   - Thereafter (from year 7 onward) = 10% per annum
3. Required Rate of Return Calculation:
   - Risk-Free Rate (Rf) = 13%
   - Market Return (Rm) = 20%
   - Correlation between Market and Z Ltd. = 0.3733
   - Standard Deviation of Market (σm) = 12%
   - Standard Deviation of Z Ltd. shares (σz) = 18%

Q1, 2 & 4

Amount in ₹

Year	Dividend	Dividend	PVF @16.92%	PVF @16.92%	PV of Dividend	
0	5	5				
1	5 x ($1+25\%)^1$	6.25	$1/(1+16.92\%)^1$	0.8553	5.346	
2	5 x ($1+25\%)^2$	7.81	$1/(1+16.92\%)^2$	0.7315	5.715	
3	5 x ($1+25\%)^3$	9.77	$1/(1+16.92\%)^3$	0.6257	6.110	
4	5 x ($1+25\%)^4$	12.21	$1/(1+16.92\%)^4$	0.5351	**6.532**	**Q1**
5	5 x ($1+25\%)^5$	15.26	$1/(1+16.92\%)^5$	0.4577	6.984	
6	5 x ($1+25\%)^6$	19.07	$1/(1+16.92\%)^6$	0.3914	7.466	
	PV of Sum of Dividends for periods 1-6 i.e explicit period				**38.152**	

Terminal period value P6		**D7 (ke-g)**	
Terminal period value P6		D6 (1+g) / (ke-g)	
Terminal period value P6		(19.07 × 1.10) / (0.1692 - 0.10)	
Terminal period value P6		**303.136**	**Q4**
PV of TV		303.136 x 0.3914	
PV of TV		118.647	
Price of Share P0=	PV of explicit Period Dividends (Year 1-6) + PV of terminal Value		
P0 =	38.152 + 118.647		
P0 =	**156.799**		**Q2**

Question 3: What is the Expected Return of Z Ltd Shares?
To find the expected return (k) for Z Ltd., we use the Capital Asset Pricing Model (CAPM):
k = Rf + β(Rm - Rf)
where:

β = (Correlation × σz) / σm
Substituting the values:
β = (0.3733 × 18%) / 12% = 0.56
k = 13% + 0.56 × (20% - 13%) = 16.92%

Therefore, the expected return of Z Ltd. shares is 16.92% (Answer: Option C).

Question 4: What is the Value in Perpetuity at the Start of the 6th Year?
From our intrinsic value calculations in Question 2, the value of dividends from year 7 onward at the start of year 7 (end of year 6) is:
Value in perpetuity at the start of the 6th year = (19.06 × 1.10) / (0.1692 - 0.10) = ₹303.14
Answer: Option B (₹ 303.14).

Question 5: If Current Market Price of the Shares is ₹ 315, is the Stock Overvalued, Undervalued, or Fairly Valued?
Since the intrinsic value is ₹156.69, which is less than ₹315, the stock is overvalued.
Answer: Option A

49) Illustration (RTP Sep'25)

XYZ company has current earnings of ₹ 3 per share with 5,00,000 shares outstanding. The company plans to issue 40,000, 7% convertible preference shares of ₹ 50 each at par. The preference shares are convertible into 2 shares for each preference shares held. The equity share has a current market price of ₹ 21 per share.

i. **What is preference share's conversion value?**
ii. **What is conversion premium?**
iii. **Assuming that total earnings remain the same, calculate the effect of the issue on the basic earning per share**
   (a) before conversion
   (b) after conversion.
iv. **If profits after tax increases by ₹ 1 million what will be the basic EPS**
   (a) before conversion and
   (b) on a fully diluted basis?

(RTP Sep'25)

Solution:
Part 1: Conversion value of preference share
Conversion Ratio x Market Price
2 × ₹21 = ₹42

Part 2: Conversion Premium
(₹ 50 / ₹ 42) – 1 = 19.05%

Part 3: Effect of the issue on basic EPS

Particulars	₹
Before Conversion	
Total (after tax) earnings ₹ 3 × 5,00,000	15,00,000

Dividend on Preference shares	1,40,000
Earnings available to equity holders	13,60,000
No. of shares	5,00,000
EPS	2.72
On Diluted Basis	
Earnings	15,00,000
No. of shares (5,00,000 + 80,000)	5,80,000
EPS	2.59

Part 4: EPS with increase in Profit

Particulars	₹
Before Conversion	
Earnings	25,00,000
Dividend on Preference shares	1,40,000
Earnings for equity shareholders	23,60,000
No. of equity shares	5,00,000
EPS	4.72
On Diluted Basis	
Earnings	25,00,000
No. of shares	5,80,000
EPS	4.31

50) Illustration

ABC Ltd.'s share is currently traded at the price of ₹ 192.50 per share. Mr. Roni is planning to purchase the shares of the company. For this purpose, he has taken the services of a financial analyst to know whether the price of ABC Ltd. is fairly priced. The analyst has assembled the following information:

- **The before-tax required rates of return on ABC Ltd. debt, preferred stock, and common stock are 8.60%, 11%, and 13%, respectively.**
- **The company's target capital structure is 20% debt, 30% preferred stock and 50% Common stock.**
- **The market value of the company's debt is ₹ 275 million and its preferred stock are valued at ₹ 120 million.**
- **ABC Ltd.'s free cash flow to the firm (FCFF) for the year just ended is ₹ 125 million. FCFF is expected to grow at a constant rate of 8% for the foreseeable future.**
- **The tax rate is 30%.**
- **ABC Ltd. has 20 million outstanding common shares. You are required to -**

(i) As a financial analyst, on the basis of value per share, advise Mr. Roni whether he should purchase the shares of the company at market price or not.

(ii) Assume, we are to get same value of equity as calculated in (i) for using FCFE approach, calculate free cash flow to the equity (FCFE) for the year just ended, if FCFE is expected to grow at a constant rate of 8.50% for the foreseeable future.

Calculation up to 2 decimal points. *(Sep -25, 7 Marks)*

Solution:

(1)

Cost	Pre Tax return	Target Ratio	MV (₹ Mn)
Ke	13.00%	0.5	20 x 192.5 = 385
Kd	8.60%	0.2	275
Kp	11.00%	0.3	120

WACC = 8.6% x (1-0.3) x 0.2 +0.5 x 13% x 0.3 x 11% = 11%

EV = $FCFF_0$ (1+g) / (WACC – g)
= ₹125 Mn x (1.08)/(11%-8%)
= ₹4500 Mn

MV of Debt & Pref Shares = ₹395 Mn
Intrinsic Value of Equity = ₹4500 Mn – ₹395 Mn = ₹ 4105 Mn
Share count = 20 Mn
Intrinsic Value per share = ₹4105 Mn / 20 Mn = ₹205.25

CMP is ₹192.50 and IV is ₹205.25; Buy as share is undervalued

(2) Growth rate of FCFE is 8.5% and IV of Equity = ₹4105 Mn, Ke = 13%. $FCFE_0$= ?
₹ 4105 Mn = $FCFE_0$ x 1.085/(13% - 8.5%)
$FCFE_0$ = ₹4105 Mn x 0.045/1.085
$FCFE_0$ = ₹ 170.25 Mn

1) Illustration

The following data are available for a bond:

- Face Value ₹10,000 to be redeemed at par on maturity
- Coupon rate 8.5 % p.a.
- Years to Maturity 5 years
- Yield to Maturity (YTM) 10%

You are required to calculate:

(i) Current market price of the Bond
(ii) Macaulay's Duration
(iii) Volatility of the Bond
(iv) Convexity of Bond
(v) Expected market price if there is decrease in YTM by 200 basis points
  a. By Macaulay's Duration based estimate
  b. By Intrinsic Value Method

Years	1	2	3	4	5
PVIF (10%, n)	0.909	0.826	0.751	0.683	0.621
PVIF (8%, n)	0.926	0.857	0.794	0.735	0.681

(Nov'20 QP 7 marks, RTP May'23)

Solution 1

(a) (i) **Current Market Price of Bond**

$$P_0 = \sum_{t=1}^{n} \frac{C}{(1+y)^t} + \frac{M}{(1+y)^n}$$

= 850 (PVIAF 10%, 5) + 10,000 (PVIF 10%, 5)

= 850 (3.79) + 10,000 (0.621) = 3,221.50 + 6,210 = **₹9,431.5**

(ii) **Macaulay's Duration**

Year	Cash flow (b)	PVF@ 10% (c)	Present Value (b) * (c) = (d)	Proportion of bond value (e)	Proportion of bond value x time (years) (e) *(a) = f
1	850	0.909	772.65	0.082	0.082
2	850	0.826	702.10	0.074	0.148
3	850	0.751	638.35	0.068	0.204
4	850	0.683	580.55	0.062	0.248
5	10,000 + 850	0.621	6737.85	0.714	3.57
			9431.50	1.000	**4.252**

Duration of the bond = **4.252 years**

Alternatively,

$$\sum_{t=1}^{n} \frac{t * c}{(1+i)^t} + \frac{n * M}{(1+i)^n}$$

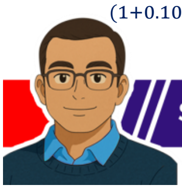
Part -1

$$\frac{1*850}{(1+0.10)} + \frac{2*850}{(1+0.10)^2} + \frac{3*850}{(1+0.10)^3} + \frac{4*850}{(1+0.10)^3} + \frac{5*850}{(1+0.10)^5}$$

= 1*850(0.909) + 2*850(0.826) + 3*850(0.751) + 4*850(0.683) + 5*850(0.621) = **9053.35**

Part-2

$\frac{5*10,000}{(1+0.10)^5}$ = **31,050**



Macaulay's Duration = $\frac{9053.35+31050}{9431.50}$ = **4.252 years**

(iii) **Volatility of Bond**

Volatility of Bonds = Modified Duration = $\frac{\text{Duration}}{(1+y)}$ = $\frac{4.252}{1.10}$ = 3.865

(iv) Convexity of Bond

$C^* \times (\Delta y)^2 \times 100$

$C^* = \frac{V_+ + V_- - 2V_0}{2V_0\,(\Delta y)^2}$

Year	Cash flow	PVF@ 8%	Present Value	PVF@ 12%	Present Value
1	850	0.926	787.10	0.082	758.20
2	850	0.857	728.45	0.074	677.45
3	850	0.794	674.90	0.068	605.20
4	850	0.735	624.75	0.062	540.60
5	10,000 + 850	0.681	7388.85	0.714	6151.95
			10204.05		**8733.40**

$C^* = \frac{10{,}204.05 + 8{,}733.40 - 2 \times 9{,}431.50}{2 \times 9{,}431.50 \times (0.02)^2} = \frac{74.45}{7.5452} = 9.867$

So, $C^* \times (\Delta y)^2 \times 100 = 9.867 \times (0.02)^2 \times 100$ = **0.395%**

(v) The expected market price if decrease in YTM by 200 basis points

(A) <u>By Macaulay's duration-based estimate</u>

9431.50 x 2 (3.865/100) = ₹729.05

Hence expected market price is 9431.50 + 729.05 = **10,160.55**

Hence, the market price will increase.

(B) <u>By Intrinsic Value method</u>

Intrinsic Value at YTM of 10%	₹9,431.50
Intrinsic Value at YTM of 8%	₹10,204.05
Price increased by	₹772.55

Hence, expected market price is ₹10,204.05

2) Illustration

The following data are available for three bonds A, B and C. These bonds are used by a bond portfolio manager to fund an outflow scheduled in 6 years.

Current yield is 9%. All bonds have face value of ₹100 each and will be redeemed at par. Interest is payable annually.

Bond	Maturity (years)	Coupon rate
A	10	10%
B	8	11%
C	5	9%

(i) Calculate the duration of each bond.

(ii) The bond portfolio manager has been asked to keep 45% of the portfolio money in Bond A. Calculate the percentage amount to be invested in bonds B and C that need to be purchased to immunise the portfolio.

(iii) After the portfolio has been formulated, an interest rate change occurs, increasing the yield to 11%. The new duration of these bonds are: Bond A = 7.15 Years, Bond B = 6.03 Years and Bond C = 4.27 years.

Is the portfolio still immunized? Why or why not?

(iv) Determine the new percentage of B and C bonds that are needed to immunize the portfolio. Bond A remaining at 45% of the portfolio. *(Nov'18 QP 12 marks, MTP Mar'21 New & Old, MTP May'20 Old)*

Solution 2

(i) Macaulay's Duration

$$\sum_{t=1}^{n} \frac{t * c}{(1+i)^t} + \frac{n * M}{(1+i)^n}$$

Bond A

Year (a)	Cash flow (b)	PVF@9% (c)	Present Value(a*b*c)
1	10	0.917	**9.17**
2	10	0.842	16.83
3	10	0.772	23.17
4	10	0.708	28.34
5	10	0.650	32.50
6	10	0.596	35.78
7	10	0.547	38.29
8	10	0.502	40.15
9	10	0.460	41.44
10	**10+100**	0.422	464.65
			∑=730.32

PVAF = 6.417 * 10 = 64.17
+ 0.422*100 = 42.24
106.41

So, D = $\frac{730.32}{106.41}$ = 6.86 Years

Bond B

Year (a)	Cash flow (b)	PVF@9% (c)	Present Value(a*b*c)
1	**11**	0.917	10.08
2	**11**	0.842	18.52
3	**11**	0.772	25.48
4	**11**	0.708	31.15
5	**11**	0.650	35.75
6	**11**	0.596	39.33
7	**11**	0.547	42.12
8	**111**	0.502	445.776
			648.2

PVAF = 5.53* 11 = 60.88
+ 0.502*100 = 50.20
111.08

So, D = $\frac{648.2}{111.08}$ = **5.835Years**

Bond C

Year (a)	Cash flow (b)	PVF@9% (c)	Present Value (a*b*c)
1	9	0.917	8.253
2	9	0.842	15.156
3	9	0.772	20.084
4	9	0.708	25.49
5	**109**	0.650	354.25
			423.991

PVAF = 3.88* 9 = 35
+ 0.65*100 = 65
100

So, D = $\frac{423.991}{100}$ = **4.24 Years**

(ii) Now if,

Bond	Duration	Weight
A	6.86 Years	45%
B	5.84 Years	b
C	4.24 Years	55% - b

Target duration = 6 Years
6.86 x 0.45 + 5.83 x b + 4.24(0.55 – b) = 6
3.087 + 5.83b + 2.332 – 4.24b = 6
5.419 + 1.59b = 6
1.59b = 6 – 5.419
b = 0.581/1.59 = 36.54%
Then, 55% - 36.54% = **18.46%**

(iii)

Bond	Duration	Weight	Duration * Weight
A	7.15 Years	45%	3.218
B	6.03 Years	36.54 %	2.203
C	4.27 Years	18.46%	0.788
			6.209

Duration of the portfolio has increased as the duration of the individual bonds has increased. Hence the portfolio is not immunized as the portfolio duration is 6.2 vs maturity of 6 years

(iv)

Bond	Duration	Weight	Weighted duration
A	7.15 Years	45%	3.2175
B	6.03 Years	b	6.03b
C	4.27 Years	55% - b	2.35 - 4.27b
			∑ = 5.566 + 1.76b

5.566 + 1.76b = 6 years
1.76b = 6 - 5.566
b = 0.434/1.76 = **24.66%**
c = 55% - b = 55% - 24.66% = **30.34%**

So, A = 45% B = 24.66% C = 30.34%

3) Illustration

A convertible bond with a face value of ₹1,000 is issued at ₹1,350 with a coupon rate of 10.5%. The conversion rate is 14 shares per bond. The current market price of bond and share is ₹1,475 and ₹80 respectively. What is the premium over conversion value?

(ICAI SM, Old PM)

Solution 3

Given, FV =₹1000 Issue price = ₹1350 coupon rate = 10.5%
Conversion rate is 14 shares per bond. Market price of share ₹80
Conversion Value = 14 x 80 = ₹1120
Market price of bond = ₹1475
Premium over Conversion Value (1475- 1120) = 355/1120 x 100 = **31.7%**

4) Illustration

Nominal value of 10% bonds issued by a company is ₹100. The bonds are redeemable at ₹110 at the end of year 5.
Determine the value of the bond if required yield is
(i) 5%
(ii) 5.1%
(iii) 10% and
(iv) 10.1%

[ICAI SM, Nov'19 QP (Old), Old PM]

Solution 4

Case (i) Required yield rate = 5%

Year	Cash flow	DF (5%)	Present Value ₹
1-5	10	4.3295	43.295
5	100+10	0.7835	86.185
	Value of bond		**129.48**

Case (ii) Required yield rate = 5.1%

Year	Cash flow	DF (5.1%)	Present Value ₹
1-5	10	4.3175	43.175
5	100+10	0.7798	85.778
	Value of bond		**128.953**

Case (iii) Required yield rate = 10%

Year	Cash flow	DF (10%)	Present Value ₹
1-5	10	3.7908	37.908
5	100+10	0.6209	68.299
	Value of bond		**106.207**

Case (iv) Required yield rate = 10.1%

Year	Cash flow	DF (10.1%)	Present Value ₹
1-5	10	3.7811	37.811
5	100+10	0.6181	67.991
	Value of bond		**105.802**

5) Illustration

An investor is considering the purchase of the following Bond:

Face value	₹100
Coupon rate	11%

Maturity 3 years

(i) If he wants a yield of 13%, what is the maximum price, he should be ready to pay for?

(ii) If the Bond is selling for ₹97.60, what would be his yield?

(ICAI SM, Old PM)

Solution 5

Given, Face value = ₹100 Coupon rate = 11% Maturity = 3 years

(i) Calculation of Maximum price

= ₹11 × PVIFA (13%,3) + ₹100 × PVIF (13%,3)
= ₹11 × 2.361 + ₹100 × 0.693 = ₹25.97 + ₹69.30 = ₹95.27

(ii) Calculation of yield

At 12% the value = ₹11 × PVIFA (12%,3) + 100 × PVIF (12%,3)
= ₹11×2.402 + ₹100×0.712 = ₹26.42 + ₹71.20 = ₹97.62
It the bond is selling at ₹97.60 which is more than the fair value, the YTM of the bond would be less than 13%. This value is almost equal to the amount price of ₹97.60. Therefore, the YTM of the bond would be 12%.

Alternatively,

YTM = $\frac{11+\frac{(100-97.60)}{3}}{\frac{(100+97.60)}{2}}$ = 0.1194 or 11.94% say 12%

6) Illustration

Calculate Market Price of:

(i) **10% Government of India security currently quoted at ₹110, but yield is expected to go up by 1%.**

(ii) **A bond with 7.5% coupon interest, Face Value ₹10,000 & term to maturity of 2 years, presently yielding 6%. Interest payable half yearly.**

(ICAI SM, Old PM)

Solution 6

(i) Current yield = (Coupon Interest / Market Price) x 100
(10/110) x 100 = 9.09%
If current yield go up by 1% i.e. 10.09 the market price would be
10.09 = 10 / Market Price x 100
Market Price = ₹99.11
(ii) Market Price of Bond = P.V. of Interest + P.V. of Principal
= ₹1,394 + ₹8,885 = **₹10,279**

7) Illustration

(7a) There is a 9% 5-year bond issue in the market. The issue price is Rs. 90 and the redemption price Rs. 105. For an investor with marginal income tax rate of 30% and capital gains tax rate of 10% (assuming no indexation), what is the post-tax yield to maturity?

(Old PM, MTP Sept'15)

Solution 7a

Coupon = 9% Maturity = 5yrs Issue price = ₹90 Redemption Price = ₹105 Tax rate = 30% /10%

Year	Cash flow	Post tax cash flow	PVF@9%	PVCF	PVF@10%	PVCF
0	-90	-90	1	-90	1	-90
1	9	6.3	0.917	5.7771	0.909	5.727
2	9	6.3	0.842	5.3046	0.826	5.204
3	9	6.3	0.772	4.8636	0.751	4.731
4	9	6.3	0.708	4.4604	0.683	4.303

5	105 + 9	109.8	0.650	71.37	0.621	68.186
				+ 1.7757		- 1.8495

Assumption

Face Value = ₹100 Issue Price = ₹90 Discount on issue ₹10 Premium on redemption ₹5 Redemption price 105 Annual Coupon 9% on ₹100

Working Note

Post tax cash flow Yr 1-4

Coupon = ₹9 Tax rate 30% Post tax cash flow = 9 x 70% = ₹6.3

On redemption value : Issue price – Redemption value = 90-105 = 15

₹15 is capital gain net off tax@10% which is ₹1.5

On Coupon ₹9 less tax @30% Net Cash flow = 6.3

Cash flow in Yr 5 for principal value = 105 less tax 1.5 = 103.5
Total Cash flow post tax Yr 5 = 103.5 + 6.3 = **₹109.8**

Using Interpolation YTM =

$9\% + \frac{1.7757}{1.7757-(-1.8495)} \times (10\% - 9\%) = 9\% + 0.49 = \mathbf{9.49\%}$

(7b) If the market price of the bond is ₹95; years to maturity = 6 yrs: coupon rate = 13% p.a. (paid annually) and issue price is ₹100. What is the yield to maturity?

(Old PM, MTP Feb'14)

Solution 7b

Present Value of Cashflow @14%

PV = 13PVIF(14% , 6yrs) + 100PVF(14% , 6TH yr)
= 3.888 x 13 + 0.455 x 100 = 50.55 + 45.55 = 96.10
Market Price is 95
SO, NPV = 96.10 – 95 = +1.10

Present Value of Cashflow @15%

PV = 13PVIF(1% , 6yrs) + 100PVF(15% , 6TH yr)
13 x 3.7844 + 100 x 0.4323 = 49.498 + 43.23 = 92.43
NPV = 92.43 – 95 = - 2.57

Using Interpolation,

$14\% + \frac{1.10}{1.10-(-2.57)} \times (15\% - 14\%) = 14\% + 0.299 = \mathbf{14.30\%}$

8) Illustration

M/s Agfa Industries is planning to issue a debenture series on the following terms:

Face value	₹100
Term of maturity	10 years
Yearly coupon rate	
Years	
1 – 4	9%
5 – 8	10%
9 – 10	14%

The current market rate on similar debentures is 15% p.a. The Company proposes to price the issue in such a manner that it can yield 16% compounded rate of return to the investors. The Company also proposes to redeem the debentures at 5% premium on maturity. Determine the issue price of the debentures.

(RTP May'20, Old PM)

Solution 8

The issue price of the debentures will be the sum of present value of interest payments during 10 years of its maturity and present value of redemption value of debenture.

Years	Cash out flow (₹)	PVIF @ 16%	PV
1	9	0.862	7.758
2	9	0.743	6.687
3	9	0.641	5.769
4	9	0.552	4.968
5	10	0.476	4.76
6	10	0.410	4.10
7	10	0.354	3.54
8	10	0.305	3.05
9	14	0.263	3.682
10	14 + 105 = 119	0.227	3.178 + 23.835
			71.327

Thus the debentures should be priced at ₹71.327

9) Illustration

Based on the credit rating of bonds, Mr. Z has decided to apply the following discount rates for valuing bonds:

Credit Rating	Discount Rate
AAA	364 day T bill rate + 3% spread
AA	AAA + 2% spread
A	AAA + 3% spread

He is considering to invest in AA rated, ₹1,000 face value bond currently selling at ₹1,025.86. The bond has five years to maturity and the coupon rate on the bond is 15% p.a. payable annually. The next interest payment is due one year from today and the bond is redeemable at par. (Assume the 364 day T-bill rate to be 9%).

You are required to calculate the intrinsic value of the bond for Mr. Z. Should he invest in the bond? Also calculate the current yield and the Yield to Maturity (YTM) of the bond.

(RTP May'19, RTP Nov'11, Old PM)

Solution 9

The appropriate discount rate for valuing the bond for Mr. Z is:
R = 9% + 3% + 2% = 14%

Year	CF	PVIF 14% PV (CF)	PV (CF)
1	150	0.877	131.55
2	150	0.769	115.35
3	150	0.675	101.25
4	150	0.592	88.80
5	1150	0.519	596.85
			P_0 = **1033.80**

Since, the current market value is less than the intrinsic value; Mr. Z should buy the bond.
Current yield = Annual Interest / Price = 150 / 1025.86 = **14.62%**

The YTM of the bond is calculated as follows:

<u>@15%</u>

P = 150 × $PVIFA_{15\%,4}$ + 1150 × $PVIF_{15\%\ 5}$
= = 150 × 2.855 + 1150 × 0.497 = 428.25 + 571.55 = 999.80

<u>@14%</u>

As found in sub part (a) P0 = 1033.80
By interpolation we get,

= 14% + $\frac{7\ 94}{7\ 94-(-\ 26.06)}$ × 15 -% 14%) = 14% + $\frac{7.94}{34}$ %

YTM = 14.23%

10)Illustration

On 31st March, 2013, the following information about Bonds is available:

Name of Security	Face Value ₹	Maturity Date	Coupon Rate	Coupon Date(s)
Zero coupon	10,000	31st March, 2023	N.A.	N.A.
T-Bill	1,00,000	20th June, 2013	N.A.	N.A.
10.71% GOI 2023	100	31st March, 2023	10.71	31st March
10% GOI 2018	100	31st March, 2018	10.00	31st March & 30th September

Calculate:

(i) **If 10 years yield is 7.5% p.a. what price the Zero Coupon Bond would fetch on 31st March, 2013?**

(ii) **What will be the annualized yield if the T-Bill is traded @ 98500?**

(iii) **If 10.71% GOI 2023 Bond having yield to maturity is 8%, what price would it fetch on April 1, 2013 (after coupon payment on 31st March)?**

(iv) **If 10% GOI 2018 Bond having yield to maturity is 8%, what price would it fetch on April 1, 2013 (after coupon payment on 31st March)? [MTP Aug 17 (8 marks), May 15 (8 marks)]**

(Old PM, MTP Aug'17, May'15)

Solution 10

(i) Rate used for discounting shall be yield. Accordingly ZCB shall fetch:

$\frac{10000}{(1.075)^{10}}$ = 4852

(ii) The day count basis is actual number days / 365. Accordingly annualized yield shall be :

Yield = $\frac{FV-Price}{Price} * \frac{365}{No.of\ days} = \frac{100000-98500}{98500} * \frac{365}{81}$ = 6.86%

(iii) Price GOI 2023 would fetch
= ₹10.71 PVAF (8%, 10) + ₹100 PVF (8%, 10)
= ₹10.71 x 6.71 + ₹100 x 0.4632
= ₹71.86 + ₹46.32 = ₹118.18

(iv) Price GOI 2018 Bond would fetch:
= ₹5 PVAF(4%, 10) + ₹100 PVF (4%, 10)
= ₹5 x 8.11 + ₹100 x 0.6756
= 40.55 + 67.56 = **108.11**

11)Illustration

Pet feed plc has outstanding, a high yield Bond with following features:

Face Value	**£ 10,000**
Coupon	**10%**
Maturity Period	**6 Years**
Special Feature	**Company can extend the life of Bond to 12 years**

Presently the interest rate on equivalent Bond is 8%.

(a) If an investor expects that interest will be 8%, six years from now then how much he should pay for this bond now.
Now suppose, on the basis of that expectation, he invests in the Bond, but interest rate turns out to be 12%, six years from now, then what will be his potential loss/ gain if company extends the life of bond by another 6 years. ***(Old PM, RTP Nov'18 Old)***

Solution 11

(a) If the current interest rate is 8%, the company will not extent the duration of Bond and the maximum amount the investor would ready to pay will be:
= £1,000 PVIAF (8%, 6) + £10,000 PVIF (8%, 6)
= £1,000 x 4.623 + £10,000 x 0.630
= £4,623 + £ 6,300
= £ 10,923
(b) If the current interest rate is 12%, the company will extent the duration of Bond. After six years the value of Bond will be
= £1,000 PVIAF (12%, 6) + £10,000 PVIF (12%, 6)
= £1,000 x 4.111 + £10,000 x 0.507
= £4,111 + £5,070
= £9,181
Thus, potential loss will be £9,181 - £10,923 = £1,742

12)Illustration

XYZ Ltd.'s bond (Face Value of Rs. 1,000) with 4 years maturity is currently trading at Rs. 900 carrying a coupon rate of 15%. Assuming that the reinvestment rate is 16%, you are required to calculate Realized Yield to Maturity of the Bond.

(MTP Apr'18 Old)

Solution 12

We shall compute y* (Realized Yield to Maturity) by using following equation:
PV $(1+y^*)^4$ = Future Value of Benefits and Future Value of Benefits shall be computed as follows:

	0	1	2	3	4
Investment (Rs.)	900				
Annual Interest (Rs.)		**150**	**150**	**150**	**150**
Compound Factor @ 16%		**1.56**	**1.35**	**1.16**	**1.00**
Future Value of Intermediate CF (Rs.)		234.00	202.50	174.00	150
Maturity Value (Rs.)					1000.00
	900	234.00	202.50	174.00	1150.00
Total of Future Benefits					**1760.50**

Accordingly,
900 $(1+y^*)^4$ = 1760.50
$(1+y^*)^4$= 1760.50/900
$(1+y^*)^4$= 1.956
(1+y*) = $(1.183)^{1/4}$
y* = 0.183 say 18.30%

13)Illustration

The following data are available for a bond

Face value	**₹1,000**
Coupon Rate	**16%**
Years to Maturity	**6**
Redemption value	**₹1,000**
Yield to maturity	**17%**

Calculate the duration and volatility of this bond? Calculate the expected market price if increase in required yield is by 75 basis points.

(ICAI SM, RTP Nov'20, RTP May'16, Old PM)

Solution 13

To calculate the duration first we shall calculate Market price of bond as follows:
160 (PVIFA 17%,6) + 1,000 (PVIF 17%,6)
= 160 (3.589) + 1,000 (0.390)
= 574.24 + 390
= 964.24

1. Duration

Year	Cash flow	P.V. @ 17%		Proportion of bond value	Proportion of bond value x time (years)
1	**160**	0.855	136.80	0.142	**0.142**
2	**160**	0.731	116.96	0.121	**0.242**
3	**160**	0.624	99.84	0.103	**0.309**
4	**160**	0.534	85.44	0.089	**0.356**
5	**160**	0.456	72.96	0.076	**0.38**
6	**1160**	0.390	452.40	0.469	**2.814**
			964.40	**1.000**	**4.243**

Duration of the Bond is 4.243 years

2. Volatility

Volatility of the bonds= Modified Duration = $\frac{\text{Macaulay Duration}}{(1+\text{YTM})} = \frac{4.243}{1.17} = 3.63$

Change yield = 0.75%

Change in price = - Modified Duration x 0.75% x Price

Change in Price = -3.63 X 0.75% X 964.40 = -26.255

Revised price = 964.4 -26.244 = 938.14421

14)Illustration

The following data is available for a bond:

Face value	**₹1,000**
Coupon rate	**11%**
Years to Maturity	**6**
Redemption Value	**₹1,000**
Yield to Maturity	**15%**

(Round-off your answers to 3 decimals)

Calculate the following in respect of the bond:

(i) Current Market Price
(ii) Duration of the Bond
(iii) Volatility of the Bond
(iv) Expected market price if increase in required yield is by 100 basis points
(v) Expected market price if decrease in required yield is by 75 basis points

(RTP May'18 Old, MTP Apr'19, Nov 15 QP, Old PM)

Solution 14

(i) & (ii)Calculation of Market price & Duration

Year	**Cash flow**	**P.V. @ 15%**	**PVCF**	**Proportion of bond value**	**Proportion of bond value x time (years)**
1	110	0.870	95.70	0.113	0.113
2	110	0.756	83.16	0.098	0.196
3	110	0.658	72.38	0.085	0.255
4	110	0.572	62.92	0.074	0.296
5	110	0.497	54.67	0.064	0.320
6	1110	0.432	479.52	0.565	3.39

			MP -> 848.293	1.000	4.575

Duration of the Bond is 4.575 years

2. Volatility

Volatility of the bonds = $\frac{\text{Duration}}{(1+\text{YTM})} = \frac{4.575}{1.15}$ = 3.979

(iv) The expected market price if increase in required yield is by 100 basis points

₹848.293 × - 1.00 (3.979/100) = - ₹33.750

Hence expected market price is ₹848.293 - ₹33.750 = ₹814.543

Hence, the market price will decrease

(v) The expected market price if decrease in required yield is by 75 basis points.

₹848.293 × -(-0.75 (3.974/100)) = ₹25.315

Hence expected market price is ₹848.293 + ₹25.315 = ₹873.608

Hence, the market price will increase.

15)Illustration

Mr. A will need ₹1,00,000 after two years for which he wants to make one time necessary investment now. He has a choice of two types of bonds. Their details are as below:

	Bond X	Bond Y
Face value	₹1,000	₹1,000
Coupon	7% payable annually	8%payable annually
Years to maturity	1	4
Current price	₹972.73	₹936.52
Current yield	10%	10%

Advice Mr. A whether he should invest all his money in one type of bond or he should buy both the bonds and, if so, in which quantity? Assume that there will not be any call risk or default risk.

(ICAI SM, RTP Nov'21, MTP Mar'22, MTP Aug'18, MTP Oct'19 Old, Old PM)

Solution 15

Duration of Bond X

Year	Cash flow	P.V. @ 10%		Proportion of bond value	Proportion of bond value x time (years)
1	1070	0.909	972.63	1.000	1.000

Duration of the Bond is **1 year**

Duration of Bond Y

Year	Cash flow	P.V. @ 10%		Proportion of bond value	Proportion of bond value x time (years)
1	80	0.909	72.72	0.077	0.077
2	80	0.826	66.08	0.071	0.142
3	80	0.751	60.08	0.064	0.192
4	1080	0.683	737.64	0.788	3.152
			936.52	1.000	3.563

Duration of the Bond is **3.563 years**

Let x_1 be the investment in Bond X and therefore investment in Bond Y shall be (1 - x_1). Since the required duration is 2 years the proportion of investment in each of these two securities shall be computed as follows:

$2 = x_1 + (1 - x_1)\ 3.563$

x_1 **= 0.61**

Amount of investment

Bond X	**Bond Y**
PV of ₹1,00,000 for 2 years @ 10% x 61% **= ₹1,00,000 (0.826) x 61%** **= ₹50,386** **No. of Bonds to be purchased** **= ₹50,386/₹972.63 = 51.80 i.e. approx. 52 bonds**	PV of ₹1,00,000 for 2 years @ 10% x 39% = ₹1,00,000 (0.826) x 39% = ₹32,214 No. of Bonds to be purchased = ₹32,214/₹936.52 = 34.40 i.e. approx. 34 bonds

Note: The investor has to keep the money invested for two years. Therefore, the investor can invest in both the bonds with the assumption that Bond X will be reinvested for another one year on same returns.

Further, in the above computation, Modified Duration can also be used instead of Duration.

16)Illustration

Mr. A is planning for making investment in bonds of one of the two companies X Ltd. and Y Ltd. The detail of these bonds is as follows:

Company	Face Value	Coupon Rate	Maturity Period
X Ltd.	₹10,000	6%	5 Years
Y Ltd.	₹10,000	4%	5 Years

The current market price of X Ltd.'s bond is ₹10,796.80 and both bonds have same Yield To Maturity (YTM). Since Mr. A considers duration of bonds as the basis of decision making, you are required to calculate the duration of each bond and give your decision.

(Old PM, RTP May'14)

Solution 16

To calculate duration of bond we need YTM, which shall be calculated as follows

Let us try NPV @ 5%

$$\frac{600}{(1+0.05)} + \frac{600}{(1+0.05)^2} + \frac{600}{(1+0.05)^3} + \frac{600}{(1+0.05)^4} + \frac{10600}{(1+0.05)^5} - 10796.80$$

=₹571.43 +₹544.22 +₹518.30 +₹493.62 +₹8,305.38 -₹10,796.80 = - ₹363.85

Let us now try NPV @ 4%

$$\frac{600}{(1+0.04)} + \frac{600}{(1+0.04)^2} + \frac{600}{(1+0.04)^3} + \frac{600}{(1+0.04)^4} + \frac{10600}{(1+0.04)^5} - 10796.80$$

=₹576.92 + ₹554.73 + ₹533.40 + ₹512.88 + ₹8,712.43 - ₹10,796.80 = ₹93.56

Let us now interpolation formula

$= 4\% + \frac{93.56}{93.56 - (-363.85)} \times (5\% - 4\%) = 4\% + \frac{93.56}{457.41} =$ **4.20%**

Duration of X Ltd.'s Bond

Year	Cash flow	P.V. @ 4.2%		Proportion of bond value	Proportion of bond value x time (years)
1	600	0.9597	575.82	0.0533	0.0533
2	600	0.9210	552.60	0.0512	0.1024
3	600	0.8839	530.34	0.0491	0.1473
4	600	0.8483	508.98	0.0472	0.1888
5	10600	0.8141	8,629.46	0.799	3.9960
			10797.20	**1.000**	**4.4878**

Duration of the Bond is 4.4878 years say **4.49 years**

Duration of Y Ltd.'s Bond

Year	Cash flow	P.V. @ 4.2%		Proportion of bond value	Proportion of bond value x time (years)
1	400	0.9597	383.88	0.0387	0.0387
2	400	0.9210	368.40	0.0372	0.0744
3	400	0.8839	353.56	0.0357	0.1071
4	400	0.8483	339.32	0.0342	0.1368
5	10400	0.8141	8,466.64	0.8542	4.2710
			9911.80	**1.000**	**4.6280**

Duration of the Bond is 4.6280 years say **4.63 years.**

Decision: Since the duration of Bond of Y Ltd. is higher hence it should be preferred. However, difference between the duration of bond is not much higher and with higher coupon. rate of X Ltd.'s bond, Mr. A should go for X Ltd.'s bond.

17)Illustration

XL Ispat Ltd. has made an issue of 14 per cent non-convertible debentures on January 1, 2007. These debentures have a face value of ₹100 and is currently traded in the market at a price of ₹90.

Interest on these NCDs will be paid through post-dated cheques dated June 30 and December 31. Interest payments for the first 3 years will be paid in advance through post-dated cheques while for the last 2 years post-dated cheques will be issued at the third year. The bond is redeemable at par on December 31, 2011 at the end of 5 years.

Required :

(i) Estimate the current yield and YTM of the bond.

(ii) Calculate the duration of the NCD.

(iii) Assuming that intermediate coupon payments are, not available for reinvestment, calculate the realised yield on the NCD.

(RTP Nov'18, Old PM)

Solution 17

(i) Current yield = $\frac{7}{90} * \frac{12}{6}$ = 0.1555 or 15.55%

YTM can be determined from the following equation

7 × PVIFA (YTM, 10) + 100 × PVIF (YTM, 10) = 90

Let us discount the cash flows using two discount rates 7.50% and 9% as follows:

Year	Cash Flows	PVF@7.50%	PV	PVF@9%	PV
0	-90	1	-90	1	-90
1	7	0.930	6.51	0.917	6.419
2	7	0.865	6.055	0.842	5.894
3	7	0.805	5.635	0.772	5.404
45	7	0.749	5.243	0.708	4.956
5	7	0.697	4.879	0.650	4.550
6	7	0.648	4.536	0.596	4.172
7	7	0.603	4.221	0.547	3.829
8	7	0.561	3.927	0.502	3.514
9	7	0.522	3.654	0.460	3.220
10	107	0.485	51.90	0.422	45.154
			6.560		**- 2.888**

Now we use interpolation formula

= 7.5% + $\frac{6.560}{6.560-(-2.888)}$× (9 - 7.5%) = 7.5% + $\frac{6.560}{9.448}$ x 1.5% = 8.54%

(ii) The duration can be calculated as follows:

Year	Cash flow	P.V. @ 8.54%	PV	Proportion of bond value	Proportion of bond value x time (years)
1	7	0.921	6.447	0.0717	0.0717
2	7	0.849	5.943	0.0661	0.1322
3	7	0.782	5.474	0.0608	0.1824
4	7	0.721	5.047	0.0561	0.2244
5	7	0.664	4.648	0.0517	0.2585
6	7	0.612	4.284	0.0476	0.2856
7	7	0.563	3.941	0.0438	0.3066
8	7	0.519	3.633	0.0404	0.3232
9	7	0.478	3.346	0.0372	0.3348
10	107	0.441	47.187	0.5246	5.2460
			89.95		**7.3654**

Duration = 7.3654 half years i.e. **3.683 years.**

(iii) Realized Yield can be calculated as follows:

$\frac{(7*10)+100}{1+R^{10}}$ = 90

$1 + R^{10}$ = $\frac{170}{90}$

R = $\left(\frac{170}{90}\right)^{1/10}$ - 1

= $(1.888)^{1/10}$ -1

= 1.065 - 1 = 0.065 or 6.50% for half yearly

Steps to compute n^{th}root of a number:

- Write the number on your calculator – Say 1.888
- Press the square root button 12 times
- Subtract 1
- Divide by n where n is the n^{th} root.= i.e 10 or multiply with 1/10
- Add 1
- Press "multiply button and then equal to button" 12 times

18) Illustration

Find the current market price of a bond having face value ₹1,00,000 redeemable after 6 year maturity with YTM at 16% payable annually and duration 4.3202 years. Given $1.16^6 = 2.4364$

(Old PM)

Solution 18

Duration = 4.3202 yrs YTM = 16% FV = 1,00,000 Maturity = 6 years

Duration = $\frac{1+y}{y} - \frac{(1+y)+t(c-y)}{c[(1+y))^t - 1]+y}$

$4.3202 = \frac{1.16}{0.16} - \frac{(1.16)+6(C-0.16)}{C[(1.16)^6 - 1]+0.16}$

$4.3202 = 7.25 - \frac{1.16+6C-0.96}{C[2.4364-1]+0.16}$

$-4.3203 + 7.25 = \frac{0.2+6C}{0.16+1.4364C}$

$2.9298(0.16 + 1.4364C) = 0.2 + 6C$

0.2688 = 1.7917C

C = 0.2688/1.7917 **= 0.15 or 15%**

Coupon rate = 15%

Price when C = 15% FV = 100000 Y = 16% T = 6yrs

P_0= 15000 PVAF (16%, 6) + 100000 PVF (16%, 10th yr)

= 3.6847 x 15000 + 0.41055 x 100000 = **₹96315.03**

19)Illustration

John inherited the following securities on his uncle's death:

Types of Security	Nos.	Annual Coupon %	Maturity Years	Yield %
Bond A (₹1,000)	10	9	3	12
Bond B (₹1,000)	10	10	5	12
Preference shares C (₹100)	100	11	*	13*
Preference shares D (₹100)	100	12	*	13*

*** likelihood of being called at a premium over par.**

Compute the current value of his uncle's portfolio. Applicable tax rate is 20% dividend is tax free.

(RTP Nov'23, MTP Oct'14, Old PM)

Solution 19

Computation of current value of John's portfolio

10 Nos. Bond A, Rs. 1,000 par value, 9% Bonds maturity 3 years:

Current value of interest on bond A 1-3 years: Rs. 900(1-0.20) × Cumulative P.V. @ 12% (1-3years) = Rs. 720 × 2.402	₹1,729
Add: Current value of amount received on maturity of Bond A End of 3rd year: Rs. 1,000 × 10 × P.V. @ 12% (3rd year) = Rs. 10,000 × 0.712	₹7,120
	₹8,849

(ii) 10 Nos. Bond B, Rs. 1,000 par value, 10% Bonds maturity 5 years:

Current value of interest on bond B 1-5 years: Rs. 1,000(1-0.20) × Cumulative P.V. @ 12% (1-5 years) = Rs. 800 × 3.605	₹2,884
Add: Current value of amount received on maturity of Bond B End of 5th year: Rs. 1,000 × 10 × P.V. @ 12% (5th year) = Rs. 10,000 × 0.567	₹5,670
	₹8,554

(iii) 100 Preference shares C, Rs. 100 par value, 11% coupon

$\frac{11\% \times 100 \text{ Nos.} \times \text{Rs. } 100}{13\%}$	**8462**

(iv) 100 Preference shares D, Rs. 100 par value, 12% coupon

$\frac{12\% \times 100 \text{ Nos.} \times \text{Rs. } 100}{13\%}$	**9231**

Total Current value of his portfolio [(i) + (ii) + (iii) + (iv)] **Rs. 35,096**

20)Illustration

MP Ltd. issued a new series of bonds on January 1, 2010. The bonds were sold at par (₹1,000), having a coupon rate 10% p.a. and mature on 31st December, 2025. Coupon payments are made semiannually on June 30th and December 31st each year. Assume that you purchased an outstanding MP Ltd. bond on 1st March, 2018 when the going interest rate was 12%.

Required:

(i) What was the YTM of MP Ltd. bonds as on January 1, 2010?

(ii) What amount you should pay to complete the transaction? Of that amount how much should be accrued interest and how much would represent bonds basic value.

(Old PM)

Solution 20

(i) As the issue @ 10% coupon is at par the YTM is also 10% p.a

(ii) Date of purchase 1/3/2018

Semi-Annual Coupon dates = 30^{th} June and 31^{st} December

On 1/03/2018 Price includes 2 months accrued interest

$P_0 = 50.PVAF_{(6\%,15\ Periods)} + 1000.PVIF_{(6\%,15th\ Period)}$ = 50 x 9.7122 + 1000 x 0.41727 = **902.88**

Two Adjustments

(i) PV of Bond by 4 months @ 12% p.a

(ii) Adjust the interest for 4 months

Price on 30^{th} June before coupon is 902.88 + 50 = ₹952.88

Now, if price on 30^{th} June 2018 is 952.88 then, what is the price on 1/03/2018?

i.e. 952.88 x $PVAF_{(6\%,4months)}$

= 952.88 x $\frac{1}{(1.06)^{4/6}}$ = $\frac{952.88}{(1.06)^{2/3}}$ = $\frac{952.88}{1.0396}$ = **₹916.57 is dirty price as on 1/03/2018**

Clean Price = Dirty Price – Interest Accrued

Interest Accrued = 1000 x $\frac{10\%}{2}$ x $\frac{2}{6}$ = 50 x $\frac{1}{3}$ = ₹16.67 (2m = 31/12 to 1/3)

Clean Price = 916.57 – 16.67 = **₹899.90**

21)Illustration

Tiger Ltd. is presently working with an Earning Before Interest and Taxes (EBIT) of ₹ 90 lakhs. Its present borrowings are as follows:

	₹ in lakhs
12% term loan	300
Working capital borrowings:	
From Bank at 15%	200
Public Deposit at 11%	100

The sales of the company are growing and to support this, the company proposes to obtain additional borrowing of ₹ 100 lakhs expected to cost 16%. The increase in EBIT is expected to be 15%. Calculate the change in interest coverage ratio after the additional borrowing is effected and comment on the arrangement made.

(Old PM)

Solution 21

Existing EBIT = ₹90 lakhs

Computation of Interest

Borrowing	Amount (₹ in lakhs)	Interest(%)	Interest(₹ in lakhs)
12% term loan	300	12%	36
From Bank at 15%	200	15%	30
Public Deposit at 11%	100	11%	11
			77

Interest Coverage Ratio = $\frac{90}{77}$ = **1.1688**

Revised Interest Coverage Ratio

	(₹ in lakhs)
Revised EB IT 90 x 1.15	103.5
Existing Interest Cost	77
New Interest Cost	16
Revised Interest Expense	**93 (77 + 16)**

Interest Coverage Ratio = $\frac{103.5}{93}$ = **1.1129**

Therefore, the interest coverage has dropped from 1.1688 to 1.1129 and hence the new borrowing cost is not recommended.

22)Illustration

Saranam Ltd. has issued convertible debentures with coupon rate 12%. Each debenture has an option to convert to 20 equity shares at any time until the date of maturity. Debentures will be redeemed at ₹ 100 on maturity of 5 years. An investor generally requires a rate of return of 8% p.a. on a 5-year security. As an investor when will you exercise conversion for given market prices of the equity share of (i) ₹ 4, (ii) ₹ 5 and (iii) ₹ 6.

Cumulative PV factor for 8% for 5 years	:	3.993
PV factor for 8% for year 5	:	0.681

(Old PM)

Solution 22

If Debentures are not converted its value is as under: -

	PVF @8%	PV₹
Interest - ₹12 for 5 years	3.993	47.916
Redemption - ₹100 in 5th year	0.681	68.100
		116.016

Value of equity shares:-

Market Price	No.	Total
4	20	80
5	20	100
6	20	120

Hence, unless the market price is ₹6 conversion should not be exercised.

23)Illustration

The data given below relates to a convertible bond :

Face value	**₹ 250**
Coupon rate	**12%**
No. of shares per bond	**20**
Market price of share	**₹ 12**
Straight value of bond	**₹ 235**
Market price of convertible bond	**₹ 265**

Calculate:

(i) Stock value of bond

(ii) The percentage of downside risk

(iii) The conversion premium

(iv) The conversion parity price of the stock

(ICAI SM, MTP Sept'23, MTP Apr'18, RTP May'19 Old, Old PM)

Solution 23

(i) Stock value or conversion value of bond

12 × 20 = ₹240

(ii) Percentage of the downside risk

$\frac{265-235}{235}$ = **12.77%** or $\frac{265-235}{265}$ = **11.32%**

This ratio gives the percentage price decline experienced by the bond if the stock becomes worthless.

(iii) Conversion Premium

$\frac{\text{Market Price Conversion Value}}{\text{Conversion Value}} \times 100 = \frac{265-240}{240} = \mathbf{10.42\%}$

(iv) Conversion Parity Price

$\frac{\text{Bond Price}}{\text{No.of shares on Conversion}} = \frac{265}{20}$ = ₹13.25

This indicates that if the price of shares rises to ₹13.25 from ₹12 the investor will neither gain nor lose on buying the bond and exercising it. Observe that ₹1.25 (₹13.25 – ₹12.00) is 10.42% of ₹12, the Conversion Premium.

24) Illustration

ABC Ltd. has ₹300 million, 12 per cent bonds outstanding with six years remaining to maturity. Since interest rates are falling, ABC Ltd. is contemplating of refunding these bonds with a ₹300 million issue of 6 year bonds carrying a coupon rate of 10 per cent.

Issue cost of the new bond will be ₹6 million and the call premium is 4 per cent. ₹9 million being the unamortized portion of issue cost of old bonds can be written off no sooner the old bonds are called off. Marginal tax rate of ABC Ltd. is 30 per cent.

You are required to analyse the bond refunding decision.

[ICAI SM, Sep-25 6M Similar, Dec'21 QP, RTP Nov'22, MTP Mar'24, RTP May 20 (Old), MTP Mar 18 (Old), May'14 QP, Nov'11 QP, Old PM]

Solution 24

(i) Calculation of initial outlay:-

	₹(million)
a. Face value	300
Add:-Call premium	12
Cost of calling old bonds	312
b. Gross proceed of new issue	300
Less: Issue costs	6
Net proceeds of new issue	294
c. Tax savings on call premium	
and unamortized cost 0.30 (12 + 9)	6.3
∴ Initial outlay = ₹312 million – ₹294 million – ₹6.3 million = ₹11.7 million	

(ii) Calculation of net present value of refunding the bond:-

	₹(million)
Saving in annual interest expenses [300 x (0.12 – 0.10)]	6.00
Less:- Tax saving on interest and amortization [0.30 x [6 + (9-6)/6]	(1.95)
Annual net cash saving	**4.05**
PVIFA (7%, 6 years)	4.766
∴Present value of net annual cash saving	19.30 million
Less:- Initial outlay	(11.70) million
Net present value of refunding the bond	₹7.60 million

Decision: The bonds should be refunded

25) Illustration

A Ltd. has issued convertible bonds, which carries a coupon rate of 14%. Each bond is convertible into 20 equity shares of the company A Ltd. The prevailing interest rate for similar credit rating bond is 8%. The convertible bond has 5 years maturity. It is redeemable at par at ₹100. The relevant present value table is as follows.

Present values	t_1	t_2	t_3	t_4	t_5
$PVIF_{0.14,\ t}$	0.877	0.769	0.675	0.592	0.519
$PVIF_{0.08,\ t}$	0.926	0.857	0.794	0.735	0.681

You are required to estimate:

(Calculations be made upto 3 decimal places)

(i) current market price of the bond, assuming it being equal to its fundamental value,

(ii) minimum market price of equity share at which bond holder should exercise conversion option; and

(iii) duration of the bond

(MTP Oct'19, RTP May'19 Old, Old PM, Nov'16 QP)

Solution 25

(i) **Current Market Price of Bond**

Year	Cashflow	PVF@8%	PV
1	14	0.926	12.964
2	14	0.857	11.998
3	14	0.794	11.116
4	14	0.735	10.290
5	114	0.681	77.634
		∑ PV (CF) i.e. P_0 =	**124.002**

(ii) **Minimum Market Price of Equity Shares at which Bondholder should exercise conversion option:** $\frac{124.00}{20}$ = ₹6.20

(iii) **Duration of the Bond**

Year	Cash flow	PVF@ 8%		Proportion of bond value	Proportion of bond value x time (years)
1	14	0.926	12.964	0.105	0.105
2	14	0.857	11.998	0.097	0.194
3	14	0.794	11.116	0.089	0.267
4	14	0.735	10.290	0.083	0.332
5	114	0.681	77.634	0.626	3.130
			124.002	**1.000**	**4.028**

Duration of the bond is 4.028

26) Illustration

Sabanam Ltd. has issued convertible debentures with coupon rate 11%. Each debenture has an option to convert to 16 equity shares at any time until the date of maturity. Debentures will be redeemed at ₹100 on maturity of 5 years. An investor generally requires a rate of return of 8% p.a. on a 5-year security. As an advisor, when will you advise the investor to exercise conversion for given market prices of the equity share of (i) ₹5, (ii) ₹6 and (iii) ₹7.10

Cumulative PV factor for 8% for 5 years: 3.993

PV factor for 8% for year 5: 0.681

(ICAI SM, May'18 QP 6 marks)

Solution 26

If Debentures are not converted its value is as under: -

	PVF @ 8 %	₹PV
Interest - ₹11 for 5 years	3.993	43.923
Redemption - ₹100 in 5th year	0.681	68.100
		112.023

Value of equity shares:-

Market Price	No.	Total
₹5	16	80
₹6	16	96
₹7	16	113.60

Recommendation

Market Price	Recommendation	Reason
₹5	Don't Covert	Current Market Price < Break Even Price
₹6	Don't Covert	Current Market Price < Break Even Price
₹7.10	Convert	Current Market Price > Break Even Price

Hence, unless the market price is (113.60/16) ₹7.10 conversion should not be exercised.

27) Illustration

Suppose Mr. A is offered a 10% Convertible Bond (par value ₹1,000) which either can be redeemed after 4 years at a premium of 5% or get converted into 25 equity shares currently trading at ₹33.50 and expected to grow by 5% each year. You are required to determine the minimum price Mr. A shall be ready to pay for bond if his expected rate of return is 11%.

(RTP May'15)

Solution 27

Bond Face Value = ₹1000 Maturity = 4yrs Redemption premium = 5% Redemption price = 1050 Coupon 10% Market rate = 11% Equity shares offered = 25 CMP = 33.5 Growth = 5%p.a

(i) Intrinsic Value of Bond assuming no immediate conversion

P_0= 100 PVAF (11%, 4) + 1050 PVF (11%, 4th yr)

= 3.1024 x100 + 0.6587 x 1050

= 310.245 + 691.635 => P_0 = **₹1001.88**

If Bond is converted to shares immediately then, value of resulted shares

F.V of shares = 25 x 33.5 x $(1.05)^4$ = 1017.99

PV of share = 1017.99 x 0.6587 = 670.54

Combined Present value of conversion option = ₹310.245 + ₹670.54 = **₹980.785**

Redemption Option is best and Conversion Option can be opted only if the PV of conversion option is > ₹1001.88

28) Illustration

XYZ company has current earnings of ₹3 per share with 5,00,000 shares outstanding. The company plans to issue 40,000, 7% convertible preference shares of ₹50 each at par. The preference shares are convertible into 2 shares for each preference shares held. The equity share has a current market price of ₹21 per share.

(i) What is preference share's conversion value?

(ii) What is conversion premium?

(iii) Assuming that total earnings remain the same, calculate the effect of the issue on the basic earning per share (a) before conversion (b) after conversion.

(iv) If profits after tax increases by ₹1 million what will be the basic EPS (a) before conversion and (b) on a fully diluted basis?

(RTP Nov'23, May'17 QP 8 marks, MTP Mar'19 Old, Old PM)

Solution 28
CMP of Equity share = ₹21
Convertible preference share issue price = ₹50
Equity share count = 500000
7% Preference shares to be issued = 40000
Conversion ratio 2:1

(i) Conversion Value = No. of equity shares/Pref shares x Market premium per equity share = 2 x 21 = ₹42

(ii) Conversion premium = FV/Issue price of pref share – Conversion price = 50 - 42 = ₹8
Conversion premium = 8/42 = 19.05%

(iii) Earning = EPS x Share Count = 3 x 5, 00,000 = 15, 00,000
Preference dividend = 7% x 40000 x 50 = 1, 40,000
Earnings available for equity shareholders = 13, 60,000 (15, 00,000 – 1, 40,000)
EPS = 13,60,000/5,00,000 = ₹2.72
<u>After Conversion</u>
PAT ₹15, 00,000
Existing equity 5, 00,000
Preference converted into equity = 40,000 x 2 = 80,000
Total share = 5, 80,000
EPS = ₹15, 00,000/5,80,000 = **₹2.586**

(iv) Assuming PAT goes up by ₹10, 00,000
Revised PAT = ₹25, 00,000
Preference dividend = 1, 40,000
Earnings available for equity shareholders = ₹23, 60,000
Share count = 5, 00,000
EPS = 4.72
Diluted EPS or EPS after conversion
PAT = ₹25, 00,000
Old equity shares = 5, 00,000
New equity shares = 80,000
Total share count = 5, 80,000
EPS = ₹25,00,000/5,80,000 = **₹4.310**

29)Illustration

The following data is related to 8.5% Fully Convertible (into Equity shares) Debentures issued by JAC Ltd. at ₹1000.

Market Price of Debenture	**₹900**
Conversion Ratio	**30**
Straight Value of Debenture	**₹700**
Market Price of Equity share on the date of Conversion	**₹25**
Expected Dividend Per Share	**1**

You are required to calculate:
(a) Conversion Value of Debenture
(b) Market Conversion Price
(c) Conversion Premium per share
(d) Ratio of Conversion Premium

(e) Premium over Straight Value of Debenture
(f) Favourable income differential per share
(g) Premium pay back period

[Similar May'18 QP (Old), RTP Nov'21, Similar MTP Apr'22, MTP Aug'18 Old, RTP Nov 16 & May 15, Old PM]

Solution 29

(a) Conversion Value of Debenture

= Market Price of one Equity Share X Conversion Ratio

= ₹30 X 25 = **₹750**

(b) Market Conversion Price

$\frac{\text{Market Price of Convertible Debenture}}{\text{Conversion Ratio}} = \frac{900}{30} = \mathbf{30}$

(c) Conversion Premium per share

Market Conversion Price – Market Price of Equity Share

= ₹30 – ₹25 = **₹5**

(d) Ratio of Conversion Premium

$\frac{\text{Conversion premium per share}}{\text{Market Price of Equity Share}} = \frac{5}{25} \times 100 = \mathbf{20\%}$

(e) Premium over Straight Value of Debenture

$\frac{\text{Market Price of Convertible Bond}}{\text{Straight Value of Bond}} - 1 = \frac{900}{700} - 1 = \mathbf{28.57\%}$

(f) Favorable income differential per share

$\frac{\text{Coupon Interest from Debenture} - \text{Conversion Ratio Dividend Per Share}}{\text{Conversion Ratio}} = \frac{85-30}{30} = \mathbf{₹\ 1.833}$

(g) Premium pay back period

$\frac{\text{Conversion premium per share}}{\text{Favourable Income Differntial Per Share}} = \frac{5}{1.833} = \mathbf{2.73\ years}$

30) Illustration

Pineapple Ltd has issued fully convertible 12 percent debentures of ₹5,000 face value, convertible into 10 equity shares. The current market price of the debentures is ₹5,400. The present market price of equity shares is ₹430.

Calculate: (i) the conversion percentage premium, and (ii) the conversion value

(Old PM)

Solution 30

(i) Conversion % premium = $\frac{5400-4300}{4300}$ = **25.58%**

(ii) Conversion Value =No. of shares x Market price per share = 10 x 430 = **4300**

31) Illustration

GHI Ltd., AAA rated company has issued, fully convertible bonds on the following terms, a year ago:

Face value of bond	₹1000
Coupon (interest rate)	8.5%
Time to Maturity (remaining)	3 years
Interest Payment	Annual, at the end of year
Principal Repayment	At the end of bond maturity
Conversion ratio (Number of shares per bond)	25
Current market price per share	₹45
Market price of convertible bond	₹1175

AAA rated company can issue plain vanilla bonds without conversion option at an interest rate of 9.5%.

Required: Calculate as of today:

(i) Straight Value of bond
(ii) Conversion Value of the bond
(iii) Conversion Premium

(iv) Percentage of downside risk
(v) Conversion Parity Price
PVIF @9.5% (Yr 1 – 0.9132, Yr 2 – 0.8340, Yr 3 – 0.7617)

(Nov'22 QP 8 marks, May'14 8 marks, MTP Aug'18, RTP May 17, Old PM)

Solution 31

(i) Straight Value of Bond
Rs. 85 x 0.9132 + Rs. 85 x 0.8340 + Rs. 1085 x 0.7617 = Rs. 974.96

(ii) Conversion Value
Conversion Ratio x Market Price of Equity Share
= Rs. 45 x 25 = ₹1,125

(iii) Conversion Premium
Conversion Premium = Market Conversion Price - Market Price of Equity Share
$= \frac{\text{Rs.}1,175}{25} - 45 = 2$
or = Rs. 1,175 - Rs. 45 x 25 = ₹ 50
Or $\frac{\text{Rs.}1,175 - \text{Rs.}1,125}{1125} = 4.44\%$

(iv) Percentage of Downside Risk
$= \frac{\text{Rs.}1,175 - \text{Rs.}974.96}{\text{Rs.}974.96} \times 100 = 20.52\%$ or $= \frac{\text{Rs.}1,175 - \text{Rs.}974.96}{\text{Rs.}1175} = 17.02\%$

(v) Conversion Parity Price
$\frac{\text{Bond Price}}{\text{No.of Share on Conversion}} = \frac{\text{Rs.}1,175}{25} = ₹47$

32)Illustration

A hypothetical company ABC Ltd. issued a 10% Debenture (Face Value of ₹ 1000) of the duration of 10 years is currently trading at ₹ 850 per debenture. The bond is convertible into 50 equity shares being currently quoted at ₹ 17 per share.
If yield on equivalent comparable bond is 11.80%, then calculate the spread of yield of the above bond from this comparable bond.
The relevant present value table is as follows:

Present Values	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
$PVIF_{0.11,\ t}$	0.901	0.812	0.731	0.659	0.593	0.535	0.482	0.434	0.391	0.352
$PVIF_{0.13,\ t}$	0.885	0.783	0.693	0.613	0.543	0.480	0.425	0.376	0.333	0.295

(RTP Nov'19 New & Old, MTP Apr'19, MTP Mar'18)

Solution 32

Conversion Price = ₹50 x 17 = ₹850
Intrinsic Value = ₹850
Accordingly the yield (r) on the bond shall be :
₹850 = ₹100 PVAF (r, 10) + ₹1000 PVF (r, 10)
Let us discount the cash flows by 11%
850 = 100 PVAF (11%, 10) + 1000 PVF (11%, 10)
850 = 100 x 5.890 + 1000 x 0.352 = 91

Now let us discount the cash flows by 13%

850 = 100 PVAF (13%, 10) + 1000 PVF (13%, 10)

850 = 100 x 5.426 + 1000 x 0.295 = -12.40

Accordingly, IRR

$11\% + \frac{90.90}{90.90-(-12.40)} \times (13\% - 11\%)$

$11\% + \frac{90.90}{103.30} \times (13\% - 11\%)$

=12.76%

The spread from comparable bond = 12.76% - 11.80% = **0.96%**

33) Illustration

M/s. Earth Limited has 11% bond worth of ₹2 crores outstanding with 10 years remaining to maturity. The company is contemplating the issue of a ₹2 crores 10-year bond carrying the coupon rate of 9% and use the proceeds to liquidate the old bonds.

The unamortized portion of issue cost on the old bonds is ₹3 lakhs which can be written off no sooner the old bonds are called. The company is paying 30% tax and its after-tax cost of debt is 7%. Should Earth Limited liquidate the old bonds?

You may assume that the issue cost of the new bonds will be ₹2.5 lakhs and the call premium is 5%.

[May'13 QP 6 marks]

Solution 33

1.Calculation of initial outlay:-	₹ (lakhs)
a. Face value	200
Add:-Call premium	10
Cost of calling old bonds	**210**
b. Gross proceed of new issue	200
Less: Issue costs	2.50
Net proceeds of new issue	197.50
c. Tax savings on call premium	
and unamortized cost 0.30 (10 + 3)	₹3.90 lakhs

∴ Initial outlay = ₹210 lakhs - ₹197.50 lakhs - ₹3.90 lakhs = ₹8.60 lakhs

1. **Calculation of net present value of refunding the bond**

Saving in annual interest expenses	**₹(lakhs)**
[₹200 x (0.11 - 0.09)]	4.000
Less:-Tax saving on interest and amortization 0.30 x [4+(3-2.5)/10]	1.215
Annual net cash saving	**2.785**
PVIFA (7%, 10 years)	7.024
∴Present value of net annual cash saving	₹19.56 lakhs
Less:- Initial outlay	₹8.60 lakhs
Net present value of refunding the bond	₹10.96 lakhs

Decision: The bonds should be refunded

34) Illustration

Tangent Ltd. is considering calling ₹3 crores of 30 years, ₹1,000 bond issued 5 years ago with a coupon interest rate of 14 per cent. The bonds have a call price of ₹1,150 and had initially collected proceeds of ₹2.91 crores since a discount of ₹30 per bond was offered. The initial floating cost was ₹3,90,000. The Company intends to sell ₹3 crores of 12 per cent coupon rate, 25 years bonds to raise funds for retiring the old bonds. It proposes to sell the new bonds at their par value of ₹1,000. The estimated floatation cost is ₹4,25,000. The company is paying 40% tax and its after tax cost of debt is 8 per cent. As the new bonds must first be sold and then their proceeds to be used to retire the old bonds, the company expects a two month period of overlapping interest during which interest must be paid on both the old and the new bonds. You are required to evaluate the bond retiring decision. [PVIFA 8%, 25 = 10.675]

(Nov'18 QP 8 marks, MTP Nov'21)

Solution 34

(a) NPV for bond refunding

	₹
PV of annual cash flow savings (W.N. 2)	
(3,49,600 x PVIFA 8%,25) i.e. 10.675	37,31,980
Less: Initial investment (W.N. 1)	31,15,000
NPV	**6,16,980**

Recommendation: Refunding of bonds is recommended as NPV is positive.

Working Notes:

(1) Initial investment:

(a) Call premium Before tax (1,150 – 1,000) × 30,000	**45,00,000**	
Less tax @ 40%	18,00,000	
After tax cost of call premium		**27,00,000**
(b) Floatation cost		4,25,000
(c) Overlapping interest Before tax (0.14 × 3 crores)× 2/12	7,00,000	
Less tax @ 40%	2,80,000	4,20,000
(d) Tax saving on unamortised discount on old bond (25/30 × 9,00,000 × (0.4)		(3,00,000)
(e) Tax savings from unamortised floatation		
Cost of old bond (25/30 × 3,90,000 × 0.4)		(1,30,000)
		31,15,000
(2) Annual cash flow savings:		
(a) Old bond		
(i) Interest cost (0.14 x 3 crores)	42,00,000	
Less tax @ 40%	16,80,000	25,20,000
(ii) Tax savings from amortisation of discount (9,00,000/30 × 0.4)		(12,000)
(iii) Tax savings from amortisation of floatation cost (3,90,000/30 × 0.4)		(5,200)
Annual after tax cost payment under old Bond (A)		**25,02,800**

(b) New bond

(i) **Interest cost before tax (0.12 × 3 crores)**	**36,00,000**
Less tax @ 40%	14,40,000
After tax interest	**21,60,000**
(ii) Tax savings from amortisation of floatation cost (0.4 x 4,25,000/25)	(6,800)
Annual after tax payment under new Bond (B)	**21,53,200**
Annual Cash Flow Saving (A) – (B)	**3,49,600**

35) Illustration

The HLL has Rs. 8.00 crore of 10% mortgage bonds outstanding under an indenture. The indenture allows additional bonds to be issued as long as all of the following conditions are met:

(1) Pre- tax interest coverage i.e., (Income before tax + Bond Interest) / Bond Interest remains greater than 4.
(2) Net depreciated value of mortgage assets remains twice the amount of the mortgage debt.
(3) Debt-to-equity ratio remains below 0.50.

The HLL has net income after taxes of Rs. 2 crores and a 40% tax-rate, Rs. 40 crores in equity and Rs. 30 crores in depreciated assets, covered by the mortgage.
Assuming that 50% of the proceeds of a new issue would be added to the base of mortgaged assets and that the company has no Sinking Fund payments until next year, how much more 10% debt could be sold under each of the three conditions? Which protective covenant is binding?

(MTP Oct'17)

Solution 35

Let x be the crores of Rupees of new 10% debt which would be sold under each of the three given conditions. Now, the value of x under each of the three conditions is as follows:

1. Pre-tax interest coverage $\frac{\text{Income before tax+bond interest}}{\text{Bond Interest}}$ remains greater than 4.

$\frac{\frac{₹2\text{crores}}{(1-0.4)} + 8\text{crores}*0.1+x*0.1}{8\text{crores}*0.1)+(x*0.1)} = 4$

Or $\frac{3\ 33\ .0 \text{ crores} + 0.80 \text{ crores} + 10x}{(\ 80 \text{ crores} + 0.10x\ )} = 4$

Or Rs. 4.13 crores + 0.10x = 4 (Rs. 0.80 crores + Rs. 0.10x)
Or Rs. 4.13 crores + 0.10x = Rs. 3.2 crores + Rs. 0.40x
Or Rs. 0.30 x = 0.93
Or x = Rs. 0.93/0.30
Or x = Rs. 3.10 crores
Additional mortgage required shall be a maximum of Rs. 3.10 crores.

2. Net depreciated value of mortgage assets remains twice the amount of mortgage debt
(Assuming that 50% of the proceeds of new issue would be added to the base of mortgaged assets)
i.e $\frac{₹30 \text{ crores}+0.5x}{₹8\text{crores}+x} = 2$
or Rs. 30 crores + 0.5x = 2 (Rs. 8 crores + x)
or Rs. 1.5x = Rs. 14 crores
or **x = 9.33 crores**
Additional mortgage required to satisfy condition No. 2 is Rs. 9.33 crores

3. Debt to equity ratio remains below 5

i.e $\frac{₹8 \text{ crores}+x}{₹40 \text{ crores}}$ = 0.5

or Rs. 8 crores + x = Rs. 20 crores

or x = Rs. 12 crores

Since all the conditions are to be met, the least i.e. Rs. 3.10 crores (as per condition – 1) can be borrowed by issuing additional bonds.

Thus, binding conditions are met and it limits the amount of new debt to 3.10 crore.

36) Illustration

ABC Ltd. issued 9%, bonds of ₹ 1,000/- each having a maturity of 3 years. The present rate of interest is 12% for one year tenure. It is expected that Forward rate of interest for one year tenure is going to fall by 75 basis points and further by 50 basis points for every next year in further for the same tenure. This bond has a beta value of 1.02 and is more popular in the market due to less credit risk.

Calculate

(i) Intrinsic value of bond

(ii) Expected price of bond in the market

[MTP May'20, MTP Apr'18, Nov'18 QP (Old), Nov'13 QP, Old PM]

Solution 36

(i) Intrinsic value of Bond

PV of Interest + PV of Maturity Value of Bond

Forward rate of interests:

1 st Year	12%
2^{nd} year	11.25%
3^{rd} Year	10.75%

PV of interest = $\frac{₹90}{(1\ 0.12)} + \frac{₹90}{(1\ 0.12)(1\ 0.1125)} + \frac{₹90}{(1\ 0.12)(1\ 0.1125)(1\ 0.1075)}$ = 217.8083

PV of Maturity Value of Bond = $\frac{₹1000}{(1+0.12)(1+0.1125)(1+0.1075)}$ = ₹724.67

Intrinsic value of Bond = ₹217.8083 + ₹724.67 = ₹942.4748

(ii) Expected Price = Intrinsic Value x Beta Value

= ₹942.4748 x 1.02 = **₹961.3243**

37) Illustration

Consider the following data for Government securities:

Face Value	**Coupon Rate (%)**	**Maturity (years)**	**Current Price**	**Yield to Maturity (%)**
10,000	12.40	1	9,987	12.546
10,000	12.75	2	9,937	13.128
10,000	13.50	3	10,035	13.351
10,000	13.50	4	9,971	13.599
10,000	13.75	5	9,948	13.901

Calculate forward rates for year 1 to 5.

(ICAI SM)

Solution 37

F_1

$F_1 = S_1$ = **12.546%**

F_2

2 Year Maturity Bond

Year	1	2
Cash flow	1275	11275
PVF	1.13128	$(1.13128)^2$
PVF	0.88396	0.7814
Present Value	1127.049	8810.285
		9937

$9937 = \frac{1275}{1 + 12.546\%} + \frac{11275}{(1.12546)(1+F_2)}$

$9937 = 1132.87 + \frac{10018.126}{(1+F_2)}$

$8804.13 = \frac{10018.126}{(1+F_2)}$

$(1 + F_2) = 1.13789$

F_2 **= 13.789%**

F_3

$10035 = \frac{1350}{1.12546} + \frac{1350}{(1.13789)(1.12546)} + \frac{11350}{(1.13789)(1.12546)(1+F_3)}$

$10035 = 1350 \times 0.88853 + 1350 \times 0.78085 + \frac{11350 \times 0.78085}{1+F_3}$

$10035 = 2253.668 + \frac{8862.6475}{1+F3}$

$1 + F_3 = 1.13896$

F_3 **= 13.896%**

F_4

$9971 = 1350 \times 0.88853 + 1350 \times 0.78085 + 1350 \times 0.68558 + 11350 \times \frac{0.68558}{1+F_4}$

$9971 = 3179.196 + \frac{7781.333}{1+F_4}$

$1 + F_4 = 1.14569$

F_4 **= 14.569%**

F_5

$9948 = 1375 \times 0.88853 + 1375 \times 0.78085 + 1375 \times 0.68558 + 1375 \times 0.5984 + 11375 \times \frac{0.5984}{1+F_5}$

$9948 = 4060.87 + \frac{6806.8}{1+F_5}$

$1 + F_5 = \frac{6806.8}{5887.13}$

F_5 **= 15.622%**

38) Illustration

RBI sold a 91-day T-bill of face value of ₹100 at an yield of 6%. What was the issue price?

(ICAI SM, Old PM)

Solution 38

Let the issue price be X

By the terms of the issue of the T-bills:

$6\% = \frac{100-x}{x} \times \frac{365}{91} \times 100$

$\frac{6 * 91 * x}{36500} = 100 - x$

$0.01496\, x = 100 - x$

$x = \frac{100}{1.01496} =$ **₹98.53**

39) Illustration

Wonderland Limited has excess cash of ₹20 lakhs, which it wants to invest in short term marketable securities. Expenses relating to investment will be ₹50,000.

The securities invested will have an annual yield of 9%.

The company seeks your advice

(i) as to the period of investment so as to earn a pre-tax income of 5%. (discuss)

(ii) the minimum period for the company to breakeven its investment expenditure overtime value of money.

[ICAI SM, May-25 Similar 4M, Nov'17 QP (Old), MTP May'20 Old, MTP Aug'18 Old, Old PM]

Solution 39

(i) Pre-tax Income required on investment of ₹20,00,000

Let the period of Investment be 'P' and return required on investment ₹1,00,000 (₹20, 00,000 x 5%)

Accordingly,

(₹20, 00,000 x $\frac{9}{100}$ x $\frac{P}{12}$) - ₹50,000 = ₹1, 00,000

P = 10 months

(ii) Break-Even its investment expenditure

(₹20, 00,000 x $\frac{9}{100}$ x $\frac{P}{12}$) - ₹50,000 = 0

P = 3.33 months

40) Illustration

Z Co. Ltd. issued commercial paper worth ₹10 crores as per following details:

Date of issue:	16th January, 2019
Date of maturity:	17th April, 2019
No. of days:	91
Interest rate:	12.04% p.a

What was the net amount received by the company on issue of CP? (Charges of intermediary may be ignored)

(ICAI SM, MTP Nov'21 New & Old, Old PM)

Solution 40

The company had issued commercial paper worth ₹10 crores

No. of days Involves = 91 days

Interest rate applicable = 12.04 % p.a.

Interest for 91 days = 12.04% × $\frac{91\text{Days}}{365\text{Days}}$ = 3.00175%

= or ₹10 cr x $\frac{3.00175}{100+3.00175}$ = ₹29, 14,274

= or ₹29.14274 Lakhs

∴ Net amount received at the time of issue:- ₹10.00 Crores – ₹0.2914274 Cr = **₹9.7085726 Cr**

Alternatively, it can also be computed as follows:

Price = $\frac{₹10\ crores}{\left(1+12.04\% \times \frac{91 days}{365 days}\right)}$ = **₹9.7085726 Cr**

41) Illustration

Bank A enter into a Repo for 14 days with Bank B in 10% Government of India Bonds 2028 @ 5.65% for ₹ 8 crore. Assuming that clean price (the price that does not have accrued interest) be ₹ 99.42 and initial Margin be 2% and days of accrued interest be 262 days. You are required to determine

(i) Dirty Price

(ii) Repayment at maturity. (consider 360 days in a year)

(ICAI SM, MTP Apr'24, MTP Oct'23, MTP Apr'21 Old, MTP Mar'21 Old, Old PM)

Solution 41

(i) Dirty Price

= Clean Price + Interest Accrued

= 99.42 + 100 x $\frac{10}{100}$ x $\frac{262}{360}$ = 106.70

(ii) First Leg (Start Proceed)

= Nominal Value x $\frac{\text{DirtyPrice}}{100}$ x $\frac{100-\text{Initial Margin}}{100}$

= ₹8,00,00,000 x $\frac{106.70}{100}$ x $\frac{100-2}{100}$ = ₹8,36,52,800

Second Leg (Repayment at Maturity) = Start Proceed x (1 + Repo Rate x $\frac{\text{No.of days}}{360}$)

= ₹8,36,52,800 x (1 + 0.0565 x $\frac{14}{360}$) = ₹8,38,36,604

42) Illustration

The Bank BK enters into a Repo for 9 days with Bank NE in 6% Government bonds 2022 for an amount of 2 crore. The other relevant details are as follows:

First Leg Payment (Start Proceeds)	2,00,06,750
Second Leg Payment (Repayment Proceeds)	2,00,31,759
Initial Margin	1.25%
Days of accrued interest	240

Assume 360 days in a year. You are required to calculate:

(i) Repo Rate

(ii) Dirty Price and

(iii) Clean Price

[RTP Nov'24, MTP Oct'22, Jul'21 QP (Old)]

Solution 42

(i) Repo Rate :

Particulars	Amt(₹)
Loan	20006750
Repayment	20031759
Difference(Interest)	25009

Interest = Loan x Repo rate x $\frac{Days}{360}$

25009 = 20006750 x r% x $\frac{9}{360}$

25009 x 40 = 20006750 x r%

r% = 5.0001 = 5%

(ii) Dirty Price

Loan = Security given – Margin

=- 1.25%

If Security is 100 then loan is 98.75% of security given.
20006750 = 98.75% of security
Security sold = 20006750/98.75% = ₹20260000
Face Value = ₹ 2,00,00,000
Market Value = ₹ 2,02,60,000
Dirty Price using Face Value of bond ₹100
If Face Value ₹100 is worth ₹20000000 Then, How much Dirty Price is worth ₹20260000?
So, $\frac{2,02,60,000}{2,00,00,000}$ x 100 = **₹101.3**
(iii) Clean Price = DP – Accrued Interest = 101.3 – $\left(100 * 6\% * \frac{240}{360}\right)$ = 101.3 – 4 = **₹97.3**

43) Illustration

The data given below relates to convertible bond of Hi -Fi Ltd.:

Face value	2,500
No. of shares per bond	20
Coupon rate	12%
Market price per share	120
Market price of convertible bond	2,650
Straight Value of the bond	2,350

You are required to calculate the following:
(i) Conversion value of bond
(ii) The percentage of downside risk
(iii) The conversion premium
(iv) Conversion parity price of the stock and also interpret the results.

(July'21 QP 8 marks)

Solution 43

(i) Stock value or conversion value of bond
120 × 20 = ₹2,400
(ii) Percentage of the downside risk
= $\frac{\text{Market Price} - \text{Straight Value}}{\text{Market Price}}$ = $\frac{2650 - 2350}{2650}$ = 0.1132 or 11.32%
This ratio gives the percentage price decline experienced by the bond if the stock becomes worthless.
(iii) Conversion Premium
$\frac{\text{Market Price} - \text{ConversionValue}}{\text{ConversionValue}}$ x 100 = $\frac{2650-2400}{2400}$ = **10.42**
(iv) Conversion Parity Price
$\frac{\text{Bond Price}}{\text{No.of Shares on Conversion}}$ = $\frac{2650}{20}$ = **₹132.50**

This indicates that if the price of shares rises to ₹132.50 from ₹120 the investor will neither gain nor lose on buying the bond and exercising it. Observe that ₹12.50 (₹132.50 – ₹120.00) is 10.42% of ₹120, the Conversion Premium.

44) Illustration

Today being 1st January 2019, Ram is considering to purchase an outstanding Corporate Bond having a face value of ₹1,000 that was issued on 1st January 2008 which has 9.5% Annual Coupon and 20 years of original maturity (i.e. maturing on 31st December 2027). Since the bond was issued, the interest rates have been on downside and it is now selling at a premium of ₹125.75 per bond.
Determine the prevailing interest on the similar type of Bonds if it is held till the maturity which shall be at Par.

(RTP Nov'20 New & Old)

Solution 44

To determine the prevailing rate of interest for the similar type of Bonds we shall compute the YTM of this Bond using IRR method as follows:

M = ₹1000 Interest = ₹95 (0.095 x ₹1000) n = 9 years

V_0 = ₹1125.75 (₹1,000 + ₹125.75)

YTM can be determined from the following equation

₹95 × PVIFA (YTM, 9) + ₹1000 × PVIF (YTM, 9) = ₹1125.75

Let us discount the cash flows using two discount rates 8% and 10% as follows:

Year	Cash Flows	PVF@6%	PV@6%	PVF@8%	PV@8%
0	- 1125.75	1	- 1125.75	1	- 1125.75
1	95	0.943	89.59	0.926	87.97
2	95	0.890	84.55	0.857	81.42
3	95	0.840	79.80	0.794	75.43
4	95	0.792	75.24	0.735	69.83
5	95	0.747	70.97	0.681	64.70
6	95	0.705	66.98	0.630	59.85
7	95	0.665	63.18	0.583	55.39
8	95	0.627	59.57	0.540	51.30
9	1095	0.592	648.24	0.500	547.50
			112.37		**-32.36**

Now we use interpolation formula

$= 6\% + \frac{112.37}{112.37-(-32.36)} \times (8\% - 6\%) = 6\% + 1.553\% = 7.55\%$

45) Illustration

The following data is available for NNTC bond:

Face value	₹1000
Coupon rate	7.50%
Market price per share	8 years
Market price of convertible bond	₹1000
YTM	8%

Calculate:

(i) The current market price, duration, and volatility of the bond.

(ii)The expected market price if the decrease in required yield by 50 bps.

(RTP Nov'20 Old)

Solution 45

(i) Current Market Price of Bond shall be computed as follows:

Year	Cash Flows	PVF@ 8%	PV@8%
1	75	0.926	69.45
2	75	0.857	64.28
3	75	0.794	59.55
4	75	0.735	55.13
5	75	0.681	51.08
6	75	0.630	47.25

7	75	0.583	43.73
8	1075	0.540	580.50
			970.97

Thus, the current market price of the Bond shall be ₹970.97.

Alternatively, using the Short-cut method the Market Price of Bond can also be computed as follows:

$$\frac{\text{Interest} + (\text{Discount/Premium})/\text{ Years to maturity}}{(\text{Face Value } + \text{ market Value})/2}$$

Let market price be X

$0.08 = \frac{75 + (1000-X)/8}{(1000+X)/2}$

Thus, Value of X i.e. the price of Bond shall be ₹969.70

For the duration of the bond, we have to see the future cash flow and discount them as follows:

Year	CF	PV@8%	Discounted CF	Proportion	Prop*Time(years)
1	75	0.926	69.45	0.071	0.071
2	75	0.857	64.28	0.066	0.132
3	75	0.794	59.55	0.061	0.183
4	75	0.735	55.13	0.057	0.228
5	75	0.681	51.08	0.053	0.265
6	75	0.630	47.25	0.049	0.294
7	75	0.583	43.73	0.045	0.315
8	1075	0.540	580.50	0.598	4.784
		Total	**970.97**	1.000	**6.272**

Volatility of the bond = Duration / (1+ Yield) = 6.272/1.08 = 5.81

If there is decrease in required yield by 50 bps the expected market price of the Bond shall be increased by: ₹970.97 × - 0.50 (-5.81/100) = ₹28.21

Hence expected market price is ₹970.97 + ₹28.21 = ₹999.18

As yield approaches coupon, market price will equal redemption value.

Alternatively, this portion using Bond Price as per Short-cut method can also be computed as follows:

0.50 (5.81/100) = ₹28.17×= ₹969.70

Then, the market price will be = ₹969.70 + ₹28.17 = **₹997.87**

Duration can also be computed as : $\frac{1+y}{y} - \frac{(1+y)+t(c-y)}{c[(1+y))^t - 1]+y} = \frac{1.08}{0.08} - \frac{(1.08)+8(0.075-0.08)}{0.075[(1.108)^8 - 1]+0.08}$

$= 13.5 - \frac{1.08 + 8(-0.005)}{0.075(1.85-1)+0.08} = 13.5 - \frac{1.08-0.04}{0.075*0.85+0.08}$

$= 13.5 - \frac{1.04}{0.14375}$ = 13.5 - 7.235 = **6.72 years**

46) Illustration

Following are the yields on Zero Coupon Bonds (ZCB) having a face value of ₹ 1,000.

Maturity (Years)	Yield to Maturity (YTM)
1	10%
2	11%
3	12%

Assume that the term structure of interest rate will remain the same.

You are required to

(i) Calculate the implied one year forward rates

(ii) Expected Yield to Maturity and prices of one year and two-year Zero Coupon Bonds at the end of the first year.

(Jan'21 QP 4 marks)

Solution 46

(i) Implied 1 year forward rates

Rate for Yr 1 is YTM for Yr 1 = 10%

Maturity (Years)	Yield to Maturity (YTM)	Froward rate
1	10%	10%
2	11%	?
3	12%	?

In ZCB, there is no coupon cashflow.

P_0 = PVF $1000_{(1,10\%)}$ = $\frac{1000}{1.10}$ = 909.09

P_0= PVF $1000_{(2,11\%)}$ = 811.62

P_0 = PVF$1000_{(3,12\%)}$ = 711.78

811.62 = $\frac{1000}{1.10}$ X $\frac{1}{(1+F_2)}$

1 + F_2 = $\frac{1000}{811.62 \times 1.10}$

1+ F_2 =1.120094

F_2 = 12.01%

Now to find F_3

711.78 = $\frac{1000}{(1.10)(1.1201)(1+F_3)}$

1 + F_3 = $\frac{1000}{711.78 * 1.1 * 1.1201}$ = $\frac{1000}{1.1403}$

F_3= 14.03%

(ii) **Price of one year bond at end of Yr 1**

P_0 = $\frac{1000}{1.1201}$

P_0 = ₹892.78 YTM = 12.01%

Then **price of two year bond** at the end of year 1 shall be:

P_0 = $\frac{1000}{1.1201 * 1.1403}$ = **₹782.93**

YTM of two year bond after year 1 when P_0 = 782.93 is computed as :

P_0 = $\frac{1000}{(1+y)^2}$

$(1+y)^2$= 1000/782.93 = 1.27725 ,

So, 1 + y = $\sqrt{1.27725}$ = 1.1302

y= 13.02%

47) Illustration

Following is the information for the free options bond:

Face value of the bond	₹1,000
Coupon rate	7%
Terms of Maturity	7 years
Yield to Maturity	8%

You are required to calculate:

(i) Market price of the bound and duration

(ii) If there is an increase in yield by 35 basis points, what would be the price of bond?

Present Values	t_1	t_2	t_3	t_4	t_5	t_6	t_7
$PVIF_{0.07,\,t}$	0.935	0.874	0.817	0.764	0.714	0.667	0.623
$PVIF_{0.08,\,t}$	0.926	0.857	0.794	0.735	0.681	0.631	0.584

Solution 47

(i) (1) Market price and duration of Bond
= 70 (PVIAF 8%,7) + 1,000 (PVIF 8%,7)
= 70 (5.208) + 1,000 (0.584) = 364.56 + 584.00 = 948.56

(2) Duration of Bond

Period	Cash flows	PVF@8%	PV (₹)	Proportion	Proportion *Time
1	70	0.926	64.82	0.0683	0.0683
2	70	0.857	59.99	0.0632	0.1264
3	70	0.794	55.58	0.0586	0.1758
4	70	0.735	51.45	0.0542	0.2168
5	70	0.681	47.67	0.0503	0.2515
6	70	0.631	44.17	0.0466	0.2796
7	1,070	0.584	624.88	0.6588	4.6116
			948.56		**5.73**

Duration of the Bond is 5.73 years

(ii) Price of Bond if increase in yield by 35 basis points

Period	Cashflow (₹)	PVF@8.35%	PV (₹)
1	70	0.923	64.61
2	70	0.852	59.64
3	70	0.786	55.02
4	70	0.726	50.82
5	70	0.670	46.90
6	70	0.618	43.26
7	1,070	0.570	609.90
			930.15

Alternatively, if the same increase in yield is linked with duration as computed in sub part (i), then answer will be computed as follows:

Volatility of Bond = $\frac{\text{Duration}}{1 + \text{YTM}} = \frac{5.73}{1+0.08} = 5.306$

The expected market price if increase in yield is by 35 basis points. : p = - M.D x Δy x P
= ₹948.56 x 0.35 (5.306/100) = **₹17.62**
Hence expected market price is ₹948.56 – ₹17.62 = **₹930.94**
Hence, the market price will decrease.

48) Illustration

Bank A enters into a Repo for 21 days with Bank B in 8% Government of India Bonds 2020 @ 6.10% for ₹5 crore. Assuming that clean price is ₹97.30 and initial margin is 1.50% and days of accrued interest are 240 days (assume 360 days in a year).

Compute:

(i) The dirty price

(ii) The repayment at maturity

[Jan'21 QP (Old) 5 marks]

Solution 48

(i) Dirty Price = Clean Price + Interest Accrued

= 97.30 + 100 x 8% x $\frac{240}{360}$ = **₹102.63**

(ii) First Leg (Start Proceed)

= Nominal Value x $\frac{\text{Dirty Price}}{100}$ x $\frac{100-\text{Initial Margin}}{100}$

= ₹5,00,00,000 x $\frac{102.63}{100}$ x $\frac{100-1.50}{100}$ = **₹5,05,45,275**

Second Leg (Repayment at Maturity) = Start Proceed x (1 + Repo Rate x $\frac{\text{No.of days}}{360}$)

= ₹5,05,45,275 x (1 + 0.0610 x $\frac{21}{360}$) = **₹5,07,25,132**

OR, ₹5,05,45,275 x 6.1% x $\frac{21}{360}$ = 1,79,856.94

Hence, Re purchase price = S.P + Repo Interest = 5,05,45,275 + 1,79,856.94 = **₹5,07,25,132**

49) Illustration

A bond is currently trading for 98.722 per 100 of par value. If the bond's yield to-maturity (YTM) rises by 10 basis points, the bond's full price is expected to fall to 98.669. If the bond's YTM decreases by 10 basis points, the bond's full price is expected to increase to 98.782. The bond's approximate convexity is closest to:

A. 35.453

B. 70.906

C. 1,144.628

Solution 49

Δy = 10bps = 1% = 0.001 V_0= 98.722

Y_- V_+ 98.782 : for Y_- 10bps

Y_+ V_- 98.669 : for Y_+ 10bps

Approximate convexity = $\frac{V_+ + V_- - 2V_0}{2\,x\,V_0(\Delta y)^2}$ = $\frac{98.782+98.669-2\times 98.722}{2\,x\,(0.001)^2\times 98.722}$ = $\frac{0.007}{0.000098722\ X\ 2}$ = **35.453**

50) Illustration

A bond has an annual modified duration of 7.020 and annual convexity of 65.180. If the bond's yield-to-maturity decreases by 25 basis points, the expected percentage price change is closest to:

A. 1.73%

B. 1.76%

C. 1.80%

Solution 50

Given, MD = 7.02 Convexity = 65.18↓ YTM = 25 bps = 0.25% = 0.0025

So, determine actual change in price **in terms of ₹** , the formula is :

Δprice = - MD x Δy x P_0 + $(\Delta y)^2$ x Conv x P_0

And to determine actual change in **price in terms of %** , the formula is :

Δprice % = - MD x Δy + $(\Delta y)^2$ x Conv

= -7.020 x – 0.0025 + $(0.0025)^2$ x 65.180

= 0.01755 + 0.000404 = 0.01795

= 1.795% or 1.8%

51) Illustration

A bond has an annual modified duration of 7.140 and annual convexity of 66.200. The bond's yield-to-maturity is expected to increase by 50 basis points.
The expected percentage price change is closest to:

A. -3.40%
B. -3.49%
C. -3.57%

Solution 51

Δprice% = - MD x Δy + $(\Delta y)^2$ x Conv = -7.14 x 0.005 + $(0.005)^2$ x 66.2
= -0.0357 + 0.001655
= -0.034045
%change in price = - 3.405% or **-3.40%**

52) Illustration

Based on the following data, estimate the Net Asset Value (NAV) 1st July 2016 on per unit basis of a Debt Fund:

Name of Security	Face Value (₹)	Purchase Price (₹)	Maturity Date	No. of Securities	Coupon Date(s)	Duration of Bonds
10.71% GOI 2028	100	104.78	31st March, 2028	100000	31st March	7.3494
10% GOI 2023	100	100.00	31st March 2023	50000	31st March & 30th Sep	5.086
9.5% GOI 2021	100	97.93	31st Dec 2021	40000	30th June & 31st Dec	4.3949
8.5% SGL 2025	100	91.36	30th June 2025	20000	30th June	6.5205

Number of Units (₹ 10 face value each): 100000
All securities were purchased at a time when applicable Yield to Maturity (YTM) was 10%. On NAV date, the required yield increased by 75 basis point and Cash in hand and accrued expenses were ₹ 6,72,800 and ₹ 2,37,400 respectively.

(RTP Nov'17)

Solution 52

<u>Computation of Change in price of the bond</u>

Name of Security	Purchase Price (₹) (A)	No. of Securities (B)	Value of the bond P = (A * B)	Duration of Bonds (C)	Modified Duration(M.D) = (C)/1.1 or 1.05	Δ Bond Δp = - Δy x M.D x P
10.71% GOI 2028 (Annual coupon)	104.78	100000	10478000	7.3494	6.6813	- 525048 (10478000 x 6.6813 x 0.75%) Or $\left(\frac{10478000 \times 7.3494}{1.1}\right) 0.75\%$
10% GOI 2023 (Semi-annual	100.00	50000	5000000	5.086	4.8438	- 181643 (5000000 x 5.086 x 0.75%) Or $\left(\frac{5000000 \times 5.086}{1.05}\right) 0.75\%$

coupon)						
9.5% GOI 2021 (Semi-annual coupon)	97.93	40000	3917200	4.3949	4.1856	- 122969 (3917200 x 4.3949x 0.75%) Or $\left(\frac{3917200 \times 4.3949}{1.05}\right) 0.75\%$
8.5% SGL 2025 (Annual coupon)	91.36	20000	1827200	6.5205	5.9277	- 81234 (1827200 x 6.5205 x 0.75%) Or $\left(\frac{1827200 \times 6.5205}{1.1}\right) 0.75\%$
			Σ=21222400			

Computation of Expected Market Price

Name of Security	Value of the bond (P)	Δ in price	Expected Market Price P - Δp
10.71% GOI 2028 (Annual coupon)	10478000	- 525048	9952952 (10478000 – 525048)
10% GOI 2023 (Semi-annual coupon)	5000000	- 181643	4818357 (5000000 – 181643)
9.5% GOI 2021 (Semi-annual coupon)	3917200	- 122969	3794231 (3917200 – 122969)
8.5% SGL 2025 (Annual coupon)	1827200	- 81234	1745966 (1827200 – 81234)
	Σ = 21222400	Σ = - 910894	Σ = 20311506

Computation of NAV per unit

Particulars	Amount(₹)
Closing Value of the bond	20311506
Add : Cash in hand	672800
Less : Accrued Expenses	(237400)
Add :Interest Accrued (WN : 1)	392750
Value of net assets	21139656
Total units	100000
NAV/No. of units	211.3965

Working Note 1: Computation of Interest Accrued

Name of Security	Computation	Interest Accrued(₹)
10.71% GOI 2028 (Annual coupon)	100 x 100000 x 10.71% x3/12	267750

10% GOI 2023 (Semi-annual coupon)	100 x 50000 x 5% x 3/6	125000
		∑ = 392750

53) Illustration

Following is the information for the options free bond

Face value of the bond	₹ 1,000
Coupon rate	7%
Terms of Maturity	7 years
Yield to Maturity	8%

You are required to calculate:

i. Market price of the bond and duration.

ii. If there is an increase in yield by 35 basis points, what would be the price of bond?

Present Value	t_1	t_2	t_3	t_4	t_5	t_6	t_7
PVIF0.07,t	0.935	0.874	0.817	0.764	0.714	0.667	0.623
PVIF0.08,t	0.926	0.857	0.794	0.735	0.681	0.631	0.584

[Jan'21 QP (Old)]

Solution 53

(i) (1) Market price and duration of Bond

= 70 (PVIAF 8%,7) + 1,000 (PVIF 8%,7)

= 70 (5.208) + 1,000 (0.584) = 364.56 + 584.00 = **₹948.56**

(2) Duration of Bond

Period (A)	Cash flow (₹) (B)	PVF@ 8% (C)	PV (₹) (D) = (B) x (C)	(E) = (A) x (D)
1	70	0.926	64.82	64.82
2	70	0.857	59.99	119.98
3	70	0.794	55.58	166.74
4	70	0.735	51.45	205.80
5	70	0.681	47.67	238.35
6	70	0.631	44.17	265.02
7	1,070	0.584	624.88	4374.16
			948.56	**5434.87**

Duration of the Bond is 5434.87/948.56 = 5.73 years

(ii) Price of Bond if increase in yield by 35 basis points

Year (A)	Cash flow (₹) (B)	PVF@ 8% (C)	PV (₹) (D) = (B) x (C)
1	70	0.923	64.61
2	70	0.852	59.64
3	70	0.786	55.02
4	70	0.726	50.82
5	70	0.670	46.90
6	70	0.618	43.26
7	1,070	0.570	609.90
			930.15

Alternatively, if the same increase in yield is linked with duration as computed in sub part (i), then answer will be computed as follows:

Volatility of Bond = $\frac{\text{Duration}}{1+\text{YTM}} = \frac{5.73}{1+0.08}$ = 5.306

The expected market price if increase in yield is by 35 basis points. : Δp = - Δy x M.D x P

0.35% x 5.306 x 948.56 = **₹17.62**

Hence expected market price is ₹948.56 – ₹17.62 = **₹930.94**

Hence, the market price will decrease with increase in the yield.

54) Illustration

Mr. X wants to invest ₹ 1,00,000 in the 7 years 8% bonds in the market (Face Value ₹100) which were issued 2 years ago.

(i) You are requested to advise him what is the maximum price for bonds to be paid in the following scenarios:

(1) If Mr. X is expecting minimum 9% return on the bonds
(2) If Mr. X is expecting minimum 7% return on the bonds
(3) If the present rate of similar bonds issued is 8.25%
(4) If the present rate of similar bonds issued is 7.75%

(ii) If the bonds are available at par and 1% is the transaction cost, what is the effective yield?

(iii) Find the number of days required to breakeven transaction cost if the bonds are available at par and 2% is the transaction cost.

(Nov'22 QP 8 marks)

Solution

Assumption: Bonds are annual coupon only
Balance period = 5 years

(i) Maximum price for bonds to be paid

1) Price of Bond if Minimum return is 9%

Coupon Payment = 100* 8% = 8
Principal = 100
Discount = 9%
Bond Price = $\frac{8}{1.09^1} + \frac{8}{1.09^2} + \frac{8}{1.09^3} + \frac{8}{1.09^4} + \frac{108}{1.09^5}$

(Alternatively) =100* PVIF (9%,5th year) + 8* PVAF (9%, 5 years)
= **₹ 96.1072**

2) Price of Bond if Minimum return is 7%

Coupon Payment = 100* 8% = 8
Principal = 100
Discount = 7%
Bond Price = $\frac{8}{1.07^1} + \frac{8}{1.07^2} + \frac{8}{1.07^3} + \frac{8}{1.07^4} + \frac{108}{1.07^5}$
(Alternatively) = 100* PVIF (7%,5th year) + 8* PVAF (7%, 5 years)
= 100* 0.7129 + 8* 4.1002
= **₹ 104.09**

3) Price of Bond if present rate on similar Bonds is 8.25%

P_0 = 100* PVIF (8.25%,5th year) + 8* PVAF (8.25%, 5 years)
= 100* 0.6727 + 8* 3.9665
= **₹ 99.002**

4) Price of Bond if present rate on similar Bonds is 7.75%
P_0 = 100* PVIF (7.75%,5th year) + 8* PVAF (7.75%, 5 years)
= 100* 0.6885 + 8* 4.0191
= **₹ 101.0032**

(ii) Effective Yield

P_0 = 100
Coupon = ₹ 8
Face Value = 100
Transaction Cost = 1%
Market Rate = 8%
Yield = 8%
Investment = 100 (1+ 1%) = ₹ 101

Of ₹ 101 invested, ₹ 1 goes to the transaction cost and the remaining to the Bond.

Effective Yield $= \frac{Coupon}{Investment}$ $= \frac{8}{101}$ = **7.92%**

(i) Number of days required to breakeven transaction cost

P_0 = 100
Coupon = ₹ 8
Face Value = 100
Transaction Cost = 2%

Coupon = ₹ 8
Face Value = 100
Transaction Cost = 2%* 100 = ₹ 2
No. of Days $= \frac{2*365}{8}$ = **91.25 days**

For a bond with annual coupon of ₹ 8, it will take 91.25 days to breakeven transaction cost, assuming 365 days in the year.

55) Illustration

An investor, in the beginning of 2022, purchased substantial number of 8-year 7.50% ₹ 1000 bond with 5 % premium on maturity at a required Yield to Maturity (YTM) of 8.50 %. However, due to the continuing war in Europe, inflation is running very high in the economies of the countries. The yield on the bonds is decreasing. The risk averse investor wants to protect himself from further loss and decides to sell the bonds in 2023. He has got a proposal from another investor who is willing to purchase these bonds by shelling out a maximum amount of ₹ 797.50 per bond. Investors follow intrinsic value method for valuation of the Bonds. You are required to determine. The Market price, Duration and Volatility of the bond.
Will it be the right decision of the new investor if he is looking for Required Yield to Maturity (YTM) as 12% p.a.?

Period	1	2	3	4	5	6	7
PVIF(8.50%,n)	0.9217	0.8495	0.7829	0.7216	0.6650	0.6129	0.5649

(May'23 QP 9 marks, Similar MTP Mar'23)

Solution

Market Price

Tenure	= 8 years
Time of Purchase	= 2022
Face Value	= 10,000
Coupon %	= 7.5%
Premium on redemption	= 5%
Regular YTM	= 8.5%
Proposed Selling Price	= ₹ 797.5

Bond price at the time of investment

P_0 = 1050* PVIF (8.5%,8th year) + 75* PVAF (8.5%, 8 years)
= 1050* 0.5649 + 75* 5.1185
= **977.0325**

Assuming the 8- year bond was purchased at balance maturity of 7 years.

Duration = $\sum CF * PV * T$

Year	Cash Flow	PV		$\sum CF * PV * T$
1	75	0.9127		69.1275
2	75	0.8495		127.425
3	75	0.7829		176.1525
4	75	0.7216		216.48
5	75	0.6650		249.375
6	75	0.6129		275.805
7	1125	0.5649		4448.5875
				5562.9525

Duration in Years = 5562.9525/977.0325 = **5.6937 years**

Volatility = $\frac{Duration}{1+YTM}$

= $\frac{5.6934}{1+8.5\%}$ = **5.2777**

Price of Bond in 2023 i.e., at balance maturity of 6 years.

P_{2023} = 1050* PVIF (12%,6th year) + 75* PVAF (12%, 6 years)
= 1050* 0.50663 + 75* 4.1114
= **₹ 840.3170**

For new investor, intrinsic value is ₹ 840.3170 but he/she is acquiring at ₹ 797.50, hence it is a great deal, but for an existing investor it is a bad deal.

56) Illustration Nov'24 Case Based MCQ

The following information is available in respect of Bond 1 and Bond 2:

	Bond 1	Bond 2
Face value, redeemable value at par	₹ 1000	₹ 1000
Coupon rate, payable annually (%)	6%	10%
Time to maturity (years)	5	3

An investor has the portfolio consisting of 75% of Bond 1 and 25% of Bond 2. The current YTMs prevailing in the market is 10%.

Year (n)	1	2	3	4	5
PVIF(10%, n)	0.9091	0.8264	0.7513	0.6830	0.6209

(Case Based MCQ Nov'24 QP 2 marks each)

From the information given above, choose the correct answer to the Question no. 1 to 4:

What should be the price and duration of Bond – 2?

A. ₹ 826.43 and 2.49
B. ₹ 1,000 and 2.74
C. ₹ 924.85 and 2.74
D. ₹ 1000 and 2.49

New price of the portfolio if YTM changes from 10% to 10.5% based on the duration is:

A. ₹ 870.12
B. ₹ 902.36
C. ₹ 1832.23
D. ₹ 1864.45

What should be the price and duration of Bond – 1?

A. ₹ 848.34 and 4.43
B. ₹ 811.09 and 4.38
C. ₹ 1,227.14 and 4.43
D. ₹ 658.15 and 3.90

What will be the price sensitivity of the portfolio?

A. -4.027
B. -2.491
C. -3.643
D. -3.981

Solutions:

Given

	Bond 1	Bond 2
FV	1000	1000
Coupon	6%	10%
YTM	10%	10%
Bond Tenure (Years)	5	3
Weight	75%	25%

Price = Sum of PV of CF

Macaulay Duration = Sum of PV F x CF x T / Price

Modified Duration / Price sensitivity = Macaulay Duration (1+y)

Price Computation **Duration Computation**

Bond 1							
Year (T)	CF	PVF	CF x PVF	CF x PVF x T			
1	60	0.9091	54.546	54.546			
2	60	0.8264	49.584	99.168			
3	60	0.7513	45.078	135.234			
4	60	0.683	40.98	163.92			
5	106	0.620	658.154	3290.77			

	0	9					
			848.342	3743.638	Mac Duration	3743.64/848.342	
			Q3		Mac Duration	4.41	Q 3
Bond 2							
Year (T)	CF	PVF	CF x PVF	CF x PVF x T			
1	100	0.9091	90.91	90.91			
2	100	0.8264	82.64	165.28			
3	1100	0.7513	826.43	2479.29			
				0			
				0			
			999.98	2735.48	Mac Duration	2735.48/1000	
			Q1		Mac Duration	2.74	Q1

Q2

	Bond1	**Bond 2**	
Weight	75%	25%	
Price ₹	848.342	1000	
WA Portfolio Value	636.2565	250	886.2565
Price will fall if Yield Increases			
Answer is A 870.12 - Only option where price is lower than Weighted Average Portfolio Price			

Q4

	Bond1	**Bond 2**	
Weight	75%	25%	
Mac Duration	4.41	2.74	
WA Portfolio Duration	3.31	0.68	3.99
WA Modified Duration			3.99/1.10
WA Modified Duration			3.63
Answer		Option D	Q9

57) Illustration

TK Ltd. has ₹ 600 Lakh 12% Debenture outstanding with 5 years remaining to redemption. Since interest rates are decreasing, company is planning to redeem these debentures with a ₹ 600 Lakh issue of 5 years 10% Debenture at par. Issue cost of 10% Debenture will be ₹ 10 Lakh. Premium paid on redemption of 12% Debenture is 5%. Tax rate applicable to company is 20%.
You are required to advise on the 12% Debenture Redemption Decision. PVF @ 10% & 8% are as under:

Rate	1	2	3	4	5
8%	0.93	0.86	0.79	0.74	0.68
10%	0.91	0.83	0.75	0.68	0.62

Make all calculations up to 2 decimals (Sep-25 6 Marks)

Solution:

Interest Exp on Old Debentures of ₹600 Lacs @ 12%	₹72 Lacs
Interest Exp on New Debentures of ₹600 Lacs @ 10%	₹60 Lacs
Savings in Interest Expense p.a	**₹12 Lacs**

Costs of Refinancing

Issue Expenses	₹10 Lacs
Premium on Redemption (600 x5% = 30)	₹30 Lacs
Total Expenses	**₹40 Lacs**

PV of Costs & benefits

PV taken at Post tax cost of new borrowing i.e 10% less 20% = 8%

Year	Particulars	CF Before Tax ₹ L	Tax	CF after Tax ₹ L	PVF @ 8%	PVCF ₹ L
0	Issue Exp & Prem on Red	-40	20%	-32	1	-32
1	Reduction in Int Exp	12	20%	9.6	0.93	8.93
2	Reduction in Int Exp	12	20%	9.6	0.86	8.26
3	Reduction in Int Exp	12	20%	9.6	0.79	7.58
4	Reduction in Int Exp	12	20%	9.6	0.74	7.10
5	Reduction in Int Exp	12	20%	9.6	0.68	6.53
	NPV					**6.40**

*Assumption – tax benefit considered on both Issue Expenses & Premium right away

NPV of Refinancing decision is positive. Go ahead

Year	Particulars	CF Before Tax ₹ L	Tax	CF after Tax ₹ L	Tax benefit on Issue Exp ₹ Lacs *	Net CF	PVF @ 8%	PVCF ₹ L
0	Issue Exp	-10	0	-10		-10	1	-10
0	Premium on Redemption	-30	20%	-24		-24	1	-24
1	Reduction in Int Exp & Tax benefit on Issue Expenses	12	20%	9.6	0.4	10	0.93	9.3
2	Reduction in Int Exp & Tax benefit on Issue Expenses	12	20%	9.6	0.4	10	0.86	8.6
3	Reduction in Int Exp & Tax benefit on Issue Expenses	12	20%	9.6	0.4	10	0.79	7.9
4	Reduction in Int Exp & Tax benefit on Issue Expenses	12	20%	9.6	0.4	10	0.74	7.4

5	Reduction in Int Exp & Tax benefit on Issue Expenses	12	20%	9.6	0.4	10	0.68	6.8
	NPV							**6.00**

*Assumption – tax benefit considered on Issue Expenses over 5 years & Premium right away

Issue Expenses ₹10 Lacs

Annual Amortization ₹ 2 Lacs

Annual tax benefit @ 20% = ₹2 Lacs x 20 % = ₹0.4 Lacs

NPV of Refinancing decision is positive. Go ahead

Example 1

The probability distribution of possible returns from an investment in shares is given. Compute expected return and the risk of the investment.

Possible returns (in %) x_i	Probability of occurrence P_i
20	0.20
30	0.20
50	0.40
60	0.10
70	0.10

Solution:

x_i	P_i	x_iP_i	$x-\bar{x}$	$(x-\bar{x})^2$	$(x-\bar{x})^2 * P_i$
20	0.20	4	-23	529	105.8
30	0.20	6	-13	169	33.8
50	0.40	20	7	49	19.6
60	0.10	6	17	289	28.9
70	0.10	7	27	729	72.9
230	**1.0**	**43**			**261**

$E(R)$ = $\sum_{i=1}^{n} P_i x_i$ = **43%**

Variance = $\sum(x-\bar{x})^2 * P_i$ = 261

Standard Deviation = $\sqrt{variance}$ = **16.16%**

Example 2

From the given data, compute covariance and correlation:

Day	ICICI Bank (X)	HDFC Bank (Y)
1	10 %	11%
2	8%	7%
3	14%	3%
4	-3%	-7%
5	-1%	0%

Solution:

Day	x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$y-\bar{y}$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
1	10%	4.4	19.36	11%	8.2	67.24	36.08
2	8%	2.4	5.76	7%	4.2	17.64	10.08
3	14%	8.4	70.56	3%	0.2	0.04	1.68
4	-3%	-8.6	73.96	-7%	-9.8	96.04	84.28
5	-1%	-6.6	43.56	0%	-2.8	7.84	18.48
Total	**28%**		**213.2**	**14%**		**188.8**	**150.6**
Mean	5.6%			2.8%			

Variance = $\frac{\sum(x-\bar{x})^2}{n}$ = $\frac{213.2}{5}$ = 42.64

S. D = $\sqrt{variance}$ = $\sqrt{42.63}$ = **6.53**

Variance = $\frac{\sum(y-\bar{y})^2}{n}$ = $\frac{188.8}{5}$ = 37.76

S. D = $\sqrt{variance}$ = $\sqrt{37.76}$ = **6.15**

cov_{xy} $= \frac{\sum(x-\bar{x})(y-\bar{y})}{n} = \frac{150.6}{5} =$ **30.12**

Correlation Coefficient, $r = \frac{cov_{xy}}{\sigma_x \sigma_y} = \frac{30.12}{6.53*6.15} =$ **0.75**

Example 3

From the given data, compute Beta (β) using all four formulae:

Day	ICICI Bank (i or y)	NIFTY (m or x)
1	10 %	11%
2	8%	7%
3	14%	3%
4	-3%	-7%
5	-1%	0%

Solution:

x	y	$(x-\bar{x})$	$(y-\bar{y})$	$(x-\bar{x})(y-\bar{y})$	xy	y^2
11	10	8.2	4.4	36.08	110	100
7	8	4.2	2.4	10.08	56	64
3	14	0.2	8.4	1.68	42	196
-7	-3	-9.8	-8.6	84.28	21	9
0	-1	-2.8	-6.6	18.48	0	1
14	**28**			**150.6**	**229**	**370**

$\bar{x}$ $= \frac{\sum x}{n} = \frac{28}{5} = 5.6$

$\bar{y}$ $= \frac{\sum y}{n} = \frac{14}{5} = 2.8$

σ_x $= \sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{213.2}{5}} = 6.15$

σ_x^2 $= 37.76$

σ_y $= \sqrt{\frac{\sum(y-\bar{y})^2}{n}} = \sqrt{\frac{188.8}{5}} = 6.53$

σ_y^2 $= 42.64$

Equation 1:

β $= \frac{cov_{xy}}{\sigma_x^2}$

cov_{xy} $= \frac{\sum(x-\bar{x})(y-\bar{y})}{n} = \frac{150.6}{5} = 30.12$

β $= \frac{30.12}{37.76} =$ **0.7976**

Equation 2:

β $= \frac{r_{xy}\sigma_y}{\sigma_x}$

r_{xy} $= \frac{cov_{xy}}{\sigma_x \sigma_y} = \frac{30.12}{6.53*5.15} = 0.75$

β $= \frac{0.75*6.53}{6.15} =$ **0.7963**

Equation 3:

β $= \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2} = \frac{5*229-28*14}{5*228-14^2} = \frac{1145-392}{1140-196} =$ **0.79767**

Equation 4:

β $= \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n(\bar{x})^2} = \frac{229-5*5.6*2.8}{228-2.8^2} = \frac{229-78.4}{228-39.2} =$ **0.79767**

Example 4

Compute portfolio risk for the portfolio consisting of ICICI Bank and HDFC Bank securities. The expected returns of both the securities are given as below:

Day	ICICI Bank (X)	HDFC Bank (Y)
1	10 %	11%
2	8%	7%
3	14%	3%
4	-3%	-7%
5	-1%	0%

Compute the portfolio risk when correlation is 0.75

(i) Securities are held in the ratio 1:1

(ii) Securities are held in the ratio 3:7

Solution:

	ICICI Bank	HDFC Bank
	10%	11%
	8%	7%
	14%	3%
	-3%	-7%
	-1%	0%
Total	**28**	**14**
Average	**5.6**	**2.8**
S. D	**6.52993**	**6.14492**

Portfolio @ 50:50 ratio of x and y

Return = 0.5*5.6% + 0.5*2.8% = 4.2%

Risk, σ_p^2 = $\sigma_1^2 * w_1^2 + \sigma_2^2 * w_2^2 + 2 * \sigma_1\sigma_2 w_1 w_2 r$

= $6.53^2 * 0.5^2 + 6.15^2 * 0.5^2 + 2 * 30.12 * 0.5 * 0.5$

= 10.66+ 9.46+ 15.06 = **35.18**

σ_p = **5.93**

Portfolio @ 30:70 ratio of x and y

Return = 0.3*5.6% + 0.7*2.8% = 3.64%

Risk, σ_p^2 = $\sigma_1^2 * w_1^2 + \sigma_2^2 * w_2^2 + 2 * \sigma_1\sigma_2 w_1 w_2 r$

= $6.53^2 * 0.3^2 + 6.15^2 * 0.7^2 + 2 * 30.12 * 0.3 * 0.7$

= 3.84+ 18.38+ 12.65 = **34.87**

σ_p = **5.90**

Example 5

Compute portfolio risk for a portfolio with three securities based on the following information

Security	Weight	Standard Deviation (σ)
X	0.2	10%
Y	0.3	11%
Z	0.5	16%

The correlation for securities in relation to another security is as follows:

	$x \,\&\, y$	$y \,\&\, z$	$x \,\&\, z$
Correlation - 'r'	0.2	0.4	-0.1

Solution:

Return = $\sum_{i=1}^{n} w_i r_i$ = 10%* 50% + 5%* 10% + 6%* 40% = **7.9%**

Variance Covariance Matrix

	x 0.2	y 0.3	z 0.5
x 0.2	0.2*0.2*1*10*10	0.2*0.3*0.2*10*11	0.2*0.5*-0.1*10* 16
y 0.3	0.3*0.2*0.2*11*10	0.3*0.3*1*11*11	0.5*0.3*11*16*0.4
z 0.5	0.2*0.5*-0.1*16*10	0.3*0.5*0.4*16*11	0.5*0.5*1*16*16

σ_p^2 = 99.45

σ_p = **9.972**

Example 6

Compute portfolio risk for a portfolio with three securities in the portfolio (weights of A, B and C in the portfolio are 0.2, 0.3 and 0.5 respectively) based on the following information;

	Security A	Security B	Security C
Return	15	20	12
SD	20	25	16

The correlation for securities in relation to another security is as follows:

	AB	BC	AC
Correlation - 'r'	0.2	0.3	0.5

Solution:

Refer the Excel Sheet provided.

Illustrations

1. Illustration

A company's beta is 1.40. The market return is 14% and the risk-free rate is 10%.

(i) What is the expected return of the company's stock based on CAPM.

(ii)If the risk premium on the market goes up by 2.50% points, what would be the revised expected return on this stock?

(ICAI SM)

Solution:

β = 1.4

R_m = 14%

R_f = 10%

(i) $E(R)$ = $R_f + \beta\,(R_m - R_f)$ = 10% + 1.4(14%-10%) = 10% + 5.6%

$\boldsymbol{E(R)}$ = **15.6%**

(ii) $E\,(R)$ when Market Risk Premium $(R_m - R_f)$goes up by 2.5%

Market Risk Premium

$(R_m - R_f)$ = 4% + 2.5% = 6.5%

$E\,(R)$ = 10% + 1.4(6.5%) = 10% + 9.1% = **19.1%**

2. Illustration

The risk premium for the market is 10%.
Assuming Beta values of Security K are 0, 0.25, 0.42, 1.00 and 1.67.
Compute the risk premium on Security K.

(ICAI SM)

Solution:

Market Risk Premium,

$(R_m - R_f)$ = 10%

$\beta\ (R_m - R_f)$ = Risk Premium on Security

B value of Security (k) (a)	Market Risk Premium (b)	Security Risk Premium (a)*(b)
0	10%	0
0.25	10%	2.5%
0.42	10%	4.2%
1.00	10%	10%
1.67	10%	16.7%

3. Illustration

Treasury Bills give a return of 5%. Market Return is 13%

(i) What is the market risk premium

(ii) Compute the Beta Value and required returns for the following combination of investments.

Treasury Bill	100	70	30	0
Market	0	30	70	100

(ICAI SM)

Solution:

Rm = 13%

Rf = 5%

(i) $(R_m - R_f)$ = 13% - 5% = **8%**

(ii)

Market Portfolio	Risk-Free TB	R_m	R_f	β_m	β_p	Return of Portfolio
0%	100%	13%	5%	1	0	5%
30%	70%	13%	5%	1	0.3	7.4%
70%	30%	13%	5%	1	0.7	10.6%
100%	0%	13%	5%	1	1	13%

a) β of Portfolio

β_p = $\sum_{i=1}^{n} w_i \beta_i$

Beta of Market = 1

Beta of R_f = 0 (no risk)

Weight of Market * β +	Weight of Risk-Free TB *β	= β_p
0* 1	100* 0	0
30* 1	70* 0	0.3
70* 1	30* 0	0.7
100* 1	0* 0	1

b) Return of Portfolio

R_p = $\sum_{i=1}^{n} w_i R_i$

Weight of Market * Return + Weight of Risk-Free TB *Return = R_p

0* 13%	100* 5%	**5%**
30* 13%	70* 5%	**7.4%**
70* 13%	30* 5%	**10.6%**
100* 13%	0* 5%	**13%**

Alternate Formula

R_p = R_f + β (Market Risk Premium)

4. Illustration

Pearl Ltd. expects that considering the current market prices, the equity shareholders as per Moderate Approach, should get a return of at least 15.50% while the current return on the market is 12%.

RBI has closed the latest auction for ₹ 2500 crores of 182-day Treasury bills for the lowest bid of 4.3% although there were bidders at a higher rate of 4.6% also for lots of less than ₹ 10 crores. What is Pearl Ltd.'s Beta?

(ICAI SM)

Solution:

$E\ (R)$ = 15.5%

R_m = 12%

R_{f1} = 4.3%

R_{f2} = 4.6% (<10 crores)

$R_{f\ avg}$ = 4.45%

$$E(R) = R_f + \beta\,(R_m - R_f)$$

15.5% = 4.45% + β (12%-4.45%)

15.5% = 4.45% + β (7.55%)

11.05% = β (7.55%)

β = **1.4636**

(Given two risk free rates, we have assumed their average in computing the beta of Pearl Ltd.)

5. Illustration

The following information is available with respect of Jaykay Ltd.

Year	Jay Kay Limited: Average Share Price (₹)	Jay Kay Limited: DPS (₹)	Market: Average Index	Market: Dividend Yield (%)	Return on Govt. Bonds
2002	242	20	1812	4	6
2003	279	25	1950	5	5
2004	305	30	2258	6	4
2005	322	35	2220	7	5

Compute Beta Value of the company as at the end of 2005. What is your observation?

(ICAI SM)

Solution:

$E\ (R)$ of stock $= \frac{\text{dividend}}{\text{opening value}} + \frac{\text{capital gain}}{\text{opening value}}$

	Op Stk Price	Closing Stk Price	Increase	Stk Return = (Cl - OP)/ OP	Dividend	Div / OP	Total Return (Stk + Div Yld)
2002-03	242	279	37	15.29%	25.00	10.33%	25.62%
2003-04	279	305	26	9.32%	30.00	10.75%	20.07%
2004-05	305	322	17	5.57%	35.00	11.48%	17.05%

	Opening Index Value	Closing Index Value	Increase	Index Return = (Cl - OP)/ OP	Div Yield	Total Return Index + Div
2009-10	1812	1950	138	7.62%	5%	12.62%
2010-11	1950	2258	308	15.79%	6.00%	21.79%
2011-12	2258	2220	-38	-1.68%	7.00%	5.32%

Average Return on Treasury Bond, $= \frac{6\% + 5\% + 4\% + 5\%}{4}$

R_f = 5%

Beta = $Covariance\ (x, y)$ / $Variance\ Y$

Beta = $\sum(x - \bar{x})\ (y - \bar{y})$ /n / $\sum(y - \bar{y})\ ^2$ / n

	X - Stock Return	Y - Market Return	x-x mean (A)	y- y mean (B)	AB	B^2
2002-03	25.62	12.62	4.71	-0.63	-2.95	0.39
2003-04	20.07	21.79	-0.84	8.55	-7.20	73.14
2004-05	17.05	5.32	-3.86	-7.93	30.63	62.81
Total	**62.74**	**39.73**			**20.48**	**136.35**
Average	20.91	13.24				

Beta = (20.458/136.38) = 0.15

Expected Return across years based on Beta

CAPM Return	Actual Return	Action
5% + 0.15 (12.62%-5%)= 6.14%	25.62%	Buy
5% + 0.15 (21.8%-5%)= 7.52%	20.07%	Buy
5% + 0.15 (5.32%-5%)= 5.05%	17.05%	Buy

In all 3 years CAPM based Expected return is lower than actual return

6. Illustration

Information related to an investment is as follows:

- **Risk free rate 10%**
- **Market Return 15%**
- **Beta 1.2**

(i) What would be the return from this investment?

(ii) If the projected return is 18%, is the investment rightly valued?

(iii) What is your strategy?

(ICAT SM)

Solution:

(i) $E(R)$ = $R_f + \beta(R_m - R_f)$ = 10%+ 1.2 (15%-10%)
= 10%+ 1.2* 5 = **16% (CAPM)**

(ii) CAPM = 16%
Projected return = 18%
Project is **undervalued**

(iii) If projected return is 18%, **invest in the project** because return is greater than what is expected based on CAPM.

7. Illustration

The expected returns and Beta of three stocks are given below

Stock	A	B	C
Expected Return (%)	18	11	15
Beta Factor	1.7	0.6	1.2

If the risk-free rate is 9% and the expected rate of return on the market portfolio is 14% which of the above stocks are over, under or correctly valued in the market? What shall be the strategy?

(ICAI SM, MTP Mar'24)

Solution:

Stock	CAPM Return = $R_f + \beta(R_m - R_f)$	Return	Expected Return	Value and Action
A	9%+ 1.7 (14%-9%)	17.5%	<18%	**Undervalued Buy**
B	9%+ 0.6 (14%-9%)	12%	>11%	**Overvalued Sell**
C	9%+ 1.2 (14%-9%)	15%	=15%	**Rightly Valued Hold**

8. Illustration

A stock costing ₹ 120 pays no dividends. The possible prices that the stock might sell for at the end of the year with the respective probabilities are:

Price	Probability
115	0.1
120	0.1
125	0.2
130	0.3
135	0.2
140	0.1

Required:

(i) **Calculate the expected return.**

(ii) **Calculate the Standard deviation of returns.**

(ICAI SM, Old PM)

Solution:

(i) Expected Return

Year End price	Cost (a)	Absolute Return (b)	Return % = [(b)/(a)] *100
115	120	-5	- 4.16
120	120	0	0
125	120	5	4.16%
130	120	10	8.33%
135	120	15	12.5%
140	120	20	16.67%

$E\ (R)$ $= \sum Probability * Return$

= 0.1* (-4.16%) +0.1*0% + 0.2*4.16% + 0.3*8.33% + 0.2*12.5% + 0.1*16.67%

= 7.082

(ii)

Scenario	Probability P_i	Return, x	$(x - \bar{x})$	$(x - \bar{x})^2$	$(x - \bar{x})^2 * P_i$
1	0.1	- 4.16	- 11.242	126.38	12.638
2	0.1	0	- 7.082	50.15	5.015
3	0.2	4.16%	- 2.922	8.538	1.707
4	0.3	8.33%	1.248	1.5575	0.467
5	0.2	12.5%	5.418	29.354	5.87
6	0.1	16.67%	9.588	91.93	9.192
					34.892

$\bar{x}$ = 7.082

Variance = 34.892

SD $=\sqrt{variance}$ = **5.907**

9. Illustration

Following information is available in respect of expected dividend, market price and market condition after one year.

Market condition	Probability	Market Price	Dividend per share
		₹	₹
Good	0.25	115	9
Normal	0.50	107	5
Bad	0.25	97	3

The existing market price of an equity share is Rs. 106 (F.V. ₹ 1), which is cum 10% bonus debenture of ₹ 6 each, per share. M/s. X Finance Company Ltd. had offered the buy-back of debentures at face value.

Find out the expected return and variability of returns of the equity shares if buyback offer is accepted by the investor.

And also advise-Whether to accept buy-back offer?

(ICAI SM, RTP May'23, MTP Oct'19 Old, Old PM)

Solution:

Current M. P = 106

Net Price = 100 (post buy back)

Face Value = 6 (when buyback is accepted)

Condition	P_i	Market Price Post Buyback	Future Price	Dividend
Good	0.25	100	115	9
Normal	0.5	100	107	5
Bad	0.25	100	97	3

Absolute Return Computation	Expected Return (Absolute*Probability)
115+9-100 = 24	24* 0.25 = 6%
107+5-100 =10	12* 0.5 = 6%
97+3-100 =0	0* 0.25 = 0%

$E\ (R)$ = $\sum_{i=1}^{n} w_i R_i$ = 6+ 6+ 0

$\bar{x}$ = 12%

σ_p^2 = $\sum_{i=1}^{n}(x-\bar{x})^2 w_i$

= (24%- 12%)² * 0.25 + (12%- 12%)²* 0.5 + (0%- 12%)²* 0.25

= 144* 0.25 + 144*0.25 = **72**

σ_p = $\sqrt{\sum_{i=1}^{n}(x-\bar{x})^2 w_i}$ = $\sqrt{72}$ = **8.48%**

Return (Debt) = 10%

$E\ (R)$ Equity = 12%

Depending on investor's risk preference, he or she may choose to sell the debenture back or hold on to it, as its return is slightly lower than that of equity shares.

10. Illustration

Mr. A is interested to invest ₹ 1,00,000 in the securities market. He selected two securities B and D for this purpose. The risk return profile of these securities are as follows:

Security	Risk (SD)	Expected Return (ER)
B	10%	12%
D	18%	20%

Co-efficient of correlation between B and D is 0.15.

You are required to calculate the portfolio return of the following portfolios of B and D to be considered by A for his investment.

(i) 100 percent investment in B only;

(ii) 50 percent of the fund in B and the rest 50 percent in D;

(iii) 75 percent of the fund in B and the rest 25 percent in D; and

(iv) 100 percent investment in D only.

Also indicate that which portfolio is best for him from risk as well as return point of view?

(ICAI SM, RTP Nov'21, MTP Mar'22, Old PM)

Solution:

	R	σ
B	12%	10%
D	20%	18%

r = 0.15

Portfolio B:D	Expected Return	Risk Computation	Risk
100:0	12 %	Only B - B's σ	10%
50:50	16%	WN 2	10.93%
75:25	14%	WN 4	9.307%
100:0	20%	Only D - D's σ	18%

(i) CASE 2

$E(R)$ = $\sum_{i=1}^{n} w_i R_i$

$E(R)$ = 12%* 0.5 + 20% * 0.5 = **16%**

(ii) σ_p^2 = $\sigma_B^2 * w_B^2 + \sigma_D^2 * w_D^2 + 2 * \sigma_B \sigma_D w_B w_D r$

= $10^2 0.5^2 + 18^2 0.5^2 + 2 * 10 * 18 * 0.5 * 0.5 * 0.15$

= 25+ 81+ 13.5 = 119.5

σ_p = $\sqrt{119.5}$ = **10.93**

(iii) CASE 3

$E(R)$ when ratio is 75:25

= $0.75 * 12\% + 0.25 * 20\%$ = 9% + 5% = **14%**

(iv) CASE 3

σ_p^2 = $\sigma_B^2 w_B^2 + \sigma_D^2 w_D^2 + 2\sigma_B \sigma_D w_B w_D r$

= $0.75^2 10^2 + 0.25^2 18^2 + 2 * 10 * 18 * 0.75 * 0.25 * 0.15$

= 56.25+ 20.25+ 10.125 = **86.625**

σ_p = $\sqrt{86.625}$ = **9.307**

Purely from risk perspective Portfolio 3 has lowest risk and very low return, and from return perspective Portfolio 4 is the best but with high risk. However, considering the need for diversification we would recommend Portfolio 2 over 3 as it gives better return for marginal increase in risk.

11. Illustration

Consider the following information on two stocks, A and B:

Year	Return on A (%)	Return on B (%)
2006	10	12
2007	16	18

You are required to determine:

(i) The expected return on a portfolio containing A and B in the proportion of 40% and 60% respectively.

(ii) The Standard Deviation of return from each of the two stocks.

(iii) The covariance of returns from the two stocks.

(iv) Correlation coefficient between the returns of the two stocks.

(v) The risk of a portfolio containing A and B in the proportion of 40% and 60%.

(ICAI SM, May'18 QP 10 marks, RTP Nov'19, RTP May'20 Old, Old PM)

Solution:

(i) $E(R)$ when A: B = 40:60

$E(R)$ of A = $\frac{10+16}{2}$ =13%

$E(R)$ of B = $\frac{12+18}{2}$ =15%

$E(R)_p$ = 13*0.4 + 15*0.6 = 5.2+ 9 =**14.2%**

(ii) σ of each of the stocks

For A:

$\sigma_A^2 = \sum_{i=1}^{n} \frac{(x-\bar{x})^2}{n} = \frac{(10-13)^2 + (16-13)^2}{2} = 9$

$\sigma_A = \sqrt{9} = \mathbf{3}$

For B:

$\sigma_B^2 = \frac{(12-15)^2 + (18-15)^2}{2} = 9$

$\sigma_B = \sqrt{9} = \mathbf{3}$

(iii) Covariance $= \sum \frac{(x-\bar{x})(y-\bar{y})}{n} = \frac{(10-13)(12-15)+(16-13)(18-15)}{2}$

$= \frac{9+9}{2} = \mathbf{9}$

(iv) Correlation

$r\sigma_A\sigma_B = cor_{AB}$

$r* 3* 3 = 9$

$\boldsymbol{r} = \mathbf{1}$

(v) $\sigma_p^2 = \sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + 2\sigma_A\sigma_B w_A w_B r$

$= 9 *0.4^2 + 9* 0.6^2 + 2* 3* 3* 0.4* 0.6* 1 = 9$

$\sigma_p = \sqrt{9} = \mathbf{3}$

12. Illustration

Following is the data regarding six securities:

	A	B	C	D	E	F
Return (%)	8	8	12	4	9	8
Risk (Standard deviation)	4	5	12	4	5	6

(i) Assuming three will have to be selected, state which ones will be picked.

(ii) Assuming perfect correlation, show whether it is preferable to invest 75% in A and 25% in C or to invest 100% in E

(ICAI SM, Old PM, MTP Mar'18, MTP Apr'21 Old)

Solution:

(i) Given that A has better return than D for the same risk, A is chosen.

Also, A is better compared to B and F.

E is better than F.

Here the stocks A, E are chosen and B, D and F are not chosen.

C is chosen for highest return despite highest risk.

(ii) Portfolio 1, A:C = 75:25

Portfolio 2, E = 100%

Return (P1) = 0.75* 0.8 + 0.25* 12 = 6+ 3 = 9%

$\sigma_p^2 = \sigma_A^2 * w_A^2 + \sigma_C^2 * w_C^2 + 2 * \sigma_A\sigma_C w_A w_C r$

$= 4^2 0.75^2 + 12^2 0.25^2 + 4 * 12 * 0.75 * \ 0.25 * 1$

$= 9+9+18 = 36$

$\sigma_p = \sqrt{36} = \mathbf{6}$

Comparing	**A and C**	**E**
R	9	9
$\boldsymbol{\sigma_p}$	6	5

It is better to invest in E as one gets same return for lower risk.

13. Illustration

The historical rates of return of two securities over the past ten years are given. Calculate the Covariance and the Correlation coefficient of the two securities:

Years:	1	2	3	4	5	6	7	8	9	10
Security 1: (Return %)	12	8	7	14	16	15	18	20	16	22
Security 2: (Return %)	20	22	24	18	15	20	24	25	22	20

(ICAI SM, May'22 QP 8 marks, Old PM)

Solution:

x	y	$(x-\bar{x})$	$(y-\bar{y})$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
12	20	-2.8	-1	7.84	1	2.8
8	22	-6.8	1	46.24	1	-6.8
7	24	-7.8	3	60.84	9	-23.4
14	18	-0.8	-3	0.64	9	2.4
16	15	1.2	-6	1.44	36	-7.2
15	20	0.2	-1	0.04	1	-0.2
18	24	3.2	3	10.24	9	9.6
20	25	5.2	4	27.04	16	20.8
16	22	1.2	1	1.44	1	1.2
22	20	7.2	-1	51.84	1	-7.2
148	**210**			**207.6**	**84**	**-8**

$\bar{x} = \frac{148}{10} = 14.8$

$\bar{y} = \frac{210}{10} = 21$

$\sigma_X = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{207.6}{10}} = 4.556$

$\sigma_Y = \sqrt{\frac{\Sigma(y-\bar{y})^2}{n}} = \sqrt{\frac{84}{10}} = 2.898$

Covariance $= \frac{\Sigma(x-\bar{x})(y-\bar{y})}{n} = -\frac{8}{10} = -0.8$

$r = \frac{cov_{xy}}{\sigma_x \sigma_y} = -\frac{0.8}{4.556 * 2.898} = -0.0606$

14. Illustration

An investor has decided to invest to invest ₹ 1,00,000 in the shares of two companies, namely, ABC and XYZ. The projections of returns from the shares of the two companies along with their probabilities are as follows:

Probability	XYZ (%)	ABC (%)
0.20	16	12
0.25	10	14
0.25	28	-7
0.30	-2	28

You are required to

(i) Comment on return and risk of investment in individual shares.

(ii) Compare the risk and return of these two shares with a Portfolio of these shares in equal proportions.

(iii) Find out the proportion of each of the above shares to formulate a minimum risk portfolio.

(ICAI SM, May'24 QP, RTP May'24, RTP May'19, Old PM)

Solution:

For ABC

p_i	r_i	$p_i r_i$	$(x-\bar{x})$	$(x-\bar{x})^2$	$(x-\bar{x})^2 p_i$
0.2	12	2.4	-0.55	0.3025	0.0605
0.25	14	3.5	1.45	2.1025	0.52563
0.25	-7	-1.75	-19.55	382.2	95.5506
0.3	28	8.4	15.45	238.7	71.6108
		12.55		623.31	**167.7475**

σ^2_{ABC} = 167.7475

σ_{ABC} = $\sqrt{167.7475}$ = 12.9517

For XYZ

p_i	r_i	$p_i r_i$	$(x-\bar{x})$	$(x-\bar{x})^2$	$(x-\bar{x})^2 p_i$
0.2	16	3.2	3.9	15.21	3.042
0.25	10	2.5	-2.1	4.41	1.1025
0.25	28	7	15.9	252.81	63.2025
0.3	-2	-0.6	-14.1	198.81	59.643
		12.1		471.24	**126.99**

σ^2_{XYZ} = 126.99

σ_{XYZ} = $\sqrt{126.99}$ = 11.269

(i) XYZ has a better risk return portfolio.

(ii) σ^2_p = $\sigma^2_A w^2_A + \sigma^2_B w^2_B + 2 * \sigma_A \sigma_B w_A w_B r$

w_A = w_B

$r\sigma_A\sigma_B$ = $\frac{cov_{AB}}{\sigma_A\sigma_B}$

$cov_{AB}\ (\sigma_{AB})$ = $\sum(x-\bar{x})(y-\bar{y})p_i$

$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})(y-\bar{y})$	p_i	$(x-\bar{x})(y-\bar{y})p_i$
-0.55	3.9	-2.145	0.2	-0.429
1.45	-2.1	-3.045	0.25	-0.761
-19.55	15.9	-310.845	0.25	-77.71
15.45	-14.1	-217.845	0.3	-65.35
		-533.880		**-144.255**

r = $-\frac{-144.255}{12.952*11.269}$ = -0.9883

σ^2_p = $(12.952)^2 * (0.5)^2 + (11.269)^2 * (0.5)^2 + 2* -144.255 * 0.5* 0.5$=

= 1.54042

σ_p = 1.2411

$E(R)$ = 0.5* 12.55 + 0.5*12.1 = **12.33%**

The shares have negative correlation and hence the 50:50 portfolio has very low risk/ variance but at almost same return.

(iii) Minimum Risk Portfolio

W_{ABC} at minimum variance, = $\frac{\sigma^2_B - \sigma_{AB}}{\sigma^2_A + \sigma^2_B - 2\sigma_{AB}}$

= $\frac{11.269^2 - (-144.255)}{11.269^2 + 12.952^2 - 2*(-144.255)}$ = $\frac{271.51}{583.254}$ = 0.465

W_{XYZ} = 1-W_{ABC} = 1- 0.465 = 0.535

The minimum risk portfolio will have ABC and XYZ in the proportion 46.5% and 53.5%

$E(R)$ = 12.55* 0.465 + 0.535* 12.1 = 12.30925

σ_p^2 = 126.99* $(0.535)^2$+ 167.754* $(0.465)^2$ + 2(-144.25) *0.535* 0.465

= **0.848**

σ_p = **0.9213**

15. Illustration

The following information are available with respect of Krishna Ltd.

Year	Krishna Ltd. Average share price (Rs.)	Dividend per Share (Rs.)	Average Market Index	Dividend Yield	Return on Govt. bonds
2012	245	20	2013	4%	7%
2013	253	22	2130	5%	6%
2014	310	25	2350	6%	6%
2015	330	30	2580	7%	6%

Compute Beta Value of the Krishna Ltd. at the end of 2015 and state your observation.

(ICAI SM, MTP Mar'19, Old PM)

Solution:

Period	Equity Return	Market Return	R_f
2012-13	12.25%	10.81%	6%
2013-14	32.41%	16.33%	6%
2014-15	16.13%	16.79%	6%

	$\frac{D}{P_0}+\frac{P_1-P_0}{P_0}$	**Equity Return**
2012-13	(22+253-245)/245	12.25%
2013-14	(25+310-253)/253	32.41%
2014-15	(30+330-310)/310	16.13%

	Market Return		**Dividend**
2012-13	(2130-2013)/2013	5.8%	5%
2013-14	(2350-2130)/2130	10.33%	6%
2014-15	(2580-2350)/2350	9.79%	7%

	x	y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})(y-\bar{y})$
2012-13	10.81%	12.25%	-3.83%	-8.01%	30.68
2013-14	16.33%	32.41%	1.69%	12.15%	20.5335
2014-15	16.79%	16.13%	2.15%	-4.13%	-8.8795
	43.93%	**60.79%**			**42.33**

$\bar{x}$ = 14.64%

$\bar{y}$ = 20.26%

(cov_{xy}) = $\frac{\sum(x-\bar{x})(y-\bar{y})}{n} = \frac{42.3323}{3}$ = 14.11076

Market Variance = $\frac{\sum(x-\bar{x})^2}{n} = \frac{22.1475}{3}$ = 7.3825

β = $\frac{\sigma_{xyz,m}}{var_m} = \frac{14.11076}{7.3825}$ = 1.911

$E(R)$ = $R_f + \beta\,(R_m - R_f)$

Period		$E(R)$	Actual Return
2012-13	6%+ 1.911(10.81-6)	15.20%	12.25%
2013-14	6%+ 1.911(16.33-6)	27.54%	32.41%
2014-15	6%+ 1.911(16.79-6)	26.62%	16.13%

In period 2012-13 and 2014-15, Actual Return< Expected Return – Stock is a sell in this period
Whereas, in 2013-14 Actual Return> Expected Return – Stock is a buy in this period

16. Illustration

The distribution of return of security 'F' and the market portfolio 'P' is given below:

Probability	Return (%)	
	F	P
0.30	30	-10
0.40	20	20
0.30	0	30

You are required to calculate the expected return of security 'F' and the market portfolio 'P', the covariance between the market portfolio and security and beta for the security.

(ICAI SM, RTP May'20, Old PM)

Solution:

Probability	Stock return	Market return	$(F-\bar{F})$	$(P-\bar{P})$	$(F-\bar{F})(P-\bar{P})$	$(P-\bar{P})^2$
0.3	30	-10	13	-24	-312	576
0.4	20	20	3	6	18	36
0.3	0	30	-17	16	-272	256

$E(R)$ = $\sum P_i R_i$

$E(R)$ of Stock F,

$\bar{F}$ = 0.3* 30+ 0.4* 20 + 0.3* 0 = **17%**

$E(R)$ of Stock P

$\bar{P}$ = 0.3* -10+ 0.4* 20 + 0.3* 30 = **14**

Covariance = $\sum(F-\bar{F})(P-\bar{P}) * P_i$

= 0.3* -312 + 0.4* 18 + 0.3* -272 = -168

Variance of Market Portfolio P = 576* 0.3 + 36* 0.4+ 256* 0.3 = **264**

β = $\frac{\sigma_{FP}}{\sigma_P^2}$ = - $\frac{168}{264}$ = **- 0.64**

17. Illustration

Given below is information of market rates of Returns and Data from two Companies A and B:

	Year 2007	Year 2008	Year 2009
Market (%)	12.0	11.0	9.0
Company A (%)	13.0	11.5	9.8
Company B (%)	11.0	10.5	9.5

You are required to determine the beta coefficients of the Shares of Company A and Company B.

(ICAI SM, MTP Apr'19, Old PM)

Solution:

Year	Market Return	$(R_M - \overline{R_M})$	$(R_M - \overline{R_M})^2$
2007	12	1.33	1.7689
2008	11	0.33	0.1089
2009	9	-1.67	2.7889
	32		**4.667**

$\bar{R}_M$ = $\frac{32}{3}$ = 10.671

Variance of Market,

σ_M^2 = $\sqrt{\frac{(R_M - \bar{R}_M)^2}{n}}$ = $\frac{4.667}{3}$ = 1.556%

Year	R_A	$(R_A - \overline{R_A})$	$(R_M - \overline{R_M})$	$(R_A - \overline{R_A})(R_M - \overline{R_M})$
2007	13	1.57	1.33	2.0881
2008	11.5	0.07	0.33	0.0231
2009	9.8	-1.63	-1.67	2.7221
	34.3			**4.8323**

$\overline{R_A}$ = 11.43

cov_A = $\frac{4.833}{3}$ = 1.6111

β_A = $\frac{\sigma_{AM}}{\sigma_M^2}$ = $\frac{1.611}{1.556}$ = 1.036

Year	R_B	$(R_B - \overline{R_B})$	$(R_M - \overline{R_M})$	$(R_B - \overline{R_B})(R_M - \overline{R_M})$
2007	11	0.67	1.33	0.8911
2008	10.5	0.17	0.33	0.0561
2009	9.5	-0.83	-1.67	1.3861
	31			**2.3333**

$\overline{R_B}$ = 10.3

cov_B = $\frac{2.333}{3}$ = 0.7776

β_B = $\frac{\sigma_{AM}}{\sigma_M^2}$ = $\frac{0.7776}{1.556}$ = **0.50**

18. Illustration

Mr. Tempest has the following portfolio of four shares:

Name	Beta	Investment (Lakhs)
Oxy Rin Ltd.	0.45	0.80
Boxed Ltd.	0.35	1.50
Square Ltd.	1.15	2.25
Ellipse Ltd.	1.85	4.50

The risk-free rate of return is 7% and the market rate of return is 14%. Required.

(i) Determine the portfolio return.

(ii) Calculate the portfolio Beta.

(ICAI SM, Old PM, MTP Oct'18, RTP Nov'18 Old, RTP Nov'19 Old)

Solution:

R_f = 7%

R_m = 14%

MRP = $(R_m - R_f)$ = 14%- 7% = 7%

Total Investment = 9.05 Lakh

Stock	β	$E\ (R)$ $= R_f + \beta\ (R_m - R_f)$	$E\ (R)$
Oxy Rin Ltd	0.45	7+ 0.45*7	10.15
Boxed Ltd.	0.35	7+ 0.35*7	9.45
Square Ltd	1.15	7+ 1.15*7	15.05
Ellipse Ltd.	1.85	7+ 1.85*7	19.95

(i) $E\ (R)$ of Portfolio $= 10.15 * \frac{0.8}{9.05} + 9.45 * \frac{1.5}{9.05} + 15.05 * \frac{2.25}{9.05} + 19.95 * \frac{4.5}{9.05}$

= 0.897+ 1.566+ 3.74+ 9.92 = 16.125

(ii) β of Portfolio $= \sum_{i=1}^{n} w_i \beta_i$

$= 0.45 * \frac{0.8}{9.05} + 0.35 * \frac{1.5}{9.05} + 1.15 * \frac{2.25}{9.05} + 1.85 * \frac{4.5}{9.05}$

= 0.0397+ 0.058+ 0.2859+ 0.91989 = 1.3034

19. Illustration

Amal Ltd. has been maintaining a growth rate of 12% in dividends. The company has paid dividend @ ₹3 per share. The rate of return on market portfolio is 15% and the risk -free rate of return in the market has been observed as10%. The beta co-efficient of the company's share is 1.2. You are required to calculate the expected rate of return on the company's shares as per CAPM model and the equilibrium price per share by dividend growth model.

(ICAI SM, RTP Nov'22, Old PM)

Solution:

g = 12%

d_0 = 3

d_1 = 3* 1.12 = 3.36

R_m = 15%

R_f = 10%

β = 1.2

$E\ (R)$ = $R_f + \beta\ (R_m - R_f)$ = 10% + 1.2(15-10) = 10+ 6 = **16% (=k_e)**

Price of Amal's Share,

P_0 = $\frac{d_1}{ke-g}$ = $\frac{3.36}{16\%-12\%}$ = **₹ 84 /share**

20. Illustration

The following information is available in respect of Security X

Equilibrium Return	-	15%
Market Return	-	15%
7% Treasury Bond Trading at	-	$140
Covariance of Market Return and Security Return	-	225%
Coefficient of Correlation	-	0.75

You are required to determine the Standard Deviation of Market Return and Security Return.

(ICAI SM, Old PM, RTP May'18 Old, MTP Mar'19 Old)

Solution:

σ_{MX} = 225%

r = 0.75

R_M = 15%

R_X = 15%

Since Security return = Market Return,

β = 1

CY = $\frac{Coupon}{MP}$ (Assuming Bond Price= 100, C= 7)

= $\frac{7}{140}$ = 5% (Assume R_f= 5%)

$E(R)_X = R_f + \beta\,(R_m - R_f)$

$15\% = 5\% + \beta\,(15-5)$

$\beta = 1$

$\beta = \frac{cov_{x,m}}{\sigma_m^2}$

$1 = \frac{225}{\sigma_m^2}$

$\sigma_m^2 = 225$

$\boldsymbol{\sigma_m} = \sqrt{225} =$ **15%**

$\beta = \frac{r * \sigma_x}{\sigma_m}$

$1 = \frac{(0.75 * \sigma_x)}{15}$

$\boldsymbol{\sigma_x} = \frac{15\%}{0.75} =$ **20%**

21. Illustration

Assuming that shares of ABC Ltd. and XYZ Ltd. are correctly priced according to Capital Asset Pricing Model. The expected return from and Beta of these shares are as follows:

Share	Beta	Expected return
ABC	1.2	19.8%
XYZ	0.9	17.1%

You are required to derive Security Market Line.

(ICAI SM, Old PM)

Solution:

Return $= R_f + \beta\,(MRP)$

$19.8 = R_f + 1.2\,(MRP)$ (1 for ABC)

$17.1 = R_f + 0.9\,(MRP)$ (2 for XYZ)

Subtracting equations (1) and (2)

$2.7 = 0.3$ MRP

MRP $= \frac{2.7}{0.3} = 9$

Substituting MRP is Equation (1)

$19.8 = R_f + 1.2\,(MRP)$

$19.8 = R_f + 1.2 * 9$

$R_f = 19.8 - 10.8 =$ **9%**

SML $= 9\% + \beta\,(9\%)$

22. Illustration

A Ltd. has an expected return of 22% and Standard deviation of 40%.
B Ltd. has an expected return of 24% and Standard deviation of 38%.
A Ltd. has a beta of 0.86 and B Ltd. a beta of 1.24.
The correlation coefficient between the return of A Ltd. and B Ltd. is 0.72.
The Standard deviation of the market return is 20%.

Suggest:

(i) Is investing in B Ltd. better than investing in A Ltd.?

(ii) If you invest 30% in B Ltd. and 70% in A Ltd., what is your expected rate of return and portfolio Standard deviation?

(iii) What is the market portfolios' expected rate of return and how much is the risk -free rate?

(iv) What is the beta of Portfolio if A Ltd.'s weight is 70% and B Ltd.'s weight is 30%?

(ICAI SM, RTP Nov'22, MTP Oct'22, Old PM)

Solution:

(i) Yes, Investing in B Ltd. is better than as it has lower S.D and higher return.

(ii) A= 70%, B= 30%

$E(R)$ = 70% * 22% + 30% * 24% = 15.4% + 7.2% = 22.6%

σ_p^2 = $\sigma_A^2 + w_B^2$ + $\sigma_A^2 + w_B^2$ + $2 * \sigma_A \sigma_B w_A w_B r$

= $(0.7)^2$ * $(40)^2$ + $(0.3)^2$ * $(38)^2$ + 2* 0.7* 0.3* 40*38* 0.72

= 1373.608

σ_p = $\sqrt{1373.608}$ = **37.0622%**

(iii) 22% = $R_f + 0.86\,(MRP)$ (1) Return of A

24% = $R_f + 1.24\,(MRP)$ (2) Return of B

Subtracting, (2) – (1)

2 = 0.38 MRP

MRP = $\frac{2}{0.38}$ = 5.263

Substituting MRP in equation (1),

22% = $R_f + 0.86 * 5.263$

R_f = 22- 4.526 = 17.474

$R_m - R_f$ = 5.263

R_m = 5.263 + 17.474 = **22.737**

(iv) β_p = $\sum_{i=1}^{n} w_i \beta_i$ = 0.7* 0.86+ 0.3* 1.24 = **0.974**

23. Illustration

XYZ Ltd. has substantial cash flow and until the surplus funds are utilised to meet the future capital expenditure, likely to happen after several months, are invested in a portfolio of short - term equity investments, details for which are given below:

Investment	No. of shares	Beta	Market price per share	Expected yield
I	60,000	1.16	4.29	19.50%
II	80,000	2.28	2.92	24.00%
III	1,00,000	0.90	2.17	17.50%
IV	1,25,000	1.50	3.14	26.00%

The current market return is 19% and the risk-free rate is 11%. Required to:

(i) Calculate the risk of XYZ's short-term investment portfolio relative to that of the market;

(ii) Whether XYZ should change the composition of its portfolio.

(ICAI SM, MTP Apr'18, Old PM, RTP May'20 Old)

Solution:

Investment	No. Of Shares	Price	Value	Beta
I	60000	4.29	257400	1.16
II	80000	2.92	233600	2.28
III	100000	2.17	217000	0.9
IV	125000	3.14	392500	1.5
			1100500	

(i) β_p $= \sum_{i=1}^{n} w_i \beta_i$

$= (1.16*257400/1100500) + (2.28*233600/1100500) + (0.9*217000/1100500) + (1.5*392500/1100500)$

$=$ **1.4677**

The portfolio is risky compared to market as it has a beta of 1.4677 v/s market beta of 1.

(ii) Return as per CAPM, = $R_f + \beta(MRP)$

	Calculation	Return	$E(R)$
I	11 + 1.16* 8%	20.28	19.5
II	11 + 2.28* 8%	29.24	24
III	11 + 0.9* 8%	18.2	17.5
IV	11 + 1.5* 8%	23	26

Only investment IV has an expected return more than CAPM Return, hence it is under-priced, should be bought and I, II, and III be sold, which are over-priced.

24. Illustration

A company has a choice of investments between several different equity oriented mutual funds. The company has an amount of ₹1 crore to invest. The details of the mutual funds are as follows:

Mutual Fund	Beta
A	1.6
B	1.0
C	0.9
D	2.0
E	0.6

Required:

(i) If the company invests 20% of its investment in each of the first two mutual funds and an equal amount in the mutual funds C, D and E, what is the beta of the portfolio?

(ii) If the company invests 15% of its investment in C, 15% in A, 10% in E and the balance in equal amount in the other two mutual funds, what is the beta of the portfolio?

(iii) If the expected return of market portfolio is 12% at a beta factor of 1.0, what will be the portfolios expected return in both the situations given above?

(iv) If the Company changes its policy to invest in any 3 securities with a minimum of 20% in each of these 3 securities to diversify risk, you are requested to advise the company to have a right mix of securities to maximize the return in the following two scenarios and calculate the expected return:

(1) Bull Phase: Expected Market returns 10%

(2) Bear Phase: Expected Market returns — 5%

(ICAI SM, Similar Nov'22 QP 8 marks, Jul'21 QP 8 marks, RTP Nov'19, MTP Mar'23, RTP Nov'18 Old, Old PM)

Solution:

Mutual Fund	β	Situation 1		Situation 2	
		Weights	Weights β	Weights	Weights β
A	1.6	0.2	0.32	0.15	0.24
B	1.0	0.2	0.2	0.3	0.3
C	0.9	0.2	0.18	0.15	0.135
D	2.0	0.2	0.4	0.3	0.6
E	0.6	0.2	0.12	0.10	0.06
		1.0	**1.22**	**1.0**	**1.335**

(i) β of Portfolio 1 =1.22

(ii) β of Portfolio 2 =1.335
R_M = 12%

(iii) Expected Return of Portfolios:
Portfolio 1 = 12% *1.22 = 14.64%
Portfolio 2 = 1.335* 12% = 16.02%

(iv) **Bull Phase:** High Beta Stocks are chosen.
Stocks A,B and D with Betas 1.6, 1 and 2 respectively in 20%, 20% and 60%, so that maximum can be allocated to the stock with highest beta.

	Investment	Investment	β	Market Return	Stock Return	Weight	Weighted Return
A	20%	20 lakhs	1.6	10%	16%	20%	3.2%
B	20%	20 lakhs	1	10%	10%	20%	2%
D	60%	60 lakhs	2	10%	20%	60%	12%
		1 Crore					**17.2%**

Weighted Average β = 2*60% + 1*20% + 1.6*20%
= **1.72**
Market Return = 10*1.72 = **17.2%**

Bear Phase: Low Beta Stocks are chosen.
Stocks B, C and E with Betas 1, 0.9 and 0.6 respectively in 20%, 20% and 60%, so that maximum can be allocated to the stock with lowest beta.

	Investment	Investment	β	Market Return	Stock Return	Weight	Weighted Return
B	20%	20 lakhs	1	-5%	-5%	20%	-1%
C	20%	20 lakhs	0.9	-5%	-4.5%	20%	-0.9%
E	60%	60 lakhs	0.6	-5%	-3%	60%	-1.8%
		1 Crore					**-3.7%**

In the Bear Phase, the portfolio will generate –3.7% against the market return of –5%.
Weighted Average β = 0.6*60% + 0.9*20% + 1*20%
= 0.36+ 0.18+ 0.2
= **0.74**
Market Return = 0.74* -5 = **- 3.7%**

25. Illustration

Mr. FedUp wants to invest an amount of ₹520 lakhs and had approached his Portfolio Manager. The Portfolio Manager had advised Mr. FedUp to invest in the following manner:

Security	Moderate	Better	Good	Very Good	Best
Amount (in ₹ Lakhs)	60	80	100	120	160
Beta	0.5	1.00	0.80	1.20	1.50

You are required to advise Mr. FedUp in regard to the following, using Capital Asset Pricing Methodology:

(i) Expected return on the portfolio, if the Government Securities are at 8% and the NIFTY is yielding 10%.

(ii) Advisability of replacing Security 'Better' with NIFTY.

(ICAI SM, MTP Apr'18, RTP Nov'18, Old PM)

Solution:

(i)

Security	Investment in Lakhs	β	Weight*β
Moderate	60	0.5	30
Better	80	1.00	80
Good	100	0.8	80
Very Good	120	1.20	144
Best	160	1.50	240
	520		**574**

β $= \frac{574}{520} = 1.1038$

R_p $= R_f + \beta\,(R_m - R_f)$

$= 8\% + 1.1038\ (10\% - 8\%) = 8\% + 1.1038 * 2\% =$ **10.207%**

(ii) It is better replaced with NIFTY. Both have a β of 1 so ideally either one is okay, but NIFTY has higher liquidity and lower impact cost. Hence it is advised to replace Better with NIFTY.

26. Illustration

Your client is holding the following securities:

Particulars of Securities	Cost	Dividends/Interest	Market price	Beta
	₹	₹	₹	
Equity Shares:				
Gold Ltd.	10,000	1,725	9,800	0.6
Silver Ltd.	15,000	1,000	16,200	0.8
Bronze Ltd.	14,000	700	20,000	0.6
GOI Bonds	36,000	3,600	34,500	0.01

Average return of the portfolio is 15.7%, calculate:

(i) Expected rate of return in each, using the Capital Asset Pricing Model (CAPM).

(ii) Risk free rate of return.

(ICAI SM, MTP Apr'24, Old PM)

Solution:

(i) Market Return = 15.7%

Weighted Average Return of Portfolio = $\frac{\text{dividend+closing−opening}}{\text{opening value}}$

$= \frac{7025 + 80500 - 75000}{75000}$ = **16.7%**

(ii) Risk free rate of return

GOI Bond Coupon = 10%

GOI Bond Return = $\frac{34500 - 36000 + 3600}{36000}$ = 5.83%

Assuming R_f= 5.83% because that is the return on GOI Bonds.

R_m= 15.7%

CAPM Return,

Gold Ltd. = 5.83+ 0.6 (15.7- 5.83) = **11.75%**

Silver Ltd. = 5.83+ 0.8 (15.7- 5.83) = **13.726%**

Bronze Ltd. = 5.83+ 0.6 (15.7- 5.83) = **11.75%**

27. Illustration

A holds the following portfolio:

Share/Bond	Beta	Initial Price	Dividends	Market Price at end of year
		₹	₹	₹
Epsilon Ltd.	0.8	25	2	50
Sigma Ltd.	0.7	35	2	60
Omega Ltd.	0.5	45	2	135
GOI Bonds	0.01	1,000	140	1,005

Calculate:

(i) The expected rate of return of each security using Capital Asset Pricing Method (CAPM)

(ii) The average return of his portfolio. Risk-free return is 14%.

(ICAI SM, Old PM)

Solution:

(i) Assumption:

Portfolio Return = Market Return

Security	Price (1)	MP (2)	CG (1)-(2)	Dividend	Total Return	$E(R) = R_f + \beta(R_m - R_f)$
Epsilon Ltd	25	50	25	2	27	14% + 0.8 (26.33%-14%) = **23.864%**
Sigma Ltd	35	60	25	2	27	14% + 0.7 (12.33%) = **22.631%**
Omega Ltd	45	135	90	2	92	14% + 0.5 (12.33%) = **20.165%**
GOI Bonds	1000	1005	5	140	145	14% + 0.01 (12.33%) = **14.1233%**
	1105	**1450**	**145**	**146**	**291**	

Portfolio Return = $\frac{291}{1105}$ = 26.33%

Average Return = $\frac{23.864+22.631+20.165+14.123}{4}$ = **20.19575%**

28. Illustration

An investor is holding 1,000 shares of Fatlass Company. Presently the rate of dividend being paid by the company is ₹ 2 per share and the share is being sold at ₹ 25 per share in the market. However, several factors are likely to change during the course of the year as indicated below:

	Existing	Revised
Risk free rate	12%	10%
Market risk premium	6%	4%
Beta value	1.4	1.25
Expected growth rate	5%	9%

In view of the above factors whether the investor should buy, hold or sell the shares? And why?
(Sep'25 Similar 6M, MTP Mar'18 Old, Old PM)

Solution:

Current MP = 25
D_0 = 2

k_e of existing = $R_f + \beta(R_m - R_f)$ = 12+ 1.4* 6 = 20.4%
k_e of revised = $R_f + \beta\ (R_m - R_f)$ = 10+ 1.25* 4 = 15%

Intrinsic Value of Shares

Existing	Revised
D_0= 2	D_0= 2
g= 5%	g= 9%
D_1= 2.1	D_1 = 2* 1.09 =2.18
k_e = 20.4%	k_e = 15%
P_0= 2.1/ (20.4%-5%) = 2.1/ 0.154 = 13.636	P_0= 2.18/ (0.15- 0.09 = 2.18/ 0.06 **= 36.33**

Currently the shares are trading at Rs 25 compared to its intrinsic value Rs 13.636. They are overpriced. However, if the investor is confident of the revised estimates, then CMP of Rs 25 is cheap as compared to the expected price of Rs 36.33. So, investor may retain and buy in such a case.

29. Illustration

An investor is holding 5,000 shares of X Ltd. Current year dividend rate is ₹ 3/ share. Market price of the share is ₹ 40 each. The investor is concerned about several factors which are likely to change during the next financial year as indicated below:

	Current Year	Next Year
Dividend paid /anticipated per share (₹)	3	2.5
Risk free rate	12%	10%
Market Risk Premium	5%	4%
Beta Value	1.3	1.4
Expected growth	9%	7%

In view of the above, advise whether the investor should buy, hold or sell the shares.
(ICAI SM, May 24'QP, Old PM)

Solution:

k_e under existing/ revised scenario

Existing = $R_f + \beta\ (R_m - R_f)$ = 12% + 1.3 (5%) = 18.5%

Next Year = 10% +1.4(4) = 15.6%

P_0 based on Current Year Data

$P_0 = \frac{d_1}{ke-g}$ (D_1=2.5, since next year dividend is already given)

$= \frac{2.5}{15.6\%-7\%}$ = **₹ 29.069**

P_0 based on Revised Data

$P_0 = \frac{d_1}{ke-g}$ (D_0= 3, g= 9%, D_1= 3* 1.09= 3.27)

$= \frac{3.27}{18.5\%-9\%}$ = **₹ 34.42**

Current Market Price= ₹40, Intrinsic Value under existing/ revised circumstance are ₹ 34.42 and ₹29.069 respectively and are lower than CMP. So, the investor is advised to sell the securities of X Ltd

30. Illustration

X Co., Ltd., invested on 1.4.2009 in certain equity shares as below:

Name of Co.	No. of shares	Cost (₹)
M Ltd.	1,000 (₹ 100 each)	2,00,000
N Ltd.	500 (₹ 10 each)	1,50,000

In September, 2009, 10% dividend was paid out by M Ltd. and in October, 2009, 30% dividend paid out by N Ltd. On 31.3.2010 market quotations showed a value of ₹ 220 and ₹ 290 per share for M Ltd. and N Ltd. respectively.

On 1.4.2010, investment advisors indicate (a) that the dividends from M Ltd. and N Ltd. for the year ending 31.3.2011 are likely to be 20% and 35%, respectively and (b) that the probabilities of market quotations on 31.3.2011 are as below:

Probability factor	Price/share of M Ltd.	Price/share of N Ltd.
0.2	220	290
0.5	250	310
0.3	280	330

You are required to:

(i) Calculate the average return from the portfolio for the year ended 31.3.2010;

(ii) Calculate the expected average return from the portfolio for the year 2010 -11; and

(iii) Advise X Co. Ltd., of the comparative risk in the two investments by calculating the standard deviation in each case.

(ICAI SM, Similar Dec'21 QP, RTP May'19, MTP Apr'22, MTP Oct'20 New and Old, MTP Apr'19, RTP Nov'18 Old, Old PM)

Solution:

(i)

	M	N	Total
Shares	1000	500	1500
Face Value (FV)	100	10	
FV Investment	100000	5000	
Investment made	200000	150000	350000
Cost per share	200	300	
Dividend	10%	30%	
Dividend Received	10000	1500	11500

Market Price	220	290	
Market price- Cost	20	-10	
Capital Gain	20* 1000	500* -10	
	20000	**-5000**	**15000**

Return = $\frac{CapitalGain + Dividend}{Opening\ Investment}$ = $\frac{15000 + 11500}{350000}$

R_e = **7.57%**

(ii) $E(R)$ = $\sum_{i=1}^{n} R_i P_i$

	M	N
Opening Share Price	220	290
Expected Price	0.2*220	0.2*290
	0.5*250	0.5*310
	0.3*280	0.3*330
	253	**312**
Capital gain	33	22
Dividend Per Share	20	3.5
Return per Share	53	25.6
No. Of shares	1000	500
Total absolute Return	53000	12750
Investment Value 01/04/2010	220000	145000

Return = $\frac{CapitalGain + Dividend}{Opening\ Investment}$ = $\frac{53000 + 12750}{365000}$

R_e = **18.014%**

(iii)

	Stock Return	Yield	Total Return
A	0	9.09%	9.09%
B	13.63	9.09%	22.72%
C	27.27	9.09%	36.36%

$\bar{x}$ = average return = 9.09* 0.2 + 22.72* 0.5 + 36.36* 0.3 = 24.086%

σ_M^2 = $\sum_{i=1}^{n}(x - \bar{x})P_i$

= (9.09%) -24.086%)X 0.2 + (22.72%-24.086%) X 0.5+(36.36%-24.086%) X 0.3

=91.10%

σ_M = 9.54%

Probability	Closing	Opening	Gain	Div	Total	Return %
0.2	290	290	0	3.5	3.5	1.20%
0.5	310	290	20	3.5	23.5	8.10%
0.3	330	290	40	3.5	43.5	15%

$E(R)$ = 1.2* 0.2 + 8.1* 0.5 + 15*0.3 = 8.79%

σ_N^2 = $\sum_{i=1}^{n}(x - \bar{x})P_i$

= (1.2%- 8.79%) *0.2 +(8.1%- 8.79%) *0.5 + (15%- 8.79%) *0.3

= 11.52+ 0.238+ 11.569 = **23.32%**

σ_p = **4.83%** Hence, M Ltd is riskier.

30.A Illustration

Mr. A, a HNI invested on 1.4.2014 in certain equity shares as below:

Name of Co.	No. of shares	Cost (₹)
X Ltd.	1,00,000 (₹ 100 each)	2,00,00,000
Y Ltd.	50,000 (₹ 10 each)	1,50,00,000

In September 2014, 10% dividend was paid out by X Ltd. and in October 2014, 30% dividend paid out by Y Ltd. On 31.3.2015 market quotations showed a value of ₹ 220 and ₹ 290 per share for X Ltd. and Y Ltd. respectively.

On 1.4.2015, a technical analyst indicated as follows:

1. that the probabilities of dividends from X Ltd. and Y Ltd. for the year ending 31.3.2016 are as below:

Probability factor	Dividend from X Ltd. (%)	Dividend from Y Ltd. (%)
0.2	10	15
0.3	15	20
0.5	20	35

(2) that the probabilities of market quotations on 31.3.2016 are as below:

Probability factor	Price/share of X Ltd.	Price/share of Y Ltd.
0.2	220	290
0.5	250	310
0.3	280	330

You are required to:

1. Analyze the average return from the portfolio for the year ended 31.3.2015
2. Analyze the expected average return from the portfolio for the year 2015-16; and
3. Advise Mr. A, of the comparative risk in the two investments.

(RTP Sep'25)

Solution:

(i) Average return from the portfolio for the year ended 31.3.2015

Calculation of return on portfolio for 2014-15

Particulars	X Ltd. (₹/share)	Y Ltd. (₹/share)
Dividend received during the year	10	3
Market value by 31.03.15	220	290
Cost of investment	200	300
Gain/loss	20	(-10)
Yield	30	(-7)
Cost	200	300
% return	15%	-2.33%
Weight in the portfolio	2/3.5 = 57.14	1.5/3.5 = 42.86
Weighted average return		7.56%

(ii) Average return from the portfolio for the year ended 2015-16

To be calculated using the concept of joint probability as follows:

X Ltd.

Path	Income from Dividend (₹)	Gain from Market Price (₹)	Total Yield (₹)	Joint Prob.	Exp. Yield (₹)
1	10	0	10	0.20 × 0.20 = 0.04	0.40
2	10	30	40	0.20 × 0.50 = 0.10	4.00
3	10	60	70	0.20 × 0.30 = 0.06	4.20
4	15	0	15	0.30 × 0.20 = 0.06	0.90
5	15	30	45	0.30 × 0.50 = 0.15	6.75
6	15	60	75	0.30 × 0.30 = 0.09	6.75
7	20	0	20	0.50 × 0.20 = 0.10	2.00
8	20	30	50	0.50 × 0.50 = 0.25	12.50
9	20	60	80	0.50 × 0.30 = 0.15	12.00
Total					49.50

Expected Yield (₹): 49.50 | Market Value on 01.04.2015 (₹): 220

% Return = 49.50/220= 22.50%

Y Ltd.

Path	Income from Dividend (₹)	Gain from Market Price (₹)	Total Yield (₹)	Joint Prob.	Exp. Yield (₹)
1	1.50	0	1.50	0.20 × 0.20 = 0.04	0.06
2	1.50	20	21.50	0.20 × 0.50 = 0.10	2.15
3	1.50	40	41.50	0.20 × 0.30 = 0.06	2.49
4	2.00	0	2.00	0.30 × 0.20 = 0.06	0.12
5	2.00	20	22.00	0.30 × 0.50 = 0.15	3.30
6	2.00	40	42.00	0.30 × 0.30 = 0.09	3.78
7	3.50	0	3.50	0.50 × 0.20 = 0.10	0.35
8	3.50	20	23.50	0.50 × 0.50 = 0.25	5.88
9	3.50	40	43.50	0.50 × 0.30 = 0.15	6.52
Total					24.65

Expected Yield (₹): 24.65 | Market Value on 01.04.2015 (₹): 290

% Return = 24.65/290 = 8.50%

Weight in portfolio (1,00,000 × 220) :(50,000 × 290) = 60.27: 39.73

Weighted Average (Expected) Return = (0.6027 × 22.50) + (0.3973 × 8.50) = 16.94%

(iii) Risk Analysis – Standard Deviation of Each Investment

X Ltd.

Path	Prob. (1)	Yield (₹)	Dev. (P_x - $\bar{P}_x$)	Square of dev. (2)	(1) × (2)
1	0.04	10	-39.50	1560.25	62.41
2	0.10	40	-9.50	90.25	9.03
3	0.06	70	20.50	420.25	25.22
4	0.06	15	-34.50	1190.25	71.42
5	0.15	45	-4.50	20.25	3.04
6	0.09	75	25.50	650.25	58.52
7	0.10	20	-29.50	870.25	87.03
8	0.25	50	0.50	0.25	0.06
9	0.15	80	30.50	930.25	139.54
Total					456.27

Variance (σ^2_x) = 456.27 => Standard Deviation (σ_x) = 21.36

Y Ltd.

Path	Prob. (1)	Yield (₹)	Dev. (P_γ - $\bar{P}_\gamma$)	Square of dev. (2)	(1) × (2)
1	0.04	1.50	-23.15	535.92	21.44
2	0.10	21.50	-3.15	9.92	0.99
3	0.06	41.50	16.85	283.92	17.04
4	0.06	2.00	-22.65	513.02	30.78
5	0.15	22.00	-2.65	7.02	1.05
6	0.09	42.00	17.35	301.02	27.09
7	0.10	3.50	-21.15	447.32	44.73
8	0.25	23.50	-1.15	1.32	0.33
9	0.15	43.50	18.85	355.32	53.30
Total					196.75

Variance (σ^2_γ) = 196.75 => Standard Deviation (σ_γ) = 14.03

Although Expected Return is higher in case of X Ltd., it also has higher risk due to High S.D.

31. Illustration

An investor holds two stocks A and B. An analyst prepared ex-ante probability distribution for the possible economic scenarios and the conditional returns for two stocks and the market index as shown below:

Economic scenario	Probability	Conditional Returns %		
		A	B	Market
Growth	0.40	25	20	18
Stagnation	0.30	10	15	13
Recession	0.30	-5	-8	-3

The risk-free rate during the next year is expected to be around 11%. Determine whether the investor should liquidate his holdings in stocks A and B or on the contrary make fresh investments in them. CAPM assumptions are holding true.

(ICAI SM, Old PM, RTP May'18 Old)

Solution:

Probability Based Returns

P_i	A	B	Market	P_i* A	P_i*B	P_i*M
0.4	25	20	18	10	8	7.2
0.3	10	15	13	3	4.5	3.9
0.3	-5	-8	-3	-1.5	-2.4	-0.9
			$E(R)$	11.5	10.1	10.2

CAPM

$$\beta = \frac{cov_{ms}}{var_m}$$

$$cov_{MA} = \sum_{i=1}^{n}(x-\bar{x})(y-\bar{y})P_i$$

P_i	a	$a-\bar{a}$	b	$b-\bar{b}$	$(a-\bar{a})(b-\bar{b})P_i$
0.4	25	13.5	18	7.8	42.12
0.3	10	-1.5	13	2.8	-1.26
0.3	-5	-16.5	-3	-13.2	65.34
					106.2

$\bar{a}$ = 11.5

$\bar{b}$ = 10.2

cov_{MB} = $\sum_{i=1}^{n}(x-\bar{x})(y-\bar{y})P_i$

P_i	a	$a-\bar{a}$	b	$b-\bar{b}$	$(a-\bar{a})(b-\bar{b})P_i$
0.4	20	7.8	18	7.8	30.888
0.3	15	4.9	13	2.8	4.116
0.3	-8	-18.1	-3	-13.2	71.676
					106.68

$\bar{a}$ = 10.1

$\bar{b}$ = 10.2

var_M = $\sum_{i=1}^{n}(x-\bar{x})^2P_i$ = $(7.8)^2$ *4 + $(28)^2$ *3 + $(13.2)^2$ *0.3 = 78.96

β of A = $\frac{cov_{MA}}{var_M}$ = $\frac{106.2}{78.96}$ = 1.345

β of B = $\frac{106.68}{78.96}$ =1.351

$E(R)$ of A = $R_f + \beta\,(R_m - R_f)$ = 11+ 1.345(10.2- 11) = **9.924%**

$E(R)$ of B =11+ 1.351(10.2- 11) = **9.919**

Since expected returns are higher than CAPM investor should buy these securities as they are under-priced.

32. Illustration

A Portfolio Manager (PM) has the following four stocks in his portfolio:

Security	No. of Shares	Market Price per share (₹)	B
VSL	10,000	50	0.9
CSL	5,000	20	1.0
SML	8,000	25	1.5
APL	2,000	200	1.2

Compute the following:

(i) Portfolio beta.

(ii) If the PM seeks to reduce the beta to 0.8, how much risk-free investment should he bring in?

(iii) If the PM seeks to increase the beta to 1.2, how much risk-free investment should he bring in?

(ICAI SM, RTP Nov'21, RTP May'20, Old PM)

Solution:

(i)

Security	Shares	CMP	Portfolio Value	Beta	Beta* Portfolio
VSL	10000	50	500000	0.9	450000
CSL	5000	20	100000	1	100000
SML	8000	25	200000	1.5	300000
APL	2000	2	400000	1.2	480000
			1200000		**13,30,000**

Portfolio Beta = Weighted Average of Beta Values = $\frac{1330000}{1200000}$ = **1.1083**

(ii) Let value of risk-free security be R

0.8 = $\frac{1200000*1.1083+(R*0)}{1200000+R}$ = $\frac{1330000}{1200000+R}$

0.8*(1200000+R) = 1330000

R = **462500**

The PM should invest ₹ 462500 in Risk Free securities with a Beta of 0 to ensure the portfolio Beta is 0.8.

(iii) Let value of risk-free security be R

1.2 = $\frac{1200000*1.1083+(R*0)}{1200000+R}$ = $\frac{1330000}{1200000+R}$

1.2*(1200000+R) = 1330000

R = -91667 |

In order to increase beta of portfolio to 1.2 one should short ₹ 91667 worth of risk-free investment

33. Illustration

The total market value of the equity share of O.R.E. Company is ₹60,00,000 and the total value of the debt is ₹ 40,00,000. The treasurer estimate that the beta of the stock is currently 1.5 and that the expected risk premium on the market is 10 per cent. The treasury bill rate is 8 per cent.

Required:

(i) What is the beta of the Company's existing portfolio of assets?

(ii) Estimate the Company's Cost of capital and the discount rate for an expansion of the company's present business.

(ICAI SM, MTP Sept'22, MTP Oct'21 New & Old, Old PM)

Solution:

(i) Beta of Assets = $\frac{Equity*\beta_E + Debt*\beta_D}{D+E}$ = $\frac{60*1.5+40*0}{60+40}$ = 0.9

(ii) WACC = $k_d * \frac{D}{D+E} + k_e * \frac{E}{D+E}$

Since cost of debt is not given, we are assuming treasury bill rate as cost of debt. However, there is a spread usually for a corporate borrower's borrowings as compared to treasury bill rate.

Cost of Equity as per CAPM,

= $R_f + \beta\,(MRP)$ = 8%+ 1.5 (10%) = 23%

Assuming that Beta of assets cannot be used to compute cost of equity.

WACC = $k_d * \frac{D}{D+E} + k_e * \frac{E}{D+E}$ = (23% * 60 + 8% *40)/ 100 = **17%**

WACC is 17% and this rate is used to evaluate expansion of company's present business.

34. Illustration

A stock costing Rs.150 pays no dividends. The possible prices that the Stock might sell for at the end of the year with the respective probabilities are given below.

Price	130	150	160	165	175	180
Probability	0.2	0.1	0.1	0.3	0.1	0.2

Compute the Expected Return and its standard Deviation and show computations up to three decimals.

Solution:

Price	P_i	Abs. R_i	Return %	R_iP_i	$x-\bar{x}$	$(x-\bar{x})^2$	$(x-\bar{x})^2P_i$
130	0.2	-20	-13.33%	-2.67%	-20%	400	80
150	0.1	0	0%	0	-6.67%	44.45	4.445
160	0.1	10	6.67%	0.667%	0	0	0
165	0.3	15	10%	3%	3.33%	11.08	3.324
175	0.1	25	16.67%	1.67%	10%	100	10
180	0.2	30	20%	4%	13.33%	177.689	35.5378
	1			**6.667%**			**133.307**

$E(R)$ = **6.667%** $(=\bar{x})$

σ_s^2 = $\sum_{i=1}^{n}(x-\bar{x})^2P_i$ = **133.307**

σ_s =**11.547**

35. Illustration

The return of security 'L' and security 'M' for the past five years are given below:

Year	Security – L Return %	Security – K Return %
2012	10	11
2013	04	-06
2014	05	13
2015	11	08
2016	15	14

Calculate the risk and return of portfolio consisting above information *[Nov'17 QP (Old)]*

Solution:

L(x)	K(y)	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
10	11	1	3	1	9	3
4	-6	-5	-14	25	196	70
5	13	-4	5	16	25	-20
11	8	2	0	4	0	0
15	14	6	6	36	36	36
45	40	0	0	82	266	89

$E(R)$ of L = 9%

$E(R)$ of K =8%

Variance = $\frac{\sum(x-\bar{x})^2}{n}$

S. D = $\sqrt{variance}$

S. D of L = $\sqrt{82/5}$ = 4.05

S. D of K = $\sqrt{266/5}$ = 7.29

Return of Portfolio assuming weights to be in 50:50

$E(R)$ = 9%* 0.5 + 8%* 0.5 = 8.5%

cov_{xy} = $\frac{\Sigma(x-\bar{x})(y-\bar{y})}{n}$ = $\frac{89}{5}$ = 17.8

σ_p^2 = $\sigma_x^2 * w_x^2 + \sigma_y^2 * w_y^2 + 2 * \sigma_x \sigma_y w_x w_y r$

= 16.4* $(0.5)^2$ + 53.2* $(0.5)^2$ + 2*17.8* 0.5*0.5 = **26.3**

σ_p = **5.13**

36. Illustration

If the rate of return and Standard Deviation of Market Portfolio (Index) is 8% and 6% respectively and the risk-free rate of return is 5%, you are required to:

(i) Construct an efficient portfolio which produces expected return of 7.5%.

(ii) Calculate the risk of above portfolio.

(iii) Suppose if Mr. X has Rs.1,00,000 of his personal funds, then how he would construct his portfolio giving expected return of 10% and what will be risk of this portfolio.

Solution:

(i) Expected Return of Portfolio = 7.5 %

Let Share of Risk-Free security be x

Then total will be 1 and Market will be 1- x

$E(R)$ = $W_1R_1 + W_2R_2$

7.5 = x* 5% + (1- x) * 8% = 5x+ 8 – 8x

3x = 0.5

x = $\frac{1}{6}$

= 16.67%

1- x = 83.33%

(ii)

σ_p^2 = $\sigma_x^2 * w_x^2 + \sigma_y^2 * w_y^2 + 2 * \sigma_x \sigma_y w_x w_y r$ (σ_1= SD of market, σ_2=Σ of risk-free)

= $(83.33)^2$* $(6)^2$

σ_p = 83.33* 6 = **5%**

(iii) New portfolio where return = 10%

$E(R)$ = 10% = $W_m R_m + W_{Rf} R_{Rf}$

Assuming W_{Rf}= y, then W_m= 1-y

10% = $R_m(1-y) + R_{Rf}(y)$

10% = $8\%(1-y) + 5\%(y)$

10% = 8%- 8y+ 5y

2% = -3%y

y = -66.67%

For ₹ 100000 of Portfolio -66.67% is invested in risk-free assets and 166.67% is invested in Market.
Borrowing -66.67% in risk-free from market and invest it in market.
Borrow ₹ 66667 and invest in market index a total of ₹ 166667.

Net Value of Portfolio= 100000

σ_p = 6% * 1.6667 = **10%**

Mr. Gupta is considering investment in the shares of R. Ltd. He has the following expectations of return on the stock and the market:

Probability	Return (%)	
	R. Ltd.	Market
0.35	30	25
0.30	25	20
0.15	40	30
0.20	20	10

You are required to:

(i) Calculate the expected return, variance and standard deviation for R. Ltd.

(ii) Calculate the expected return variance and standard deviation for the market.

(iii) Find out the beta co-efficient for R. Ltd. shares.

[Nov'18 QP (Old)]

Solution:

$E(R)$ = $\sum_{i=1}^{n} P_i x_i$

σ_i^2 = $\sum_{i=1}^{n} (x-\bar{x})^2 P_i$

β = $\frac{cov_{x,y}}{\sigma_y^2}$

cov = $\sum_{i=1}^{n} (x-\bar{x})(y-\bar{y})P_i$

Probability	R	$\bar{R}$	$(R-\bar{R})$	$(R-\bar{R})^2$	$(R-\bar{R})^2 P_i$
0.35	30	10.5	2	4	1.4
0.3	25	7.5	-3	9	2.7
0.15	40	6	12	144	21.6
0.2	20	4	-8	64	12.8
		28		**221**	**38.5**

Probability	M	$\bar{M}$	$(M-\bar{M})$	$(M-\bar{M})^2$	$(M-\bar{M})^2 P_i$
0.35	25	8.75	3.75	14.063	4.92188
0.3	20	6	-1.25	1.5625	0.46875
0.15	30	4.5	8.75	76.563	11.4844
0.2	10	2	-11.25	126.56	25.3125
		21.25		**218.75**	**42.1875**

Probability	$(R-\bar{R})$	$(M-\bar{M})$	$(R-\bar{R})(M-\bar{M}) P_i$
0.35	2	3.75	2.625
0.3	-3	-1.25	1.125
0.15	12	8.75	15.75
0.2	-8	-11.25	18
			37.5

(i) $E(R)$ of Stock R = **28%**

σ_i^2 = $\sum_{i=1}^{n} (x-\bar{x})^2 P_i$

For R, = 38.5%

σ_R = $\sqrt{38.5}$ = **6.205%**

(ii) $E(R)$ of Market = **21.25%**

σ_i^2 = $\sum_{i=1}^{n}(x-\bar{x})^2 P_i$

For M = 42.1875%

σ_R = $\sqrt{42.1875}$ = **6.495%**

(iii) β = $\frac{cov_{x,y}}{\sigma_y^2}$

For R and M = $\frac{\sum(R-\bar{R})(M-\bar{M})P_i}{\sum(M-\bar{M})^2 P_i}$ = $\frac{37.5}{42.1875}$ = **0.889**

38. Illustration

XYZ Ltd. paid a dividend of ₹ 2 for the current year. The dividend is expected to grow at 40% for the next 5 years and at 15% per annum thereafter. The return on 182 days T-bills is 11% per annum and the market return is expected to be around 18% with a variance of 24%.
The co-variance of XYZ's return with that of the market is 30%. You are required to calculate the required rate of return and intrinsic value of the stock.

(ICAI SM, May'24 QP)

Solution:

R_m = 18%

R_f = 11%

D_0 = 2

σ_m^2 = 24

$\sigma_{xyz,m}$ = 30%

β = $\frac{\sigma_{xyz,m}}{var_M}$ = $\frac{30}{24}$ = 1.25

$E(R)$ = $R_f + \beta(R_m - R_f)$

k_e = 11%+ 1.25(18%-11%) = 11% + 1.25(7%) = **19.75%**

Intrinsic Value Calculation,

D_0=	2	Dividend	PVIF @19.75%	PV of Dividends
D_1=	2*1.4	2.8	0.8350	2.338
D_2=	2*1.4^2	3.92	0.6973	2.733
D_3=	2*1.4^3	5.488	0.5823	3.196
D_4=	2*1.4^4	7.6832	0.4863	3.736
D_5=	2*1.4^5	10.75648	0.4061	4.368
			PV of Dividend	**16.3716**

Intrinsic Value of Stock in 2 Stage Dividend Discount Model

= PV of Dividend for Years 1-5 + $\frac{P_5}{(1+k_e)^5}$

P_5 = $\frac{d_6}{k_e-g}$ = $\frac{d_5(1+g_2)}{k_e-g_2}$ = $\frac{10.75648(1+15\%)}{19.75\%-15\%}$ = 260.42

PV of P_5 = 260.42* 0.4061 = **105.757**

Intrinsic Value = 16.3716+ 105.757 = **₹ 122.128**

39. Illustration

M/s X Ltd. has paid a dividend of ₹ 2.5 per share on a face value of ₹10 in the financial year ending on 31st March, 2009. The details are as follows:

Current market price of share	**₹ 60**
Growth rate of earnings and dividends	**10%**
Beta of share	**0.75**
Average market returns	**15%**
Risk free rate of return	**9%**

Calculate the intrinsic value of the share.

(ICAI SM)

Solution:

R_m = 15%
R_f = 9%
β = 0.75
$E(R)$ = $R_f + \beta\,(R_m - R_f)$
k_e = 9%+ 0.75(15%-9%) = 9% + 0.75(6%) = 13.5%
Intrinsic Value,
P_0 = $\frac{d_1}{k_e - g}$ (D_0 = 2.5, g= 10%, D_1= 2.75)
= $\frac{2.75}{13.5\% - 10\%}$ = **₹ 78.571**

40. Illustration

A Company pays a dividend of ₹ 2.00 per share with a growth rate of 7%. The risk-free rate is 9% and the market rate of return is 13%.
The Company has a beta factor of 1.50. However, due to a decision of the Finance Manager, beta is likely to increase to 1.75. Find out the present as well as the likely value of the share after the decision.

(ICAI SM)

Solution:

Scenario 1:
β = 1.5
k_e = $R_f + \beta(R_m - R_f)$
= 9% +1.5(13-9) = 9% + 1.5* 4% = 9% + 6% = 15%
$P_0 = \frac{d_1}{ke - g} = \frac{2(1+7\%)}{15\% - 7\%}$ = **26.75**
Scenario 2:
β = 1.75
k_e = $R_f + \beta(R_m - R_f)$
= 9% +1.75(13-9) = 9% + 1.75* 4% = 9% + 7% = 16%
$P_0 = \frac{d_1}{ke - g} = \frac{2(1+7\%)}{16\% - 7\%}$ = **23.788**

41. Illustration

Two companies A Ltd. and B Ltd. paid a dividend of ₹3.50 per share. Both are anticipating that dividend shall grow @ 8%. The beta of A Ltd. and B Ltd. are 0.95 and 1.42 respectively.
The yield on GOI Bond is 7% and it is expected that stock market index shall increase at an annual rate of 13%. You are required to determine:

(i) Value of share of both companies.
(ii) Why there is a difference in the value of shares of two companies?
(iii) If current market price of share of A Ltd. and B Ltd. Are ₹74 and ₹55 respectively. As an investor what course of action should be followed?

(Similar Dec'21 QP)

Solution:
R_f = 7%
R_m =13%
MRP = 6%

	A	B
D_0	3.5	3.5
g	8%	8%
D_1	3.78	3.78
β	0.95	1.42

k_e of A = R_f + β (MRP) = 7% + 0.95(6%) =7% + 5.7% = 12.7%

k_e of B = R_f + β (MRP) = 7% + 1.42(6%) = 7% + 8.52% = 15.52%

(i) P_A = $\frac{d_1}{ke-g}$ = $\frac{3.78}{12.7\%-8\%}$ = ₹ 80.42

P_B = $\frac{d_1}{ke-g}$ = $\frac{3.78}{15.52\%-8\%}$= ₹ 50.266

(ii) Company B has a higher beta which means its returns vary lot more or are more sensitive to market returns. So, the volatility of returns with respect to market is high which leads to higher cost of equity i.e. a higher discount rate, which implies a lower price.

(iii)

Company	CMP	Intrinsic Value	Action
A Ltd.	74	80.42	Buy the stock as it is trading below intrinsic value.
B Ltd.	55	50.266	Sell the stock as it is trading above intrinsic value.

42. Illustration

Seawell Corporation, a manufacturer of do-it-yourself hardware and housewares, reported earnings per share of € 2.10 in 2003, on which it paid dividends per share of €0.69. Earnings are expected to grow 15% a year from 2004 to 2008, during this period the dividend pay-out ratio is expected to remain unchanged. After 2008, the earnings growth rate is expected to drop to a stable rate of 6%, and the pay-out ratio is expected to increase to 65% of earnings. The firm has a beta of 1.40 currently, and is expected to have a beta of 1.10 after 2008. The market risk premium is 5.5%. The Treasury bond rate is 6.25%.

(i) What is the expected price of the stock at the end of 2008?

(ii) What is the value of the stock, using the two-stage dividend discount model?

(RTP May'19, Old PM)

Solution:

	Year	EPS	DPS	g	Pay-out Ratio
2003	0	2.1	0.69	15%	0.3285
2004	1	2.415	0.7935	15%	0.3285
2005	2	2.77725	0.91253	15%	0.3285
2006	3	3.19384	1.0494	15%	0.3285
2007	4	3.67291	1.20681	15%	0.3285
2008	5	4.22385	1.38784	15%	0.3285
2009	6	4.47728	2.91	6%	0.65

(i) β Current = 1.4

MRP = 5.5%

β 2000 = 1.1

R_f = 6.25%

k_e in 2009 $= R_f + \beta\ (MRP) = 6.25\% + 1.1(5.5\%) = 6.25\% + 6.05\% = 12.5\%$

P_0 at the end of,

2008 $= \frac{d_{2009}}{ke-g} = \frac{2.91}{12.3-6} =$ **46.19**

₹ 46.19 is price at the end of 2008

(ii) Value of stock at the end of 2003

= PV of Dividend from 2004-08+ PV of price at the end of 2008

k_e applicable for this PV computation is based on Beta for period 2003-08

k_e $= R_f + \beta\ (MRP) = 6.25\% + 1.4\ (5.5\%) = 13.95\%$

PV of Dividend from 2004-08

Year	Dividend	PVIF	Dividend* PVIF
2004	0.7935	0.8775	0.6929
2005	0.91253	0.7701	0.7027
2006	1.0494	0.6759	0.7093
2007	1.20681	0.5931	0.7157
2008	1.38784	0.5205	0.7223
			3.5463

P_0 today = 3.5463+ 4619*(0.5206) = **27.589**

43. Illustration

SRK Ltd. is a listed company and it has just announced annual dividend for the year ending 2013-14. Earnings Per Share (EPS) and Dividend Per Share (DPS) for 5 years is as follows:

	2013-14	2012-13	2011-12	2010-11	2009-10
EPS (Rs.)	14.00	13.60	13.10	12.70	12.20
DPS (Rs.)	8.20	8.10	7.90	7.80	7.70

In the opinion of MD of SRK Ltd., if current dividend policy is maintained annual growth in Earning and Dividends will be no better than the annual growth in earnings over the past years.
Since the Board of SRK Ltd. is reluctant to take debt to finance growth it is considering changing its dividend policy by retaining 50% of its earnings for investment in various projects having a post-tax rate of return of 15%. The beta of SRK Ltd. is 1.5, market risk premium is 4% and Risk-Free Rate of Return is 6%.
You are required to calculate expected market price of share, if

(1) SRK Ltd. does not announce a change in its Dividend Policy.
(2) SRK Ltd. does announce a change in its Dividend Policy by retaining 50% of its earnings.

Note: Growth Rate can be assumed to be remain stable.

(MTP Mar'15)

Solution:

(1) β = 1.5

MRP = 4%

R_f = 6%

k_e $= R_f + \beta\ (R_m - R_f) = 6\% + 1.5 * 4\% = 12\%$

Year	EPS	DPS	Payout	EPS (g)
09-10	12.2	7.7	63.11%	
10-11	12.7	7.8	61.42%	4.1%
11-12	13.1	7.9	60.3%	3.15%
12-13	13.6	8.1	59.56%	3.82%
13-14	14	**8.2**	**58.57%**	2.9%

Payout ratio is close to 60%

EPS Growth (13-14) =$(\frac{14}{12.2})^{1/4}$-1 = 1.035-1 = 3.5%

Dividend Growth = $\frac{8.2}{7.7}^{1/4}$-1 = 1.585%

P_0 = $\frac{D_1}{ke-g}$ | D1 = 14 x 1.035 x 60%= 8.694

P_0 in current payout of 60% is $\frac{8.694}{12\%-3.5\%}$ = ₹ **102.28**

(2) SRK Dividend policy changes, retention(b) and payout ratio are 50% each

g = $b * r$ (r= rate of return)

g = 50%* 15% = 7.5%

EPS = EPS_0 * growth rate = ₹ 14

EPS_1 = 14* (1+ 7.5%) = 15.05%

D_1 = EPS_1 * Payout Ratio = 15.05 * 50% = 7.525

P_0 = $\frac{D_1}{ke-g}$ = $\frac{7.525}{12\%-7.5\%}$ = **167.22**

Alternative 2 is better as it will lead to a much higher dividend growth rate in the long term

44. Illustration

Mr. A has short term investments in shares of the various companies. The details of these investments are as follows:

Name of Company	No. of shares	Geared Beta	Current Market Price (₹)	Current Dividend Yield (%)	Expected Return (%)
T Ltd. (Face Value ₹ 50)	1000	1.55	280	6.8	21.00
U Ltd. (Face Value ₹ 100)	1550	0.65	340	3.6	12.50
V Ltd. (Face Value ₹ 20)	2600	1.26	150	6.4	18.00
W Ltd. (Face Value ₹10)	4300	1.14	95	7.2	18.50

Risk Free Rate of Return and market return are 6% and 16% respectively. You are required to:

(a) Estimate the risk of Mr. A's portfolio relative to market.

(b) Whether the composition of portfolio should be changed if yes then how.

Solution:

a)

	Beta	Shares	CMP	MV	Beta* MV
T	1.55	1000	280	280000	434000
U	0.65	1550	340	527000	342550
V	1.26	2000	150	390000	491400
W	1.14	4300	95	408500	465690
				1605500	**1733640**

β_p = $\frac{1733640}{1605500}$ = 1.0798

b) Assuming Expected Return also includes dividend yield

	$E(R)$	CAPM Return		Comments
T	21	6% + 1.55(10%) =	21.50%	overvalued
U	12.5	6% + 0.65(10%) =	12.50%	rightly valued
V	18	6% + 1.26(10%) =	18.60%	overvalued
W	18.5	6% + 1.14(10%) =	17.40%	undervalued

Mr. A should sell overvalued stocks of T and V, should hold rightly valued stock of U and buy undervalued stock of W.

45. Illustration

Following data is related to Company X, Market Index and Treasury Bonds for the current year and last 4 years:

Year	Company X: Average Share Price (P)	Company X: Dividend Per Share (D)	Market Index: Average Market Index	Market Index: Market Dividend Yield	Return on Treasury Bonds
2009	₹ 139	₹ 7.00	1300	3%	7%
2010	₹ 147	₹ 8.50	1495	5%	9%
2011	₹ 163	₹ 9.00	1520	5.5%	8%
2012	₹ 179	₹ 9.50	1640	4.75%	8%
2013	₹ 203.51	₹ 10.00	1768	5.5%	8%

With the above data estimate the beta of Company X's share. *(Similar May'22 QP)*

Solution:

$E(R)$ of stock $= \frac{\text{dividend}}{\text{opening value}} + \frac{\text{capital gain}}{\text{opening value}}$

	Op Stk Price	Closing Stk Price	Increase	Stk Return = (Cl - OP)/ OP	Dividend	Div / OP	Total Return (Stk + Div Yld)
2009-10	139	147	8	5.76%	8.50	6.12%	11.87%
2010-11	147	163	16	10.88%	9.00	6.12%	17.01%
2011-12	163	179	16	9.82%	9.50	5.83%	15.64%
2012-13	179	203.5	24.5	13.69%	10.00	5.59%	19.27%

	Opening Index Value	Closing Index Value	Increase	Index Return = (Cl - OP)/ OP	Div Yield	Total Return Index + Div
2009-10	1300	1495	195	15.00%	5.00%	20.00%
2010-11	1495	1520	25	1.67%	5.50%	7.17%
2011-12	1520	1640	120	7.89%	4.75%	12.64%
2012-13	1640	1768	128	7.80%	5.50%	13.30%

Average Return on Treasury Bond, $= \frac{7\% + 9\% + 8\% + 8\% + 8\%}{5}$

R_f = 8%

	X - Stock Return	Y - Market Return	XY	Y^2
2009-10	11.87	20.00	237.41	400.00
2010-11	17.01	7.17	121.98	51.41
2011-12	15.64	12.64	197.82	159.77
2012-13	19.27	13.30	256.42	176.89
Total	**63.80**	**53.12**	**813.53**	**788.07**

Average	15.95	13.28		

Beta = $\sum XY - n\,\bar{x}\bar{y} \,/\, \sum y^2 - n\,\bar{y}^2$

Beta = (813.53 – 4 x 15.95 x 13.28) / (788.07 – 4 x (13.28)^2) = -0.408

46. Illustration

Ms. Preeti, a school teacher, after retirement has built up a portfolio of Rs. 1,20,000 which is as follow:

Stock	Shares	CMP (₹)	Beta
ABC Ltd.	1000	50	0.9
DEF Ltd.	500	20	1.0
GHI Ltd.	800	25	1.5
JKL Ltd.	200	200	1.2

Her portfolio consultant Sri Vijay has advised her to bring down the, beta to 0.8. You are required to compute:

(i) Present portfolio beta

(ii) How much risk-free investment should be bought in, to reduce the beta to 0.8?

(MTP Mar'21 New & Old, MTP May'20, May'19 QP (Old)]

Solution:

	Shares	CMP	Market Value	Beta	Beta* MV
ABC	1000	50	50000	0.9	45000
DEF	500	20	10000	1	10000
GHI	800	25	20000	1.5	30000
JKL	200	200	40000	1.2	48000
			120000		133000

(i) β_P = $\frac{133000}{120000}$ = **1.1083**

(ii) Let risk free investment be x, then value of portfolio is 1-x and total portfolio value becomes $(x + 1 - x)$ = 1

Risk free investments have NIL beta.

β_{RP} = 1.1083 (1-x) + 0*x

0.8 = 1.1083- 1.1083x

x = $\frac{0.3083}{1.1083}$

Weight of R_f Asset = 27.817%

$R_f : Eq$ = 27.82: 72.18

Existing Equity = 120000

Total Portfolio = $\frac{120000}{72.18\%}$ = 166251

R_f = 166251- 120000 = **46251**

As new investment needs to be brought in and after that is brought in the ratio should be 27.82: 72.18. So, the overall portfolio value increases to ₹ 166251 and the value of risk free investment is ₹ 46251.

47. Illustration

Assuming that two securities X and Y are correctly priced on SML and expected return from these securities are 9.40% (R_x) and 13.40% (R_y) respectively. The Beta of these securities are 0.80 and 1.30 respectively.

Mr. A, an investment manager states that the return on market index is 9%. You are required to determine,

(a) Whether the claim of Mr. A is right. If not, then what is correct return on market index.

(b) Risk Free Rate of Return

Solution:

$E(R) = R_f + \beta\,(MRP)$

$9.4 = R_f + 0.8\,(MRP)$ (1)

$13.4 = R_f + 1.3\,(MRP)$ (2)

Subtracting (2) - (1)

$4 = 0.5(MRP)$

$MRP = \frac{4}{0.5} =$ **8%**

Substituting (MRP) in equation (1)

$9.4 = R_f + 0.8\,(8)$

$R_f = 9.4 - 6.4 =$ **3%**

$MRP = R_m - R_f$

$8 = R_m - 3 \Rightarrow R_m =$ **11%**

a) Claim of Mr A is incorrect as R_m is 11 % and not 9% & b) R_f = 3%

48. Illustration

The returns of a portfolio A and market portfolio for the last 12 months are indicated as follows:

Month	Portfolio A	Market Portfolio
January	- 0.52	0.82
February	2.20	0.04
March	2.17	2.80
April	4.17	1.72
May	2.04	0.27
June	3.00	0.39
July	1.99	1.95
August	4.0	0.64
September	- 1.38	1.53
October'	2.67	2.70
November	3.99	2.52
December	1.86	2.09
Standard Deviation (σ)	1.6223	0.9498

(i) You are required to find out the monthly returns attributable to 'the sheer skill of the Portfolio Manager.

(ii) What part of the monthly return is attributable to the higher risk assumed by the Portfolio Manager?

Assume that the risk-free rate of return is 12% per annum and the portfolio is fully diversified.

[Nov'19 QP (Old)]

Solution:

x	y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})\,(y-\bar{y})$
- 0.52	0.82	-2.7025	-0.6358	1.71825
2.20	0.04	0.0175	-1.4158	-0.0248

2.17	2.8	-0.0125	1.3442	-0.0168
4.17	1.72	1.9875	0.2642	0.5251
2.04	0.27	-0.1425	-1.1858	0.16898
3.00	0.39	0.8175	-1.0658	-0.8713
1.99	1.95	-0.1925	0.4942	-0.0951
4.00	0.64	1.8175	-0.8158	-1.4827
- 1.38	1.53	-3.5625	0.0742	-0.2643
2.67	2.7	0.4875	1.2442	0.60655
3.99	2.52	1.8075	1.0642	1.92354
1.86	2.09	-0.3225	0.6342	-0.2045
26.19	**17.47**			**1.98283**

$\bar{x}$ = 2.1825%

$\bar{y}$ = 1.4558%

$\beta = \frac{cov}{var_M} = \frac{(1.98283/12)}{0.9498^2} =$ **0.1832**

$E(R) = R_f + \beta\,(R_m - R_f)$

= 1% +0.183 (1.4558- 1%) = **1.083%**

Actual Return = 2.1825%

(i) Return due to Skill of PM,

= 2.1825- 1.083 = **1.0995% per month**

(ii) Return attributable to higher risk,

= Expected Return – Market Return

= 1.083- 1.4558 = **-0.3728%**

The investor has taken stock specific risk; hence this number is negative.

49. Illustration

ABC Ltd. manufactures Car Air Conditioners (ACs), Window ACs and Split ACs constituting 60%, 25% and 15% of total market value.

The stand-alone Standard Deviation and Coefficient of Correlation with market return of Car AC and Window AC is as follows:

	S.D.	Correlation
Car AC	0.30	0.6
Window AC	0.35	0.7

No data for stand-alone SD and Coefficient of Correlation of Split AC is not available. However, a company who derives its half value from Split AC and half from Window AC has a SD of 0.50 and Coefficient of correlation with market return is 0.85. Index has a return of 10% and has SD of 0.20. Further, the risk-free rate of return is 4%.

You are required to determine:

(i) Beta of ABC Ltd.

(ii) Cost of Equity of ABC Ltd. Assuming that ABC Ltd. wants to raise debt of an amount equal to half of its Market Value then determine equity beta, if yield of debt is 5%.

(RTP Nov'17)

Solution:

(i) R_f = 4%

R_m = 10%

σ_m = 0.2

$\beta = \frac{\rho * \sigma_s}{\sigma_m}$

A/C	S. D	Correlation	$\frac{\rho * \sigma_s}{\sigma_m}$	β
Car	0.3	0.6	0.6* 0.3/0.2	0.9
Window	0.35	0.7	0.7* 0.35/0.2	1.225
Split+ Window	0.5	0.85	0.85* 0.5/0.2	2.125

$\beta_{p\,(S+W)}$ = $\beta_s\, W_s + \beta_p\, W_p$

2.125 = $\beta_s * 0.5 + 1.225 * 0.5$

2.125 = $\beta_s * 0.5 + 0.6125$

$\beta_s * 0.5$ = 1.5125

β_s = $\frac{1.5125}{0.5}$ = **3.025**

A/C	β	w_i	$w_i\beta_i$
Car	0.9	60%	0.54
Window	1.225	25%	0.30625
Split	3.025	15%	0.45375
			1.3

Weighted Average, β_p = $\sum_{i=1}^{n} w_i\beta_i$ = **1.3**

$E(R)$ of ABC Ltd. = $R_f + \beta(R_m - R_f)$ = 4% +1.3 (10%-4%) = 4% + 1.3 (6%)

$E(R)$ of ABC Ltd. = **11.8%**

50. Illustration

With the help of following data determine the return on the security X.

Factor	Risk Premium associated with the Factor	β_i
Market	4%	1.3
Growth Rate of GDP	1%	0.3
Inflation	-4%	0.2

Risk Free Rate of Return is 8%.

(ICAI SM)

Solution:

$E(R)$ = $R_f + \lambda_1\beta_1 + \lambda_2\beta_2 + \ldots\ldots\ldots\ldots \Lambda_n\beta_n$

= 8% + 4% (1.3) + 1% (0.3) -4% (0.2)

= 8% + 5.2% + 0.3% - 0.8%

$E(R)$ = **12.7%**

51. Illustration

Mr. X owns a portfolio with the following characteristics:

	Security A	Security B	Risk Free security
Factor 1 sensitivity	0.80	1.50	0
Factor 2 sensitivity	0.60	1.20	0
Expected Return	15%	20%	10%

It is assumed that security returns are generated by a two-factor model.

(i) If Mr. X has Rs. 1,00,000 to invest and sells short Rs. 50,000 of security B and purchases Rs. 1,50,000 of security A, what is the sensitivity of Mr. X's portfolio to the two factors?

(ii) If Mr. X borrows Rs. 1,00,000 at the risk-free rate and invests the amount he borrows along with the original amount of Rs. 1,00,000 in security A and B in the same proportion as described in part (i), what is the sensitivity of the portfolio to the two factors?

(iii) What is the expected return premium of factor 2?

Solution:

(i) Investment in A = 1,50,000
Investment in B = - 50,000
Total = 1,00,000
Portfolio Sensitivity to factor 1, (Weights of A & B are 1.5:-0.5)
= 1.5 * 0.8 – 0.5 * 1.5 = 1.2 – 0.75 = **0.45**
Portfolio Sensitivity to factor 2,
= 1.5 * 0.6 – 0.5 * 1.2 = 0.9 – 0.60 = **0.30**
Portfolio Sensitivity to factor 1 is **0.45** and Portfolio Sensitivity to factor 2 is **0.30**.

(ii) Total = 1,00,000 + 1,00,000
Ratio = 3: -1
Portfolio Sensitivity to factor 1, (Weights of A & B are 1.5:-0.5 i.e 3:-1)
= 3* 0.8 – 1* 1.5 = 2.4– 1.5 = **0.9**
Portfolio Sensitivity to factor 2,
= 3* 0.6 – 1* 1.2 = 1.8 – 1.2 = **0.6**

(iii) $E(R) = R_f + \lambda_1\beta_1 + \lambda_2\beta_2 + \ldots\ldots\ldots\ldots \lambda_n\beta_n$
A, 15% = 10% + λ_1 * 0.8 + λ_2 *0.6
B, 20% = 10% + λ_1 * 1.5 + λ_2 *1.2
Multiplying A with 2
A , 30% = 20% + λ_1 * 1.6 + λ_2 *1.2
B, 20% = 10% + λ_1 * 1.5 + λ_2 *1.2

A – B
10% = 10% + λ_1 * 0.1

=> λ_1 = 0
(Substituting value of λ_1 *in Equation A)* λ_2 = **8.33**

52. Illustration

Mr. Nirmal Kumar has categorized all the available stock in the market into the following types:
(a) Small cap growth stocks
(b) Small cap value stocks
(c) Large cap growth stocks
(d) Large cap value stocks

Mr. Nirmal Kumar also estimated the weights of the above categories of stocks in the market index. Further, the sensitivity of returns on these categories of stocks to the three important factors are estimated to be:

Category of Stocks	Weight in the Market Index	Factor I (Beta)	Factor II (Book Price)	Factor III (Inflation)
Small cap growth	25%	0.80	1.39	1.35
Small cap value	10%	0.90	0.75	1.25
Large cap growth	50%	1.165	2.75	8.65
Large cap value	15%	0.85	2.05	6.75
Risk Premium		6.85%	-3.5%	0.65%

The rate of return on treasury bonds is 4.5% Required:

(i) Using Arbitrage Pricing Theory, determine the expected return on the market index.

(ii) Using Capital Asset Pricing Model (CAPM), determine the expected return on the market index.

(iii) Mr. Nirmal Kumar wants to construct a portfolio constituting only the 'small cap value' and 'large cap growth' stocks. If the target beta for the desired portfolio is 1, determine the composition of his portfolio.

(ICAI SM, MTP Apr'22, RTP Nov'19 Old, Old PM)

Solution:

(i) $E\ (R)$ using APT

Small Cap Growth

$E\ (R)$ = $R_f + \lambda_1\beta_1 + \lambda_2\beta_2 + \ldots\ldots\ldots\ldots \lambda_n\beta_n$

= 4.5% + (6.85* 0.8) + (-3.5%* 1.39) + (0.65* 1.35)

= 5.9925 %

Small Cap Value

$E\ (R)$ = $R_f + \lambda_1\beta_1 + \lambda_2\beta_2 + \ldots\ldots\ldots\ldots \lambda_n\beta_n$

= 4.5% + (6.85* 0.9) + (-3.5%* 0.75) + (0.65 * 1.25)

= 8.8525 %

Large Cap Growth

$E\ (R)$ = $R_f + \lambda_1\beta_1 + \lambda_2\beta_2 + \ldots\ldots\ldots\ldots \lambda_n\beta_n$

= 4.5% + (6.85* 1.165) + (-3.5%* 2.75) + (0.65 * 8.65)

= 8.47775 %

Large Cap Value

$E\ (R)$ = $R_f + \lambda_1\beta_1 + \lambda_2\beta_2 + \ldots\ldots\ldots\ldots \lambda_n\beta_n$

= 4.5% + (6.85* 0.85) + (-3.5% * 2.05) + (0.65 * 6.75)

= 7.535%

$E\ (R)$ of Market = Weighted Average of Returns

	Return	Weights	
Small Cap G	5.9925	0.25	1.498125
Small Cap V	8.8525	0.1	0.885250
Large Cap G	8.4775	0.5	4.238750
Large Cap v	7.535	0.15	1.130250
		$E\ (R)$ of Market	**7.752375**

(ii) CAPM based Return

	$R_f + \beta\ (MRP)$	Return	Weights	Return* Weight
Small Cap G	4.5+ 0.8*6.85	9.98	0.25	2.495
Small Cap V	4.5+ 0.9*6.85	10.665	0.1	1.0665
Large Cap G	4.5+ 1.165*6.85	12.48025	0.5	6.240125
Large Cap v	4.5+ 0.85*6.85	10.3225	0.15	1.548375
				11.35%

(iii) Let weight of Small Cap Value be w_1, then of Large Cap growth w_2= 1-w_1

1 = 0.9* w_1 + 1.165* (1-w_1)

= 0.9 w_1 + 1.165- 1.165w_1

1.165 -1 = 1.165w_1 - 0.9 w_1

0.165 = 0.265w_1

w_1 = $\frac{0.165}{0.265}$ = **0.6226**

w_2 = 1- 0.6226 = **0.3774**

53. Illustration

The risk-free rate of return Rf is 9 %. The expected rate of return on the market portfolio Rm is 13 %. The expected rate of growth for the dividend of Platinum Ltd. is 7 %. The last dividend paid on the equity stock of firm A was ₹ 2.00. The beta of Platinum Ltd. equity stock is 1.2.

(i) What is the equilibrium price of the equity stock of Platinum Ltd.?

(ii) How would the equilibrium price change when

- The inflation premium increases by 2 %?
- The expected growth rate increases by 3 %?
- The beta of Platinum Ltd. equity rises to 1.3?

(ICAI SM, May'18 QP 4 marks, MTP Sept'23, MTP Mar'19, Old PM)

Solution:

(i)

R_f = 9%

R_m = 13%

D_0 = 2

D_1 = 2.14

g = 7%

β = 1.2

k_e = $R_f + \beta\ (R_m - R_f)$ = 9% +1.2* 4% = 13.8%

P_0 for Platinum = $\frac{d_1}{ke-g} = \frac{2.14}{(13.8\% - 7\%)}$ = 2.14/6.8% = ₹ 31.47

(ii) **Case 1:**

$E\ (R)$ = $R_f + \beta$ (MRP + Infl.)

= 9% +1.2 (4)+2 = 9+ 4.8 + 2 = 15.8%

P_0 when $E\ (R)$ =15.8%,

= $\frac{d_1}{ke-g} = \frac{2.14}{15.8\%-7\%}$ = **₹ 24.32**

Price falls to ₹ 24.32 when inflation premium goes up by 2%

Case 2:

D_0 = 2

D_1 = 2*1.1

= 2.2

g = 7.1%

k_e = 13.8%

P_0 when $E\ (R)$ =13.8%,

P_0 = $\frac{d_1}{ke-g} = \frac{2.2}{13.8\%-7\%}$ = **₹ 57.89**

Price becomes ₹ 57.89

Case 3:

D_1 = 2.4

MRP = 4

k_e = $R_f + \beta(R_m - R_f)$

= 9% + 1.3* 4% = 9% + 5.2 = 14.2%

P_0 = $\frac{d_1}{ke-g} = \frac{2.14}{14.2\%-7\%}$ = **₹29.72**

Price increases to ₹ 29.72.

Assumption: All factors in Part 2 affect the computation independently

54. Illustration

The following details are given for X and Y companies' stocks and the Bombay Sensex for a period of one year. Calculate the systematic and unsystematic risk for the companies' stocks. If equal amount of money is allocated for the stocks what would be the portfolio risk?

	X Stock	Y Stock	Sensex
Average return	0.15	0.25	0.06
Variance of return	6.30	5.86	2.25
β	0.71	0.685	

Correlation Co-efficient (r) 0.424

Co-efficient of determination (r^2) 0.18

(ICAI SM, Sep25 Similar 6M)

Solution:

Systematic and Unsystematic Risk for stocks X and Y

Total Risk = Systematic Risk+ Unsystematic Risk

Systematic Risk = $\beta_i^2 \sigma_m^2$

For X Ltd,

Systematic Risk = $(0.71)^2$ *2.25 = 1.134

Total Risk = 6.3

Unsystematic Risk = 6.3 – 1.134 = 5.166

For Y Ltd,

Systematic Risk = $(0.685)^2$ *2.25 = 1.056

Total Risk = 5.86

Unsystematic Risk = 5.86 – 1.056 = 4.804

Variance of a 50:50 Portfolio of X and Y

	Weight	β_i	σ_e^2	σ_m^2
X	50	0.71	5.166	2.25
Y	50	0.685	4.804	

σ_p^2 = $(\sum x_i\beta_i)^2 * \sigma_m^2 + \sum x_i^2\sigma_e^2$

$(\sum x_i\beta_i)^2 * \sigma_m^2$ = $[(0.5*0.71) + (0.5*0.685)]^2 * 2.25$ = **1.095**

$\sum x_i^2\sigma_e^2$ = $0.5^2 *4.804 + 0.5^2 *5.166$ = 1.201+ 1.2915 = **2.4925**

σ_p^2 = 1.095+ 2.4925 = **3.588**

β = $\frac{r\,\sigma_x}{\sigma_y}$ = $0.424* \frac{\sqrt{6.3}}{\sqrt{2.25}}$ = **0.70948**

Systematic Risk = $r^2 * \sigma_x^2$ = 0.18* 6.3 = **1.134**

Unsystematic Risk = 6.3(1-0.18) = **5.166**

55. Illustration

The returns on stock A and market portfolio for a period of 6 years are as follows:

Year	Return on A (%)	Return on market portfolio (%)
1	12	8
2	15	12
3	11	11
4	2	-4
5	10	9.5
6	-12	-2

You are required to determine:

- Characteristic line for stock A
- The systematic and unsystematic risk of stock A.

(ICAI SM, Old PM)

Solution:

A	M	A*M	M^2	$A-\bar{A}$	$M-\bar{M}$	$(A-\bar{A})^2$	$(M-\bar{M})^2$
12	8	96	64	5.67	2.25	32.14	5.0625
15	12	180	144	8.67	6.25	75.17	39.063
11	11	121	121	4.67	5.25	21.81	27.563
2	-4	-8	16	-4.33	-9.75	18.75	95.063
10	9.5	95	90.25	3.67	3.75	13.47	14.063
-12	-2	24	4	-18.33	-7.75	335.99	60.063
38	**34.5**	**508**	**439.25**			**497.33**	**240.88**

$\bar{A}$ = $\frac{38}{6}$ = 6.33

$\bar{M}$ = $\frac{34.5}{6}$ = 5.75

β = $\frac{\sum AM - n\bar{A}\bar{M}}{\sum M^2 - n\bar{M}^2}$ = $\frac{508-6*6.33*5.75}{439.25-(6)*(5.75)^2}$ = $\frac{289.5}{240.875}$= **1.202**

σ_m^2 = $\frac{240.875}{6}$= **40.146**

Characteristic Line

R_m = 5.75

R_A = 6.33

β = 1.202

R_A = α + β R_m

6.33 = α+ 1202*5.75

α = **-0.582**

R_s = -0.582 + 1.202 R_m

Systematic Risk of Stock A:

= $\beta_i^2 \sigma_m^2$ = $(1.202)^2$ * 40.146 = **58.003**

Total Risk of Stock = $\frac{\sum(A-\bar{A})^2}{n}$

σ_A^2 = 497.33/ 6 = **82.889**

Unsystematic Risk =82.889- 58.003 = **24.884**

56. Illustration

The rates of return on the security of Company X and market portfolio for 10 periods are given below:

Period	Return of Security X (%)	Return on Market Portfolio (%)
1	20	22
2	22	20
3	25	18
4	21	16
5	18	20
6	-5	8
7	17	-6
8	19	5
9	-7	6
10	20	11

(i) What is the beta of Security X?

(ii) What is the characteristic line for Security X?

(ICAI SM, Old PM)

Solution:

	X	M	XM	M^2
1	20	22	440	484
2	22	20	440	400
3	25	18	450	324
4	21	16	336	256
5	18	20	360	400
6	-5	8	-40	64
7	17	-6	-102	36
8	19	5	95	25
9	-7	6	-42	36
10	20	11	220	121
	150	**120**	**2157**	**2146**

$\bar{X}$ = 150/10

= 15

$\bar{M}$ = 120/10

= 12

(i) β $= \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n(\bar{x})^2}$

$= \frac{2157 - 10*15*12}{2146 - 10*144}$

= 357/706

= **0.5057**

(ii) R_s $= \beta\ R_m + \alpha$

15 = 12* 0.55057 + α

15 = 6.0680+ α

α = **8.9320**

R_s = **8.9320% + 0.5057R_M**

57. Illustration

Expected returns on two stocks for particular market returns are given in the following table:

Market Return	Aggressive	Defensive
7%	4%	9%
25%	40%	18%

You are required to calculate:

(a) The Betas of the two stocks.

(b) Expected return of each stock, if the market return is equally likely to be 7% or 25%.

(c) The Security Market Line (SML), if the risk-free rate is 7.5% and market return is equally likely to be 7% or 25%.

(d) The Alphas of the two stocks.

(ICAI SM, Nov 23'QP 8 marks, RTP Nov'18, MTP Sept'23, MTP May'20 New & Old, MTP Oct'19, RTP May'19 Old, MTP Aug'18 Old, Old PM)

Solution:

a) Beta Sharpe Model:

R_s = α+ β R_m

4% = α + β* 7% (1)

40% = α + β* 25% (2)

(2)- (1),

36% = β* 18%

$\boldsymbol{\beta}$ **= 2**

α = 4- 14 = **- 10**

9% = α + β* 7% (3)

18% = α + β* 25% (4)

(4)- (3),

9% = β* 18%

$\boldsymbol{\beta}$ **= 0.5**

α = 9-3.5 = **5.5**

b) Expected Return of Market = $\sum_{i=1}^{n} P_i R_i$ = 0.5* 7 + 0.5* 12.5 = 16%

Characteristic Line of Each security

Security 1:

R_1 = -10% + 2(R_M)

Security 2:

R_2 = 5.5% + 0.5(R_M)

At R_M= 16%,

R_1 = -10% + 2*16 = **22%**

R_2 = 5.5% + 0.5* 16 = **13.5%**

c) R_M = 0.5(7+25) = 16%

R_f = 7.5%

SML as per CAPM = 7.5% + β(16%-7.5%)

$E(R)$ = 7.5% + 8.5% β

d) α of Stock 1 = **-10**

α of Stock 2 = **5.5%**

58. Illustration

A study by a Mutual fund has revealed the following data in respect of three securities:

Security	σ (%)	Correlation with Index, P_m
A	20	0.60
B	18	0.95
C	12	0.75

The standard deviation of market portfolio (BSE Sensex) is observed to be 15%.

(i) What is the sensitivity of returns of each stock with respect to the market?

(ii) What are the covariances among the various stocks?

(iii) What would be the risk of portfolio consisting of all the three stocks equally?

(iv) What is the beta of the portfolio consisting of equal investment in each stock?

(v) What is the total, systematic and unsystematic risk of the portfolio in (iv)?

(ICAI SM, RTP Nov'20 New & Old, MTP Mar'19 Old, Old PM)

Solution:

(i) β $= \frac{r\,\sigma_s}{\sigma_m}$

Stock	r	σ_s	σ_m	β
A	0.6	10	15	0.8
B	0.95	18	15	1.14
C	0.75	12	15	0.6

(ii) Covariances amongst stocks

Co variance between 2 stocks = Beta 1 x Beta 2 x Market Variance

Stock	A	B	C
A	20^2	0.8 x 1.14 x 225	0.8 x 0.6 x 225
B	0.8 x 1.14 x 225	18^2	0.6 x 1.14 x 225
C	0.8 x 0.6 x 225	0.6 x 1.14 x 225	12^2

Stock	A	B	C
A	400	205.2	108
B	205.2	324	153.9
C	108	153.90	144

(iii) Portfolio Variance for a portfolio with greater than 2 securities is computed using variance covariance matrix.

$= (1/3)^2 * 400 + (1/3)^2 * 324 + (1/3)^2 * 144 + (1/3)^2 * 108 + (1/3)^2 * 153.9*2 + (1/3)^2 * 2* 205.2$

$= (1/3)^2 (400+324+144+216+ 307.8+ 410.4)$

Total Risk = **200.24**

(iv) β_p $= \sum_{i=1}^{n} w_i \beta_i$ = 1/3 (0.8+ 1.14+ 0.6) = **0.847**

(v) Total Risk = 200.24

Systematic Risk $= \beta_i^2 \sigma_m^2 = (0.847)^2 * (15)^2$ = **161.417**

Unsystematic Risk = 200.24 – 161.417 = **38.822**

59. Illustration

Following are the details of a portfolio consisting of three shares:

Share	Portfolio weight	Beta	Expected return in %	Total variance
A	0.20	0.40	14	0.015
B	0.50	0.50	15	0.025
C	0.30	1.10	21	0.100

Standard Deviation of Market Portfolio Returns = 10%. You are given the following additional data:

- **Covariance (A, B) = 0.030**
- **Covariance (A, C) = 0.020**
- **Covariance (B, C) = 0.040**

Calculate the following:

(i) The Portfolio Beta

(ii) Residual variance of each of the three shares

(iii) Portfolio variance using Sharpe Index Model

(iv) Portfolio variance (on the basis of modern portfolio theory given by Markowitz

[ICAI SM, May'19 QP 8 marks, RTP May'18, MTP Oct'24, MTP Aug'18, Jan'21 QP (Old), MTP Nov'21 Old, MTP Apr'18 Old, RTP Nov'19 Old, Old PM]

Solution:

(i) β_p = $\sum_{i=1}^{n} w_i\beta_i$ = 0.2*0.4+ 0.5*0.5+ 0.3*1.10= 0.66

(ii) Residual Variance = Total Variance- Systematic Variance

	σ^2	β	σ_m^2	Systematic Risk	Unsystematic Risk
A	0.015	0.4	0.01	0.0016	0.0134
B	0.025	0.5	0.01	0.0025	0.0225
C	0.1	1.1	0.01	0.0121	0.0879

(iii) σ_p^2 = Systematic Risk+ Unsystematic Risk

= $(\sum x_i\beta_i)^2 * \sigma_m^2 + \sum x_i^2\sigma_e^2$

Systematic Risk = $\beta_p^2 * \sigma_m^2$ = $(0.66)^2 + (0.10)^2$ = 0.004356

Unsystematic Risk

= $(0.2)^2$ *0.0134 + $(0.5)^2$ *0.0225 + $(0.3)^2$*0.0879 = 0.014072

σ_p^2 = 0.004356+ 0.014072 = 0.018428

(iv) Variance as per modern portfolio theory of Markowitz using variance- covariance matrix

	A (0.2)	B (0.5)	C (0.3)
A (0.2)	0.015 * $(0.2)^2$	0.03* (0.5*0.2)	0.02 *(0.2*0.3)
B (0.5)	0.03* (0.5*0.2)	0.025* $(0.5)^2$	0.04* (0.3* 0.5)
C (0.3)	0.02* (0.2*0.3)	0.04* (0.3*0.5)	0.1* $(0.3)^2$

σ_p^2 = 0.015 * $(0.2)^2$ +0.025* $(0.5)^2$ +0.1* $(0.3)^2$ + 2(0.03*0.5*0.2 + 0.02*0.2*0.3 + 0.04*0.3*0.5)

= 0.0006+ 0.00625+ 0.0009+ 2(0.0003+ 0.0012+ 0.0006) = **0.03625**

60. Illustration

A has portfolio having following features:

Security	B	Random Error σ_(ϵ_i)	Weight
L	1.60	7	0.25
M	1.15	11	0.30
N	1.40	3	0.25
K	1.00	9	0.20

You are required to find out the risk of the portfolio if the standard deviation of the market index (σ_m) is 18%.

(ICAI SM, MTP Mar'22, Old PM)

Solution:

Total Risk = Systematic Risk + Unsystematic Risk = $(\sum x_i\beta_i)^2 * \sigma_m^2 + \sum x_i^2\sigma_e^2$

Systematic Risk = [(1.6* 0.25) + (1.15* 0.3) + (1.14*0.23) + (1*0.2) $]^2$ * $(0.18)^2$

= $(1.295)^2$ * $(0.18)^2$ = **0.054336**

Unsystematic Risk = $\sum x_i^2\sigma_e^2$

= [$(0.07)^2$ *$(0.25)^2$ + $(0.11)^2$ *$(0.3)^2$ +$(0.03)^2$ *$(0.25)^2$+$(0.09)^2$ *$(0.2)^2$]

= [0.00030625+ 0.001089+ 0.00005625+ 0.000324] = **0.0017755**

Total Variance = 0.054336+ 0.0017755

σ_p^2 = **0.0561115**

σ_p = **23.6878%**

61. Illustration

The following are the data on five mutual funds:

Fund	Return	Standard Deviation	Beta
A	15	7	1.25
B	18	10	0.75
C	14	5	1.40
D	12	6	0.98
E	16	9	1.50

You are required to compute Reward to Volatility Ratio and rank these portfolios using:
- Sharpe method and
- Treynor's method assuming the risk-free rate is 6%.

(ICAI SM, RTP Nov'18, MTP Sept'22, MTP Oct'20, RTP May'19 Old, Old PM)

Solution:

Sharpe Ratio:

MF	R_i	R_f	$R_i - R_f$	S. D	$(R_i - R_f)$/S. D	Rank
A	15	6	9	7	1.2857	2
B	18	6	12	10	1.2	3
C	14	6	8	5	1.6	1
D	12	6	6	6	1	5
E	16	6	10	9	1.1111	4

Treynor ratio:

MF	R_i	R_f	$R_i - R_f$	β	$(R_i - R_f)/\beta$	Rank
A	15	6	9	1.25	7.2	2
B	18	6	12	0.75	16	1
C	14	6	8	1.4	5.7143	5
D	12	6	6	0.98	6.1224	4
E	16	6	10	1.5	6.6667	3

62. Illustration

Mr. Abhishek is interested in investing ₹ 2,00,000 for which he is considering following three alternatives:

(i) Invest ₹ 2,00,000 in Mutual Fund X (MFX)
(ii) Invest ₹ 2,00,000 in Mutual Fund Y (MFY)
(iii) Invest ₹ 1,20,000 in Mutual Fund X (MFX) and ₹ 80,000 in Mutual Fund Y (MFY)

Average annual return earned by MFX and MFY is 15% and 14% respectively. Risk free rate of return is 10% and market rate of return is 12%.
Covariance of returns of MFX, MFY and market portfolio Mix are as follows

	MFX	MFY	Mix
MFX	4.800	4.300	3.370
MFY	4.300	4.250	2.800
Mix	3.370	2.800	3.100

You are required to calculate:

(i) variance of return from MFX, MFY and market return,
(ii) portfolio return, beta, portfolio variance and portfolio standard deviation,
(iii) expected return, systematic risk and unsystematic risk; and
(iv) Sharpe ratio, Treynor ratio and Alpha of MFX, MFY and Portfolio Mix

(ICAI SM, RTP Nov'20 New & Old, MTP Mar'19, Old PM)

Solution:

(i) Variance of MFX, MFY and MPM,

Covariance of a Stock/ Portfolio with itself or its own returns is its variance.

Variance MFX = 4.8

Variance MFY = 4.25

Variance MPM = 3.1

(ii)

	Beta	Return	σ_p^2	σ_p
MFX	1.0871	12.1742	4.8	2.1909
MFY	0.9032	11.8064	4.25	2.0616
X+Y	1.101354	12.0271	4.472	2.1147

Working Notes:

1. β = Var of Stock Returns/ Var of Market Returns = $\frac{cov_{x,y}}{\sigma_y^2}$

   β MFX = 3.37/ 3.1 = **1.0871**

   β MFY = 2.8/ 3.1 = **0.9032**

   β_p of MFX and MFY is in the ratio 120: 80 i.e., 3:2

   $\sum_{i=1}^{n} x_i\beta_i$ = 3/5 * 1.0871 + 2/5 * 0.9032 = **1.01354**

2. $E\ (R)$ = $R_f + \beta(R_m - R_f)$

   $E\ (R)$ for MFX = 10% +1.0871(12-10)

   = 10+ 2.1742 = **12.1742%**

   $E\ (R)$ for MFY = 10% + 0.9032 (12-10) = **11.8064%**

   $E\ (R)$ for X+Y = 10% + 1.01354 *2% = 12.027

3. Variance

   σ_p^2 = $\sigma_x^2 + w_x^2 + \sigma_y^2 + w_y^2 + 2 * \sigma_x\sigma_y w_x w_y r$

   = 4.8* $(0.6)^2$ + 4.25* $(0.4)^2$ + 2*0.6* 0.4* 4.3

   = 4.472

   σ_p $=$ 2.1147

(iii) Calculation:

	MFX	MFY	X+Y
Beta	1.0871	0.9032	1.0135
σ_m^2	3.1	3.1	3.1
Systematic Risk (1)	3.6635	3.5289	3.18
Total Risk (2)	4.8	4.25	4.472
Unsystematic risk (2)- (1)	1.1365	1.7211	1.2875
E(R)	12.1742	11.8064	12.027

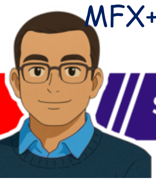
(iv) Sharpe Ratio: $(\frac{R_i - R_f}{S.D})$

MFX $= \frac{15-10}{2.1909}$ = 2.2822

MFY $= \frac{14-10}{2.0616}$ = 1.9403

MFX+ MFY $= \frac{14.6-10}{2.1147}$ (6/10 *15+ 4/10 * 14 = 14.6)

= 2.1753



Treynor Ratio: $\left(\frac{R_i - R_f}{\beta}\right)$

MFX $= \frac{15-10}{1.0871} = 4.60$

MFY $= \frac{14-10}{0.9032} = \mathbf{4.4287}$

MFX+ MFY $= \frac{14.6-10}{1.01354}$ (6/10 *15+ 4/10 * 14 = 14.6)

$= \mathbf{4.5385}$

Alpha = Actual Return – Expected Return

MFX = 15- 12.1742% = **2.8258**

MFY = 14%- 11.8064% = **2.1936**

MFX+ MFY = 14.6% - 12.0271% = **2.5729**

63. Illustration

Suppose that economy A is growing rapidly and you are managing a global equity fund and so far, you have invested only in developed-country stocks only. Now you have decided to add stocks of economy A to your portfolio.

The table below shows the expected rates of return, standard deviations, and correlation coefficients (all estimates are for aggregate stock market of developed countries and stock market of Economy A).

	Developed Country Stocks	Stocks of Economy A
Expected rate of return (annualized percentage)	10	15
Risk [Annualized Standard Deviation (%)]	16	30
Correlation Coefficient (r)	0.30	

Assuming the risk-free interest rate to be 3%, you are required to determine:

(a) What percentage of your portfolio should you allocate to stocks of Economy A if you want to increase the expected rate of return on your portfolio by 0.5%?

(b) What will be the standard deviation of your portfolio assuming that stocks of Economy A are included in the portfolio as calculated above?

(c) Also show how well the Fund will be compensated for the risk undertaken due to inclusion of stocks of Economy A in the portfolio?

(ICAI SM, MTP Apr'24, Old PM)

Solution:

a) Current R = 10%

Current σ = 16

Required return = 10.5%

Current Return = 10%

Weight of D = D

Weight of A = 1-D

10% (D)+ 15%(1-D) = 10.5%

10D+ 15- 15D = 10.5

-5d = 10.5- 15

-5D = -4.5

D = 0.9

Developing Economy A's share in portfolio is 10% (1-0.9)

b) w_1 = 90

w_2 = 10

σ_1 = 16

σ_2 = 30

r/ρ = 0.3

σ_p^2 = $\sigma_1^2 + w_1^2$ + $\sigma_2^2 + w_2^2$ + 2 $* \sigma_1\sigma_2 w_1 w_2 r$

= $(0.9*16)^2 + (0.1*30)^2 + 2*0.9*0.1*16*30*0.3$

= 207.36+ 0.9+ 25.92 = 242.28

σ_p = 15.565%

c)

	Return	σ
D	10	16
P	10.5	15.565

Due to inclusion of stocks of Economy a, Portfolio Return has increased and SD has decreased, it is beneficial.

Sharpe Ratio:

Before: (10- 3)/16 = 0.4375

After: (10.5-3)/ 15.565 = 0.4819

Sharpe Ratio has increased hence the revised portfolio is benefitted due to inclusion of stocks of economy.

64. Illustration

The five portfolios of a mutual fund experienced following result during last 10 years periods:

Portfolio	Average annual return %	Standard deviation	Correlation with the market return
A	20.0	2.3	0.8869
B	17.0	1.8	0.6667
C	18.0	1.6	0.600
D	16.0	1.8	0.867
E	13.5	1.9	0.5437

- Market Risk 1.2
- Market rate of return 14.3%
- Risk free rate 10.1%

Beta may be calculated only up to two decimals. Rank the portfolio using JENSEN'S ALPHA method.

Solution:

	σ_p	r	σ_m	$\beta = r*\sigma_p\ /\sigma_m$
A	2.3	0.8869	1.2	1.70
B	1.8	0.6667	1.2	1.00
C	1.6	0.6	1.2	0.80
D	1.8	0.867	1.2	1.30
E	1.9	0.5437	1.2	0.86

β	R_f	$(R_m - R_f$	$R_f + (R_m - R_f)$	$E(R)$	AR	AR $-E(R)$	Rank
1.70	10.1%	4.2%	0.101+1.7(0.042)	17.24	20	2.76	**2**
1.00	10.1%	4.2%	0.101+1.0(0.042)	14.3	17	2.7	**3**
0.80	10.1%	4.2%	0.101+0.8(0.042)	13.46	18	4.54	**1**
1.30	10.1%	4.2%	0.101+1.3(0.042)	15.56	16	0.44	**4**
0.86	10.1%	4.2%	0.101+0.86(0.042)	13.712	13.5	-0.212	**5**

65. Illustration

Five portfolios experienced the following results during a 7- year period:

Portfolio	Average Annual Return (Rp) (%)	Standard Deviation (SD)	Correlation with the market returns (r)
A	19.0	2.5	0.84036
B	15.0	2.0	0.540
C	15.0	0.8	0.975
D	17.5	2.0	0.750
E	17.1	1.8	0.600

- Market Risk (σ_m) - 1.2
- Market rate of Return (R_m) - 14.0
- Risk-free Rate (R_f) - 9.0

Rank the portfolios using

(a) Sharpe's method,

(b) Treynor's method and

(c) Jensen's Alpha

(ICAI SM, RTP Nov'21, MTP Oct'19 Old, Old PM)

Solution:

Sharpe Method:

	AR (1)	R_f(2)	(3) = (1)- (2)	SD	(3)/ SD	Rank
A	19	9	10	2.5	4	4
B	15	9	6	2	3	5
C	15	9	6	0.8	7.5	1
D	17.5	9	8.5	2	4.25	3
E	17.1	9	8.1	1.8	5.5	2

	σ_p	r	σ_m	$\beta = r{*}\sigma_p\ /\sigma_m$	$R_f + (R_m - R_f)$	$E(R)$
A	2.5	0.84	1.2	1.75	9+1.75(14-9)	17.75
B	2	0.54	1.2	0.9	9+0.9(14-9)	13.5
C	0.8	0.975	1.2	0.65	9+0.65(14-9)	12.25
D	2	0.75	1.2	1.25	9+1.25(14-9)	15.25
E	1.8	0.6	1.2	0.9	9+0.9(14-9)	13.5

Treynor Method:

	AR (1)	R_f(2)	(3) = (1)- (2)	β	(3)/ β	Rank
A	19	9	10	1.75	5.7	5
B	15	9	6	0.9	6.67	4
C	15	9	6	0.65	9.23	1
D	17.5	9	8.5	1.25	6.8	3
E	17.1	9	8.1	0.9	9	2

Jensen's Alpha Method:

	AR (1)	$E(R)$ (2)	α= (1)- (2)	Rank
A	19	17.75	1.25	5
B	15	13.5	1.5	4
C	15	12.25	2.75	2
D	17.5	15.25	2.25	3
E	17.1	13.5	3.6	1

66. Illustration

There are two mutual funds viz. D Mutual Fund Ltd. and K Mutual Fund Ltd. Each having close ended equity schemes.

NAV as on 31-12-2014 of equity schemes of D Mutual Fund Ltd. is Rs.70.71 (consisting 99% equity and remaining cash balance) and that of K Mutual Fund Ltd. is Rs.62.50 (consisting 96% equity and balance in cash).

Following is the other information:

Particulars	Equity Schemes	
	D Mutual Fund Ltd	K Mutual Fund Ltd.
Sharpe Ratio	2	3.3
Treynor Ratio	15	15
Standard Deviation	11.25	5

There is no change in portfolios during the next month and annual average cost is Rs.3 per unit for the schemes of both the Mutual Funds.

If Share Market goes down by 5% within a month, calculate expected NAV after a month for the schemes of both the Mutual funds.

For calculation, consider 12 months in a year and ignore number of days for particular month.

[ICAI SM, Similar Dec'21 QP 8 marks New & Old, RTP May'24, RTP Nov'20 New & Old, RTP May'19, MTP Apr'23, MTP Nov'21 New & Old, MTP Mar'18 Old, Old PM]

Solution:

	DMF	KMF
NAV	70.71	62.5
Equity	99%	96%
Cash	1%	4%
Equity	70.0029	60
Cash	0.7071	2.5

Sharpe Ratio: $(\frac{R_i-R_f}{S.D})$

For D Ltd

S, 2	$= \frac{R_i-R_f}{11.25}$	
$R_i - R_f$	= 11.25* 2 = 22.5	(1)

Substituting (1),

T, 15	$= \frac{R_i-R_f}{\beta}$	
15	= 22.5/ β	
β	= 22.5/15 = 1.5	(2)

For K Ltd,

S, 3.3	$= \frac{R_i-R_f}{5}$	
$R_i - R_f$	= 3.3* 5 = 16.51	(3)
T, 15	$= \frac{R_i-R_f}{\beta}$	
15	= 16.51/ β	
β	= 16.51/15.00 = 1.1	

If market goes down by 5%, value of D Ltd Portfolio goes down by β multiple of 5 i.e., 1.5*5= **7.5%** and for K Ltd, 1.1* 5= **5.5%.**

Revised NAV after one month

	D Ltd	K Ltd
Equity: Opening	70.0029	60
Less: Reduction in NAV	7.5%	5.5%
Closing Equity Value	**64.7527**	**56.7**
Cash: Opening	0.7071	2.5
Less: Management Expenses	-0.25	-0.25
Closing Cash value	**0.4571**	**2.25**
Closing NAV	64.7527+ 0.4571	56.7+ 2.25
	65.2098	**58.95**

67. Illustration

Ramesh wants to invest in stock market. He has got the following information about individual securities:

Security	Expected Return	Beta	σ^2 ci
A	15	1.5	40
B	12	2	20
C	10	2.5	30
D	09	1	10
E	08	1.2	20
F	14	1.5	30

Market index variance is 10 percent and the risk-free rate of return is 7%. What should be the optimum portfolio assuming no short sales?

(ICAI SM, Old PM)

Solution:

C_i = $\frac{\sigma_m^2 * \sum_{i=1}^{N}\left(\frac{(R_i - R_f)}{\sigma_{ei}^2} * \beta_i\right)}{1+\left(\sigma_m^2 * \sum_{i=1}^{N}\frac{\beta_i^2}{\sigma_{ei}^2}\right)}$

C^* = maximum value

Weights = $\frac{\beta_i}{\sigma_{ei}^2} * \left[\frac{R_i - R_f}{\beta_i} - C^*\right]$

Rank on the basis of Treynor= $\frac{R_i - R_f}{\beta_i}$

	$E(R)$	R_f	β	$(E(R) - R_f)/\beta$	Ranks
A	15	7	1.5	5.33333	1
B	12	7	2	2.5	3
C	10	7	2.5	1.2	5
D	9	7	1	2	4
E	8	7	1.2	0.83333	6
F	14	7	1.5	4.66667	2

	Rank	$R_f - R_i$ (1)	β (2)	σ_E^2 (3)	4=1*2/3	β^2	β^2/σ_E^2
A	1	8	1.5	40	0.3	2.25	0.05625
F	2	7	1.5	30	0.35	2.25	0.075
B	3	5	2	20	0.5	4	0.2
D	4	2	1	10	0.2	1	0.1
C	5	3	2.5	30	0.25	6.25	0.20833
E	6	1	1.2	20	0.06	1.44	0.072

Stock	Σ4	Σ6	C_i	C^*	
A	0.3	0.05625	(10*0.3)/(1+10*0.05625)	1.92	Optimal
F	0.65	0.13125	(10*0.65)/(1+10*0.13125)	2.811	c*
B	1.15	0.33125	(10*1.15)/(1+10*0.33125)	2.667	Ignored
D	1.35	0.43125	(10*1.35)/(1+10*0.41325)	2.630	
C	1.6	0.63958	(10*1.6)/(1+10*0.63958)	2.163	
E	1.66	0.71158	(10*1.66)/(1+10*0.71158)	2.045	

Securities F and A are part of Optimal portfolios

Z_A = $\frac{\beta_i}{\sigma_{ei}^2} * \left[\frac{R_i - R_f}{\beta_i} - C^*\right]$

= 1.5/40 (5.33- 2.811) = 0.0944625

Z_F = 1.5/30 (4.67- 2.811) = 0.09295

$Z_A + Z_F$ = 0.0944625 + 0.09295 = 0.1874125

Z_A weights = 0.0944625/ 0.1874125 = **50.40%**

Z_F weights = 100 - 50.40 = **49.60%**

The optimal portfolio with no short sales will have A and F ratio 50.40% and 49.60%

68. Illustration

Compute the Optimal portfolio out of following stocks as per Sharpe's optimal portfolio model if risk-free rate of return is 4%, Historical / Expected Market Return is 8% & Historical Market variance is 36%

The following data is given:

S. No	Name	Return	Beta β	Variance (σ^2)
1	A	-0.21	0.66	21.77
2	B	1.92	1.07	104.4
3	C	2.44	0.16	44.12
4	D	2.56	0.98	49.49
5	E	-0.43	1.01	37.00
6	F	3.18	0.77	31.04
7	G	1.69	0.15	48.30
8	H	2.69	0.77	34.63
9	I	0.39	0.88	63.60
10	J	3.15	0.89	60.34
	Mkt Return	2.18	1.00	18.52

(Source: https://www.researchgate.net/publication/336602910)

Solution:

R_f = 4% (expected)

$E(R)_m$ = 8%

σ_m^2 = 36% (historical)

C_i = $\dfrac{\sigma_m^2 * \sum_{i=1}^{N}\left(\frac{(R_i-R_f)}{\sigma_{ei}^2} * \beta_i\right)}{1+\left(\sigma_m^2 * \sum_{i=1}^{N}\frac{\beta_i^2}{\sigma_{ei}^2}\right)}$

C^* = maximum value

Weights = $\frac{\beta_i}{\sigma_{ei}^2} * \left[\frac{R_i-R_f}{\beta_i} - C^*\right]$

Rank on the basis of Treynor = $\frac{R_i-R_f}{\beta_i}$

$E(R)$ = $\propto + \beta * R_m + E_i$

$\propto$ = $AR - E(R)$

$\propto$ = $AR - \beta * R_m$

Numerator:

	AR	β	R_m	$= AR - \beta * R_m$	$\propto$
A	-0.21	0.66	2.18	-0.21-(0.66*2.18)	-1.65
B	1.92	1.07	2.18	1.92-(1.07*2.18)	-0.41
C	2.44	0.16	2.18	2.44-(0.16*2.18)	2.09
D	2.56	0.98	2.18	2.56-(0.98*2.18)	0.42
E	-0.42	1.01	2.18	-0.42-(1.01*2.18)	-2.62
F	3.18	0.77	2.18	3.18-(0.77*2.18)	1.50
G	1.69	0.15	2.18	1.69-(0.15*2.18)	1.36
H	2.69	0.77	2.18	2.69-(0.77*2.18)	1.01
I	0.39	0.88	2.18	0.39-(0.88*2.18)	-1.53
J	3.15	0.89	2.18	3.15-(0.89*2.18)	1.21

	$\propto$	β	R_m	= $\propto + \beta * R_m$	$E(R)$
A	-1.65	0.66	8	-1.6488+0.66*8	3.6312
B	-0.41	1.07	8	-0.4126+1.07*8	8.1474
C	2.09	0.16	8	2.0912+0.16*8	3.3712
D	0.42	0.98	8	0.4236+0.98*8	8.2636
E	-2.62	1.01	8	-2.6218+1.01*8	5.4582
F	1.50	0.77	8	1.5014+0.77*8	7.6614
G	1.36	0.15	8	1.363+0.15*8	2.563
H	1.01	0.77	8	1.0114+0.77*8	7.1714
I	-1.53	0.88	8	-1.5284+0.88*8	5.5116
J	1.21	0.89	8	1.2098+0.89*8	8.3298

	$R_s - R_f$	β	$R_s - R_f/\ \beta$	Rank
A	-0.3688	0.66	-0.5588	8
B	4.1474	1.07	3.87607	5
C	-0.6288	0.16	-3.93	9
D	4.2636	0.98	4.35061	3
E	1.4582	1.01	1.44376	7
F	3.6614	0.77	4.75506	2
G	-1.437	0.15	-9.58	10
H	3.1714	0.77	4.1187	4
I	1.5116	0.88	1.71773	6
J	4.3298	0.89	4.86494	1

	Variance	β	Systematic Risk ($\beta * \sigma_m^2$)	Unsystematic Risk
A	21.77	0.66	8.06731	13.70
B	104.4	1.07	21.2035	83.20
C	44.12	0.16	0.47411	43.65
D	49.49	0.98	17.7866	31.70
E	37	1.01	18.8923	18.11
F	31.04	0.77	10.9805	20.06
G	48.3	0.15	0.4167	47.88
H	34.63	0.77	10.9805	23.65
I	63.6	0.88	14.3419	49.26
J	60.34	0.89	14.6697	45.67

	$R_s - R_f$	σ_{EI}^2	β	$(R_s - R_f)*\beta/\ \sigma_E^2$	Cumulative ($\Sigma 1$)	$\Sigma 1$ * σ_m^2
J	4.33	45.6703	0.89	0.08438	0.08438	**3.024**
F	3.66	20.0595	0.77	0.14049	0.22487	**8.064**
D	4.26	31.7034	0.98	0.13168	0.35656	**12.816**
H	3.17	23.6495	0.77	0.10321	0.45977	**16.524**
B	4.15	83.1965	1.07	0.05337	0.51314	**18.432**
I	1.51	49.2581	0.88	0.02698	0.54012	**19.404**
E	1.45	18.1077	1.01	0.08088	0.62099	**22.32**
A	-0.37	13.7027	0.66	-0.0178	0.60317	**21.672**
C	-0.63	43.6459	0.16	-0.0023	0.60086	**21.60**
G	-1.44	47.8833	0.15	-0.0045	0.59635	**21.42**

Denominator:

	β	β^2	σ^2_{EI}	β^2/σ^2_m	$\Sigma 1$	$1 + \sigma^2_m * \Sigma 1$
J	0.89	0.7921	45.6703	0.017	0.17	**1.612**
F	0.77	0.5929	20.0595	0.030	0.047	**2.692**
D	0.98	0.9604	31.7034	0.030	0.077	**3.772**
H	0.77	0.5929	23.6495	0.025	0.102	**4.672**
B	1.07	1.1449	83.1965	0.014	0.116	**5.176**
I	0.88	0.7744	49.2581	0.016	0.132	**5.752**
E	1.01	1.0201	18.1077	0.056	0.188	**7.768**
A	0.66	0.4356	13.7027	0.032	0.22	**8.92**
C	0.16	0.0256	43.6459	0.001	0.221	**8.956**
G	0.15	0.0225	47.8833	0.001	0.222	**8.992**

Calculation:

	Numerator	Denominator	Numerator/Denominator	
J	3.024	1.612	1.87593	Part of Optimal Portfolio
F	8.064	2.692	2.99554	
D	12.816	3.772	3.39767	
H	16.524	4.672	3.53682	
B	18.432	5.176	3.56105	C^*
I	19.404	5.752	3.37344	Ignored
E	22.32	7.768	2.87333	
A	21.672	8.92	2.4296	
C	21.6	8.956	2.41179	
G	21.42	8.992	2.38212	

	$(R_s - R_f)*\beta$	$-C^*$	$*\beta/\ \sigma^2_E$	$= z_i$	Weights
J	4.87	-3.561	*0.89/45.67	0.026	22.03%
F	4.25	-3.561	*0.77/20.06	0.046	38.98%
D	4.35	-3.561	*0.98/31.70	0.024	20.34%
H	4.12	-3.561	*0.76/23.65	0.018	15.25%
B	3.88	-3.561	*1.07/83.20	0.004	3.39%
				0.118	**100.00%**

	Weights (1)	$\propto$ (2)	β (3)	R_m (4)	$E(R)$ (5) = (2) + (3) *(4)	War (1) * (5)
J	22.03%	1.21	0.89	8	8.33	1.83542
F	38.98%	1.5	0.77	8	7.66	2.9861
D	20.34%	0.42	0.98	8	8.26	1.68
H	15.25%	1.01	0.76	8	7.09	1.08153
B	3.39%	-0.41	1.07	8	8.15	0.27627
	100.00%				$E(R)_p$	**7.85932**

	x_i	β_i	$x_i\beta_i$	σ^2_e	$\sigma^2_e * {x_i}^2$
J	22.03	0.89	19.6067	45.67	2.21648
F	38.98	0.77	30.0146	20.06	3.04792
D	20.34	0.98	19.9332	31.70	1.31162
H	15.35	0.76	11.666	23.65	0.55724
B	3.4	1.07	3.638	83.20	0.09618
			84.8585		**7.22942**

$\beta_i x_i$ = 0.8485 = **0.85**

$(\beta_i x_i)^2 \sigma_m^2$ = $(0.85)^2$*36 = **26.01**

σ_p^2 = 26.01+7.22942 = **33.24**

	Market	Optimum Portfolio
$E(R)$	8	7.86
σ_p^2	36	33.24

69. Illustration

The following are the details of three mutual funds of MFL:

	Growth Fund	Balanced Fund	Regular Fund	Market
Average Return (%)	7	6	5	9
Variance	92.16	54.76	40.96	57.76
Coefficient of Determination	0.3025	0.6561	0.9604	

The yield on 182 days Treasury Bill is 9 per cent per annum. You are required to:

- Rank the funds as per Sharpe's measure.
- Rank the funds as per Treynor's measure.

Compare the performance with the market.

(Nov-24 Similar 8M, Nov'20 QP 8 marks, MTP Apr'23)

Solution:

Particulars	Growth Fund	Balanced Fund	Regular Fund	Market
R_s	7	6	5	9
σ^2	92.16	54.76	40.96	57.76
S. D	9.6	7.4	6.4	7.6
r^2	0.3025	0.6561	0.9604	
r	0.55	0.81	0.98	
R_f	9	9	9	9
$R_s - R_f$	-2	-3	-4	0
$\beta = r * \sigma_s/\sigma_m$	0.6947	0.7887	0.8253	1
Sharpe Ratio: $(R_s - R_f)/\ S.D$	-0.2083	-0.4054	-0.6250	0
Ranks	1	2	3	
Treynor Ratio $(R_s - R_f)/\beta$	-2.8788	-3.8038	-4.8469	0
Ranks	1	2	3	

The market has Sharpe Ratio and Treynor Ratio values at 0, higher than all three funds.
The market has performed better than all three funds.

70. Illustration

Equity of ABC Ltd. (ABCL) is ₹ 500 Crores, its debt, is worth ₹ 290 Crores.
Printer Division segments value is attributable to 64%, which has an Asset Beta (β_p) of 1.55, balance value is applied on Spares and Consumables Division, which has an Asset Beta (β_sc) of 1.40 ; ABCL Debt beta (β_D) is 0.28. You are required to calculate:

(i) Equity Beta (β_E),

(ii) Ascertain Equity Beta (β_E), if ABC Ltd. decides to change its Debt Equity position by raising further debt and buying back of equity to have its Debt-to-Equity Ratio at 1.50.
Assume that the present Debt Beta (β_D1) is 0.45 and any further funds raised by way of Debt will have a Beta (β_D2) of 0.50.

(iii) Whether the new Equity Beta (β_E) justifies increase in the value of equity on account of leverage?

(RTP May'21 New & Old)

Solution:

(i)

		Assets (%)	β	Weighted β
Printer		64	1.55	0.992
Spares		36	1.4	0.504
		100		**1.496**

Asset Beta = 1.496

$\beta_A = \frac{E}{E+D} * \beta_E + \frac{D}{E+D} * \beta_D$

$1.496 = \frac{500}{790} * \beta_E + \frac{290}{790} * 0.28$

$1.496 = 0.6329 * \beta_E + 0.1028$

$0.6329 * \beta_E = 1.496 - 0.1028$

$\beta_E = \mathbf{2.2013}$

(ii) Target Debt Equity Ratio is 1.5:1

$\frac{D}{E} = \frac{1.5}{1}$

D = 1.5 E

D+ E = 790

1.5 E + E = 790

2.5 E = 790

E = **316**

D = 790- 316 = **474**

	Existing	Revised	Change
Debt	290	474	+184
Equity	500	316	-184
Total	790	790	**0**
D/E	0.28	1.5	

(ii)

β_D for 290 is 0.45

β_D for 474- 290= 184 is 0.50

$\beta_A = \frac{E}{E+D1+D2} * \beta_E + \frac{D1}{E+D1+D2} * \beta_{D1} + \frac{D2}{E+D1+D2} * \beta_{D2}$

$1.496 = \frac{316}{790} * \beta_E + \frac{290}{790} * 0.45 + \frac{184}{790} * 0.5$

$1.496 = 0.4 * \beta_E + 0.2817$

$0.4 * \beta_E = 1.496 - 0.2817$

$\beta_E = 3.0358$

(iii)Yes, the new beta justifies increase in value of equity when market moves positively.

71. Illustration

K Ltd. has invested in a portfolio of short-term equity investments. You are required to calculate the risk of K Ltd.'s short-term investment portfolio relative to that of the market from the information given below:

Investment	A	B	C	D
No. of shares	1,20,000	1,60,000	2,00,000	2,50,000
Market price per share (₹)	8.58	5.84	4.34	6.28
Beta	2.32	4.56	1.80	3.00
Actual Return	9.50%	14.00%	7.50%	16.00%

The current market return is 20% and the risk-free return is 10%.
Advise whether K Ltd. should change the composition of its portfolio. If yes, then how.
Note: Make calculations up to 4 decimal points.

(RTP May'21 New & Old)

Solution:

	Shares	CMP	Value	Beta	Value* Beta
A	1.2	8.58	10.296	2.32	23.88672
B	1.6	5.84	9.344	4.56	42.60864
C	2	4.34	8.68	1.8	15.624
D	2.5	6.28	15.7	3	47.1
			44.02		**129.2194**

β_p $= \sum x_i \beta_i$

$= \frac{129.2194}{44.02} = 2.9355$

Compared to the market, the portfolio has higher beta of 2.9355

	Value	₹	Value* ₹
A	10.296	9.5	97.812
B	9.344	14	130.816
C	8.68	7.5	65.1
D	15.7	16	251.2
	44.02	**12.2805**	**544.928**

Actual Return = 12.28

$E(R)$ $= R_f + \beta\,(MRP)$

= 10% + 2.9355 (10%) = **39.355**

The portfolio significantly underperformed the market.

	AR	$R_f + \beta(MRP)$	$E(R)$	$AR-\ E(R)$
A	9.5	10+ 2.32(10)	33.20%	-23.70%
B	14	10+ 4.56(10)	55.60%	-41.60%
C	7.5	10+ 1.8(10)	28%	-20.50%
D	16	10+ 3(10)	40%	-24%

Yes. None of the stocks have performed or reached their E(R) as per CAPM. They all have significantly underperformed and so has the overall portfolio. Hence all the stocks should be removed from the portfolio and new stocks should be added.

72. Illustration

Suppose one of your HNI clients is holding the following portfolio as per his risk appetite:

Particulars	Securities
Equity Shares:	
G Ltd.	1000
S Ltd.	1000
B Ltd.	500
PSU Bonds	20,000

The other data related to each of these securities is as follows:

Cost	Dividends/ Interest	Market price	Beta
₹	₹	₹	
10,000	1,725	9,800	0.60
15,000	1,000	16,200	0.80
28,000	1400	28,300	0.60
1,800	180	1,725	0.10

Your client is interested in investing some more funds in Bonds issued by GOI.

(i) Estimate the minimum rate of return that your client would expect from these Bonds keeping in view his risk appetite and assuming Market Return as 12.70%.

(ii) Analyze whether this portfolio has out-performed the market or not assuming Risk Free Rate of Return as 7%.

(MTP Nov'21 New & Old)

Solution:

(i) Minimum Return Expectation is Risk-free rate or can also be bond coupon rate (10%)

As per CAPM,

$$E(R) = R_f + \beta\,(R_m - R_f)$$

Securities	Cost	Number	Value	M.P	Gain or Loss
G	10000	1000	100	9800	-200
S	15000	1000	150	16200	1200
B	28000	500	140	28300	300
Bonds	1800	20000	360	1725	-75
			750		

Securities	Total Gain	Dividend/ Interest	Total Dividend/ Interest	Gain+ Dividend
G	-200000	1725	17.25	15.25
S	1200000	1000	10	2.2
B	150000	1400	7	8.5
Bonds	-1500000	180	36	21
	-350000		**70.25**	**66.75**

Actual Return $= \dfrac{\text{capital gain+ dividend}}{\text{opening value}}$

$= \dfrac{66.75}{750} = \mathbf{8.9\%}$

Securities	Cost Value	Beta	Value* Beta
G	100	0.6	60
S	150	0.8	120
B	140	0.6	84
Bonds	360	0.1	36
	750		300

$\beta_p = \sum_{i=1}^{n} x_i \beta_i$

$= \frac{300}{750} = 0.40$

Assuming the portfolio returns follow CAPM

$E(R) = R_f + \beta(R_m - R_f)$

8.9 $= R_f$ + 5.08- 0.4 R_f

8.09 - 5.08 = 0.6 R_f

$\boldsymbol{R_f}$ **= 6.37%**

Minimum return expected is 6.37% R_f or if annual coupon is 10% then that value.

(ii) R_f = 7%

R_m = 12.7%

β = 0.4

$E(R) = R_f + \beta(R_m - R_f)$

= 7+ 0.4 (12.7- 7)

= 7+ 2.28% = **9.28%**

Actual Return = **8.90%**

The portfolio underperformed with respect to the market return of 7% and its own beta of 0.4 as it generated 8.9% v/s expected return of 9.28%.

73. Illustration

Blue Tooth Mutual Fund is planning to float a fixed income fund of Rs.100 crore on 1 January 2015 with a term of 7 years.

If the target duration of fund is $5\frac{1}{2}$ years and has expected rate of return of 8.00%, then determine the amount of interest it has to earn annually on its investment after defraying management expenses of 10% of amount income earned.

Solution:

Let coupon/ interest p.a be x% of 100

Year	Coupon/ CF	PVF @8%	Time (T)	$PV * CF * T$
1	X	0.926	1	0.926x
2	X	0.857	2	1.714x
3	X	0.794	3	2.382x
4	X	0.735	4	2.94x
5	X	0.68	5	3.4x
6	X	0.63	6	3.78x
7	x+100	0.583	7	4.081x+ 408.1
				19.223x+ 408.1

Duration $= \frac{\sum PV * CF * T}{P_0}$

5.5 $= \frac{19.223x + 408.1}{100}$

550- 408.1 = 19.223x

x $= \frac{141.9}{19.223}$ = **7.38%**

Annual return required post expenses is 7.38%

Pre-Expense Return,

$= \frac{7.38\%}{0.9}$ = **8.20%**

74. Illustration

Indira has a fund of ₹ 3 lacs which she wants to invest in share market with rebalancing target after every 10 days to start with for a period of one month from now. The present NIFTY is 5326. The minimum NIFTY within a month can at most be 4793.4. She wants to know as to how she should rebalance her portfolio under the following situations, according to the theory of Constant Proportion Portfolio Insurance Policy, using "2" as the multiplier:

(1) Immediately to start with.

(2) 10 days later-being the 1st day of rebalancing if NIFTY falls to 5122.96.

(3) 10 days further from the above date if the NIFTY touches 5539.04.

For the sake of simplicity, assume that the value of her equity component will change in tandem with that of the NIFTY and the risk-free securities in which she is going to invest will have no Beta.

(RTP Nov'23, RTP May'19, RTP Nov'24, MTP Oct'23, MTP Apr'23, Old PM)

Solution:

Current NIFTY = 5326

Max Loss = 10% (532.6)

Max Loss Level (NIFTY) = 4793.4

Assuming floor is at 10% level of loss from current level;

Current Value of Portfolio = ₹ 3,00,000

Floor = 90% of Portfolio Value = ₹ **2,70,000**

M = 2

F = 2.7

P. V = 3

S i.e., investment in Equity Shares

= 2* (3,00,000- 2,70,000)

= 2* 30,000 = ₹ **60,000**

Debt Investment = ₹ 2,40,000 i.e (3,00,000- 60,000)

Day	NIFTY	Equity	Debt	Equity Value	Debt Value	Portfolio
0	5326	20%	80%	60000	240000	300000
11	5122.96	19.39%	80.61%	57713	240000	297713
11 (After Reb) *	5122.96	18.61%	81.39%	55426	242287	297713
21	5539.04	19.83%	80.17%	59928	242287	302215
21 (After Reb) *	5539.04	21.32%	78.68%	64430	237785	302215

Working Notes:

1. Value of Equity on Day 11 when NIFTY falls to 5122.96 is
   = ₹ 60000* 5122.96/ 5326 = **57713**
2. After rebalancing on Day 11
   Equity Shares = 2* (297713- 270000) = **55426**
   On Day 11 Indira will have to rebalance the portfolio by withdrawing money of ₹ 2287 from equity investment and putting it in debt or risk-free securities.
3. Value of Equity portfolio when NIFTY rises from 5122.96 to 5539.04
   = 55426* 5539.04/ 5122.96 = **59928**
4. Equity Value = M(P-F)
   = 2 (302215- 27000) = **64430**

On Day 11 Indira will have to rebalance the portfolio by **withdrawing money of ₹ 2287 from equity investment** and putting it in debt or risk-free securities.

On Day 21 Indira will rebalance by **investing more in Equity i.e., Rs.4502 by withdrawing** this money from debt or risk-free securities.

75. Illustration

Ms. Kiran had a surplus fund of ₹ 2,00,000 on 31.03.2016. She is interested in constructing a portfolio of shares of the core sectors to be weighted equally in rupee value terms. Her friend Shaila based on her research advised her to purchase following shares:

Company	No. of Shares	Price Per Share
O Ltd.	100	400
H Ltd.	1000	40
A Ltd.	320	125
R Ltd.	400	100
T Ltd.	200	200

On April 1, 2016, the prices of these stocks were as follows:

Company	Price Per Share
O Ltd.	300
H Ltd.	60
A Ltd.	120
R Ltd.	150
T Ltd.	125

You are required to exhibit how Kiran can rebalance her portfolio on 1.4.2016 so that her exposure to individual stock is maintained at original level in terms of rupee value.

(MTP Mar'18 Old)

Solution:

On **31/03/16;**

Company	Shares	Inv. Price	Value
O	100	400	40000
H	1000	40	40000
A	320	125	40000
R	400	100	40000
T	200	200	40000
			200000

Ms Kiran invested her surplus equally in each of the securities.

On **01/04/16;**

Company	Shares	Market price	Market Value	Desired Value	Action
O	100	300	30000	42680	Buy- 12680
H	1000	60	60000	42680	Sell- 17320
A	320	120	38400	42680	Buy- 4280
R	400	150	60000	42680	Sell- 17320
T	200	125	25000	42680	Buy- 17680
			213400		0

* Desired Value per security: Ms Kiran rebalances the portfolio in such a way that the total market value in each security is equal i.e., 2,13,400/5 = 42680.

Company	Action	Total Value	Price per Share	Shares sold/bought
O	Buy	12680	300	42.27
H	Sell	-17320	60	-288.67
A	Buy	4280	120	35.67
R	Sell	-17320	150	-115.47
T	Buy	17680	125	141.44
		0		

Post Rebalancing Portfolio of Ms Kiran:

Company	Shares	Price per Share	Market Value
O	142.27	300	42680
H	711.33	60	42680
A	355.67	120	42680
R	284.53	150	42680
T	341.44	125	42680

76. Illustration

Ms. Sunidhi is working with an MNC at Mumbai. She is well versant with the portfolio management techniques and wants to test one of the techniques on an equity fund she has constructed and compare the gains and losses from the technique with those from a passive buy and hold strategy. The fund consists of equities only and the ending NAVs of the fund he constructed for the last 10 months are given below:

Month	Ending NAV (₹/unit)	Month	Ending NAV (₹/unit)
December 2008	40.00	May 2009	37.00
January 2009	25.00	June 2009	42.00
February 2009	36.00	July 2009	43.00
March 2009	32.00	August 2009	50.00
April 2009	38.00	September 2009	52.00

Assume Sunidhi had invested a notional amount of ₹ 2 lakhs equally in the equity fund and a conservative portfolio (of bonds) in the beginning of December 2008 and the total portfolio was being rebalanced each time the NAV of the fund increased or decreased by 15%.

You are required to determine the value of the portfolio for each level of NAV following the Constant Ratio Plan.

Are there any errors in the technique developed by Sunidhi? If So please explain?

(ICAI SM, Nov'22 QP 8 marks, MTP Oct'21 New & Old)

Solution:

Constant Mix Policy

Period	E:D	Equity	Debt	PV	NAV	Units
01-12-2008	50:50	100000	100000	200000	40	2500
31-12-2008	50:50	100000	100000	200000	40	2500
Before Rebalance 31/01/2009	38:62	62500	100000	162500	25	2500
After Rebalance 31/01/2009	50:50	81250	81250	162500	25	3250
Before Rebalance 28/02/2009	59:41	117000	81250	198250	36	3250
After Rebalance 28/02/2009	50:50	99125	99125	198250	36	2753.47
31/03/2009 - No action	47:53	88111.04	99125	187236.04	32	2753.47
Bef Rebal (Apr)	51:49	104631.86	991250	203765.86	38	2753.47
Aft Rebal (Apr)	50:50	101878.43	101878.4	203756.86	38	2681.01
May	49:51	99197.37	101878.4	201075.8	37	2681.01
June	53:47	112602.42	101878.4	214480.85	42	2681.01
July	53:47	115283.43	101878.4	217161.86	43	2681.01
Bef Rebal (Aug)	57:43	134050.5	101878.4	235928.93	50	2681.01
Aft Rebal (Aug)	50:50	117964.47	117964.5	235928.94	50	2359.29
September	51:49	122683.08	117964.5	**240647.55**	52	2359.29

BUY And HOLD

Month	Equity	Debt	NAV	Units	PV
Dec	100000	100000	40	2500	200000
Jan	62500	100000	25	2500	162500
Feb	90000	100000	36	2500	190000
Mar	80000	100000	32	2500	180000
Apr	95000	100000	38	2500	195000
May	92500	100000	37	2500	192500
Jun	105000	100000	42	2500	205000
Jul	107500	100000	43	2500	207500
Aug	125000	100000	50	2500	225000
Sep	130000	100000	52	2500	**230000**

Value of Buy and Hold Portfolio with 50:50 Equity and Debt is ₹ 230000 v/s ₹ 240647.55 in constant mix policy

BUY and HOLD with E:D as 100:0

Month	NAV	Equity	Units
Dec	40	200000	5000
Jan	25	125000	5000
Feb	36	180000	5000
Mar	32	160000	5000
Apr	38	190000	5000
May	37	185000	5000
Jun	42	210000	5000
Jul	43	210000	5000
Aug	50	215000	5000
Sep	52	260000	5000

If Buy and Hold of E:D 100:0 is implemented, final portfolio is ₹ 260000 v/s ₹240647.55 in constant mix and ₹ 230000 in Buy and Hold E:D 50:50 rebalancing policies. The 100:0 E:D outperforms the other two strategies.

There are no errors. However, if the market is expected to be in the bull phase, then 100% equity allocation with Buy & Hold will give best results and if markets are expected to be in the unknown territory then, 50:50 constant mix policy with give both upside & also protect the downside. If market is in recessionary phase, then Buy & Hold of 100% Debt would be best suited.

77. Illustration

Mr. A is having 1 lakh shares of K Ltd. The beta of the company is 1.40. Mr. B a financial advisor has suggested having the following portfolio:

Security	Beta	% holding
L	1.20	10
M	0.75	10
N	0.40	30
O	1.40	50
		100

Market Return is 12%
Risk free rate is 8%

You are required to calculate the following for the present investment and suggested portfolio:

(i) What is the expected return based on CAPM and also
(1) If the market goes upby 2.5%.
(2) If the market goes down by 2.5%.
(3) If the market is giving a negative return of 2.5%.

(ii) If the probability of market giving negative return is more, please advise Mr. A whether to continue the holdings of M/s. K Ltd. or to buy the portfolio as per the suggestion of Mr. B. If so, why?

[RTP May'22, MTP Oct'22, Jul'21 QP (Old)]

Solution:

Security	(β)	% Holding	$\beta * \%$
L	1.2	10%	0.12
M	0.75	10%	0.075
N	0.4	30%	0.12
O	1.4	50%	0.7
			1.015

$E(R)$ = $R_f + \beta(MRP)$

R_m = 12%

R_f = 8%

MRP = $R_m - R_f$ = 4%

Return of K Ltd. As per CAPM,

β = 1.4

$E(R)$ = 8%+ 1.4* (4%) = **13.6%**

Return of Portfolio,

= 8%+ 1.015* (4%) = **12.06%**

(i) When R_m increases by 2.5%

Return of K Ltd.

$E(R)$ = 8%+ 1.4* (12%+ 2.5%- 8%)

= 8%+ 1.4* (6.5%) = **17.1%**

Return of Portfolio,

= 8%+ 1.015* (6.5%) = **14.5975%**

(ii) When R_m changes from 12% to 9.5% i.e., 2.5% decrease

Return of K Ltd.

$E(R)$ = 8%+ 1.4* (12%- 2.5%- 8%)

= 8%+ 1.4* (1.5%) = **10.1%**

Return of Portfolio,

= 8%+ 1.015* (1.5%) = **14.5975%**

(iii) When market return is -2.5%

Return of K Ltd.

$E(R)$ = 8%+ 1.4* (- 2.5%- 8%)

= 8%+ 1.4* (-10.5%) = **- 6.7%**

Return of Portfolio,

= 8%+ 1.015* (-10.5%) = **- 2.6575%**

If market is expected to generate negative returns, then portfolio of L, M, N, O is giving lower losses @ 2.6575% v/s loss of 6.71% in case of stock k. So, it is better to go with the portfolio of stocks suggested by B as this portfolio has lower Beta and hence when market gives negative returns its returns are not too different as compared to K Ltd. which has a higher Beta and hence leads to higher losses when market has negative returns.

78. Illustration

Mayuri is interested to construct a Portfolio of Securities X and Y. She has collected the following information:

	X	Y
Expected Return (ER)	19%	23%
Risk (σ)	14%	18%

Mayuri has 5 Portfolio options of X and Y as follows:

(i) 50% of funds in each X and Y
(ii) 75% of funds in X and 25% in Y
(iii) 25% of funds in X and 75% in Y
(iv) 60% of funds in X and 40% in Y
(v) 35% of funds in X and 65% in Y

Suppose if Co-efficient of correlation (r) between X and Y is 0.16, you are required to calculate:

(i) Expected Return under different Portfolio Options.
(ii) Risk Factor associated with these Portfolio Options.
(iii) Which Portfolio is best from the point of view of Risk?
(iv) Which Portfolio is best from the point of view of Return?

[RTP May'22, MTP Oct'21 New & Old, Jan'21 QP (Old)]

Solution:

Weighted Average,

WAR = $W_A K_A + W_B K_B$

${\sigma_p}^2$ = $(W_A * \sigma A)^2 + (W_B * \sigma B)^2 + 2\,W_A W_B\,\sigma A\,\sigma B\,r$

$E(R)_X$ = 19%

$E(R)_Y$ = 23%

σX = 14%

σY = 18%

r = 0.16

Option 1:

W_x = 50%

W_Y = 50%

$E(R)_p$ = 0.5*19% + 0.5*23%
= 9.5+ 11.5 = **21%**

${\sigma_p}^2$ = $(0.5)^2(14)^2 + (0.5)^2(18)^2$ + 2*14*18*0.5*0.5*0.16
= 49+ 81+ 20.16 = **150.16**

σ_p = $\sqrt{150.16}$ = **12.25%**

Option 2:

W_x = 75%

W_Y = 25%

$E(R)_p$ = (0.75*19%) + (0.25*23%) = **20%**

${\sigma_p}^2$ = $(0.75)^2(14)^2 + (0.25)^2(18)^2$ + 2*14*18*0.75*0.25*0.16
= 110.25+ 20.25+ 15.12 = **145.62**

σ_p = $\sqrt{145.62}$ = **12.067%**

Option 3:

W_x = 25%

W_Y = 75%

$E(R)_p$ = (0.25*19%) + (0.75*23%) = **22%**

${\sigma_p}^2$ = $(0.25)^2 (14)^2 + (0.75)^2 (18)^2$ + 2*14*18*0.25*0.5*0.16

= 12.25+ 182.25+ 15.12 = **209.62**

σ_p = $\sqrt{209.62}$ = **14.478%**

Option 4:

W_x = 60%

W_Y = 40%

$E(R)_p$ = (0.60*19%) + (0.40*23%) = **20.6%**

${\sigma_p}^2$ = $(0.60)^2 (14)^2 + (0.40)^2 (18)^2$ + 2*14*18*0.60*0.40*0.16

= 70.56+ 51.84+ 19.3536 = **141.7536**

σ_p = $\sqrt{141.7536}$ = **11.906%**

Option 5:

W_x = 35%

W_Y = 65%

$E(R)_p$ = (0.35*19%) + (0.65*23%) = 21.6%

${\sigma_p}^2$ = $(0.35)^2 (14)^2 + (0.65)^2 (18)^2$ + 2*14*18*0.35*0.65*0.16

= 24.01+ 136.89+ 18.3456 = **179.24**

σ_p = $\sqrt{179.24}$ = **13.388%**

Options	**1**	**2**	**3**	**4**	**5**
$E(R)_p$	21	20	22	20.6	21.6
σ_p	12.25	12.067	14.478	11.906	13.388

Portfolio 3 has highest return and Portfolio 4 has lowest risk, these are best suited from return and risk perspective respectively.

79. Illustration (Critical Line Theory)

An investor has two portfolios known to be on minimum variance set for a population of three securities A, B and C having below mentioned weights:

	WA	WB	WC
Portfolio X	0.30	0.40	0.30
Portfolio Y	0.20	0.50	0.30

It is supposed that there are no restrictions on short sales.

(2) What would be the weight for each stock for a portfolio constructed by investing? ₹5,000 in portfolio X and ₹3,000 in portfolio Y?

(3) Suppose the investor invests ₹4,000 out of ₹8,000 in security A. How will he allocate the balance between security B and C to ensure that his portfolio is on minimum variance set?

(ICAI SM, MTP Apr'24, Old PM)

Solution

1.

	X		Y		X+Y	Ratio	Weights
A	0.3	1500	0.2	600	2100	21	26.25%
B	0.4	2000	0.5	1500	3500	35	43.75%
C	0.3	1500	0.3	900	2400	24	30%
	1	5000	1	3000	8000	80	

The weights of stocks A, B and C in combined portfolio is **26.35: 43.75:30**.

2.

w_B = $\alpha + w_A \beta$ (We are trying to find WB because we already know WA at 50% i.e 4000/8000)

y = $C + mx$

For Portfolio X, 0.4 = $\alpha + 0.3\,\beta$ **(a)**

For Portfolio Y, 0.5 = $\alpha + 0.2\,\beta$ **(b)**

(b) -(a),

0.1 = $-0.1\,\beta$

β = -1

Substituting β = -1 in Equation **(a)**,

0.4 = $\alpha - 0.3 * 1$

α = 0.7

₹ 4,000 invested in A i.e., 50%,

w_A = $\alpha + w_B\,\beta$

0.5 = $0.7 - w_B * 1$

w_B = 0.2 = 20%

$20\% * 8000$ = **1,600**

w_C = 0.3 = 30%

$30\% * 8000$ = **2,400**

80. Illustration- Sharp & Treynor Ratios

Following is the information related to three mutual funds:

Year	MF-A	MF-B	MF-C
2020	10%	5%	14%
2021	8%	10%	10%
2022	12%	8%	18%

Correlation between market and mutual fund:

	MF-A	MF-B	MF-C
Correlation with market	0.45	0.25	0.65

The variance of the market is 9% and rate of return of government bond is 7%. You are required to Rank the Mutual fund using Sharpe's ratio and Treynor's ratio.

(Nov'22 QP 8 marks)

Solution

	A	B	C	$A-\bar{A}$	$(A-\bar{A})^2$	$B-\bar{B}$	$(B-\bar{B})^2$	$C-\bar{C}$	$(C-\bar{C})^2$
2020	10	5	14	0	0	2.67	7.1289	0	0
2021	8	10	10	-2	4	2.33	5.4289	-4	16
2022	12	8	18	2	4	0.33	0.1089	4	16
	30	23	42		**8**		**12.67**		**32**
Average	10%	7.67%	14%						
σ					$\sqrt{\frac{8}{3}}$ **= 1.63**		$\sqrt{\frac{12.67}{3}}$ **= 2.05**		$\sqrt{\frac{32}{3}}$ **= 3.27**

	A	B	C
(1) Correlation	0.45	0.25	0.65
σ_m^2	9	9	9
(2) σ_m	3	3	3
(3) σ_p	1.63	2.05	3.27
$\beta = (1) * \frac{(3)}{(2)}$	0.2445	0.1708	0.708

	A	B	C
(1) R_p	10	7.67	14
(2) R_f	7	7	7
(3) σ_p	1.63	2.05	3.27
(4) β	0.2445	0.1708	0.708
Sharpe Ratio $= \frac{(1)-(2)}{(3)}$	**1.84**	**0.33**	**2.14**
Rank	**2**	**3**	**1**
Treynor Ratio= $\frac{(1)-(2)}{(4)}$	**12.26**	**3.92**	**9.88**
Rank	**1**	**3**	**2**

81. Illustration - Portfolio Return & Beta

Following is the information related to return on shares of three different companies:

Years	A Ltd.	B Ltd.	C Ltd.
2018	2%	3%	5%
2019	6%	8%	7%
2020	13%	14%	15%
2021	7%	9%	11%

Required:

a. Construct maximum number of portfolio and its return, if each portfolio consists of any two Company's shares in proportion of 65% and 35% and suggest which portfolio provides highest return.

b. Calculate portfolio return and beta (β), if Mr. X invests ₹ 65,000 in A Ltd. having beta (β) of 0.45; ₹ 20,000 in B Ltd. having beta (β) of 1.15 and ₹15,000 in C Ltd. having beta (β) of 1.8

(Nov'22 QP 8 marks)

Solution:

	A	B	C
2018	2	3	5
2019	6	8	7
2020	13	14	15
2021	7	9	11
	28	34	38
Average	**7%**	**8.5%**	**9.5%**

	A	B	C	Return
1	0	65	35	(0*7)+ (0.65*8.5)+ (0.35*9.5)= **8.85**
2	0	35	65	(0*7)+ (0.35*8.5)+ (0.65*9.5)= **9.15**
3	65	0	35	(0.65*7)+ (0*8.5)+ (0.35*9.5)= **7.875**
4	35	0	65	(0.35*7)+ (0*8.5)+ (0.65*9.5)= **8.625**
5	65	35	0	(0.65*7)+ (0.35*8.5)+ (0*9.5)= **7.525**
6	35	65	0	(0.35*7)+ (0.65*8.5)+ (0*9.5)= **7.975**

The best performing portfolio consists of **Stock B @ 35% and Stock C at 65%.**

b.

		Return (1)	Beta (2)	Weights (3)	Weighted Average Return (1)*(3)	Weighted Average Beta (2)*(3)
A	65,000	7	0.45	0.65	4.55	0.2925
B	20,000	8.5	1.15	0.2	1.7	0.23
C	15,000	9.5	1.8	0.15	1.425	0.27
					7.675	**0.7925**

The **portfolio return** consisting of Stocks A ,B and C in ratio of 65,000: 20,000: 15,000 is **7.675%** and **Beta is 0.7925.**

82. Illustration

Mr. Potential has made investments in two mutual funds. The following information is available:

Mutual Fund	Smart	Growth
Jensen Alpha	1.10%	1.50%
Treynor's Ratio	0.0714	0.0775
Actual Return	8.50%	9.10%
Risk Premium	4%	

You are required to calculate:

(i) Beta for both the funds

(ii) Risk free Rate

(iii) Security Market Line

(May 23'QP 8 marks)

Solution

(i)

Jenson's Alpha = Actual Return – Expected Return

For Smart:

1.1% = 8.5% - Expected Return

Expected Return = 8.5- 1.1 = 7.4 %

For Growth:

1.5% = 9.1% - Expected Return

Expected Return = 9.1- 1.5 = 7.6 %

Treynor Ratio $= \frac{Actual\ Return - Risk\ Free\ Rate}{\beta}$

For Smart:

$0.0714 = \frac{8.5\% - Rf}{\beta}$

$\beta = \frac{8.5\% - Rf}{0.0714}$ **(1)**

For Growth:

$0.0775 = \frac{9.1\% - R_f}{\beta}$

$\beta = \frac{9.1\% - R_f}{0.0775}$ **(2)**

CAPM,

Expected Return $= R_f + \beta(MRP)$

$= R_f + \beta(4\%)$

For Smart:

$7.4\% = R_f + \beta(4\%)$

$\beta = \frac{7.4\% - R_f}{4\%}$ **(3)**

For Growth:

$7.6\% = R_f + \beta(4\%)$

$\beta = \frac{7.6\% - R_f}{4\%}$ **(4)**

Equating **(1)** and **(3)**,

$\frac{8.5\% - R_f}{0.0714} = \frac{7.4\% - R_f}{4\%}$

0.0034 - 0.04 R_f = 0.0052836 - 0.0714 R_f

0.0034 - 0.0052836 = 0.04 R_f - 0.0714 R_f

$R_f = \frac{0.0018836}{0.0314}$ = **0.06**

(ii) For Smart, Substituting Value of R_f = 0.06 in Equation **(1)**

$\beta = \frac{8.5\% - 0.06}{0.0714} = \frac{2.5\%}{0.0714}$ = **0.35**

For Growth, Substituting Value of R_f = 0.06 in Equation **(2)**

$\beta = \frac{9.1\% - 0.06}{0.0775} = \frac{3.1\%}{0.0775}$ = **0.4**

(iii) SML i.e., CAPM

Smart: $= R_f + \beta(MRP)$

$= R_f + \beta(R_m - R_f)$

$= 6\% + 0.35(R_m - 6\%)$

$MRP = R_m - R_f$

$4\% = R_m - 6\%$

R_m = 10%

SML for Smart $= 6\% + \mathbf{0.35}\ (\boldsymbol{Rm} - 6\%)$ **&** **SML for Growth** $= 6\% + \mathbf{0.40}\ (\boldsymbol{Rm} - 6\%)$

83. Illustration

An Investor is proposing to invest 10,000/- in two Portfolios A and B in the ratio of 3:2. The Portfolios have three securities each with following weights:

	Wx	Wy	Wz
Portfolio A	0.30	0.25	0.45
Portfolio B	0.20	0.45	0.35

You are requested to

(i) Calculate the weight of each stock

(ii) Calculate the amount allocated to securities y & z if half of the funds are allocated to security x.

(May'23 QP 8 marks)

Solution:

i)

			x	y	z	
A	3	6,000	0.30 1,800	0.25 1,500	0.45 2,700	6,000
B	2	4,000	0.20 800	0.45 1,800	0.35 1,400	4,000
	5	10,000	**2,600**	**3,300**	**4,100**	

Allocation amongst securities x,y,z is in the ratio 26: 33: 41 for an amount of ₹ 10,000.
Weight of each security in the combined portfolio is 26: 33: 41.

w_y $= a + b\, w_x$

Portfolio A: 0.25 $= a + b * 0.3$

Portfolio B: 0.45 $= a + b * 0.2$

A - B: - 0.20 $= b * 0.1$

$b = -\frac{0.20}{0.1}$

b = - 2

Substituting, b = - 2 in equation of Portfolio A,

0.25 $= a - 2 * 0.3$

0.25 $= a - 0.6$

a = 0.85

w_x in new portfolio is 50 %,

$w_y = a + b\, w_x$

$= 0.85 - 2 * 0.5$

$= 0.85 - 1$ = - 0.15

Also,

$w_x + w_y + w_z = 1$

$w_z = 1 - w_x - w_y$

= 1 - 0.5 + 0.15 = 0.65

Amount Allocated

x = 0.5* 10,000 =**₹ 5,000**

y = -0.15* 10,000 = **-₹ 1,500**, shorting

x = 0.65* 10,000 = **₹ 6,500**

84. Illustration

An investor has categorized all the available stock in the market into the following types and the estimated weights of the categories of stocks in the market index are given below. Further, the sensitivity of returns of these categories of stocks to two factors Inflation and Stock market are also given below:

Category	Weight in Market Index	Factor 1 (Inflation)			Factor 2 (Stock Market)		
		Beta 1	Expected Value in %	Actual Value in %	Beta 2	Expected Value in %	Actual Value in %
Small Cap	20%	1.20	6.70	6.70	0.80	10.00	10.50
Medium Cap	30%	1.75	4.50	6.00	0.90	7.00	8.00
Large Cap	15%	1.30	6.75	8.00	1.165	9.00	10.00
Flexi Cap	35%	1.70	7.00	6.50	0.85	8.85	9.75

Risk Free Rate of Interest is 7.50%. Round off to 2 decimal.
You are required to calculate:

(i) Expected return on the market index for both the factors.
(ii) Expected return on the market index under Arbitrage Pricing Theory (Existing Scenario).
(iii) Expected return on the market index under Arbitrage Pricing Theory if the composition of the Portfolio is changed to 25% equally in all four categories.
(iii) Which alternative (Existing or Changed) will be more profitable?

(Nov'23 QP 8 marks)

Solution:

(i) **Expected Return on Market Index for Both factors**

Factor 1 - Inflation
= 0.20 x 6.70% + 0.30 x 4.50% + 0.15 x 6.75% + 0.35 x 7.00%
= 1.34% + 1.35% + 1.01% + 2.45% = 6.1525%

Factor 2 – Stock Market Return
= 0.20 x 10% + 0.30 x 7% + 0.15 x 9% + 0.35 x 8.85%
= 2% + 2.10% + 1.35% + 3.10%
= 8.5475%

(ii)

As per APT modfied by Ross,
$E(R) = R_f + B_1 (AV_1-EV_1) +B2 (AV_2-EV_2).....$

Factor 1 (Inflation)

Category	Beta	Actual value	Expected value	Difference	Beta x Diff.
	(a)	(b) (%)	(c) (%)	(b) - (c) = (d) (%)	(e)
Small Cap	1.20	6.70	6.70	0.00	0.00
Medium Cap	1.75	6.00	4.50	1.50	2.63
Large Cap	1.30	8.00	6.75	1.25	1.63
Flexi cap	1.70	6.50	7.00	(0.50)	(0.85)

Factor 2 (Stock Market)

Category	Beta	Actual value	Expected value	Difference	Beta x Diff.
	(a)	(b) (%)	(c) (%)	(b) - (c) = (d) (%)	(e)
Small Cap	0.80	10.50	10.00	0.50	0.40
Medium Cap	0.90	8.00	7.00	1.00	0.90
Large Cap	1.165	10.00	9.00	1.00	1.165
Flexi cap	0.85	9.75	8.85	0.90	0.765

Combined Expected return

Category	Weight	A = B1 (AV1-EV1)	B = B2 (AV2-EV2)	Rf	E (R)= A + B + Rf
Small Cap	0.20	0.00%	0.40%	7.5%	7.9%
Medium Cap	0.30	2.63%	0.90%	7.5%	11.025%
Large Cap	0.15	1.63%	1.165%	7.5%	10.29%
Flexi cap	0.35	-0.85%	0.765%	7.5%	7.415%
	Weighted Average	=	Sum of (weights x E(R))		9.02625%

(iii)

Expected return with weights of 25% each

Category	Weight	A = B1 (AV1-EV1)	B = B2 (AV2-EV2)	Rf	E (R)= A + B + Rf
Small Cap	0.25	0.00%	0.40%	7.5%	
Medium Cap	0.25	2.63%	0.90%	7.5%	
Large Cap	0.25	1.63%	1.165%	7.5%	
Flexi cap	0.25	-0.85%	0.765%	7.5%	
	Weighted Average	=	Sum of (weights x E(R))		9.1575

(iv) The revised allocation where all weights are 25% each is better compared to existing scenario

85. Illustration

Following is the data regarding Three Securities:

Stock	Expected Return (%)	Std. Deviation	Correlation with the Market return
A	19%	2.50	0.840
B	13.50%	2.00	0.540
C	11.00%	0.80	0.975
Market risk	-	1.20	-
Market rate of return	14.00%	-	-
Risk free rate	9.00%	-	-

(i) Advise which of the above stocks are over, under or correctly valued in the market?
(ii) What will be strategy you would like to recommend?

(Nov'24 QP 6 marks)

Solution:

Beta Calculation	$\beta i = \rho \times (\sigma i / \sigma m)$
Stock A	$\beta A = 0.840 \times (2.5 / 1.2) = 1.75$
Stock B	$\beta B = 0.540 \times (2.0 / 1.2) = 0.90$
Stock C	$\beta C = 0.975 \times (0.8 / 1.2) = 0.65$
Expected Return Using CAPM	Expected Return = Rf + βi (Rm - Rf)
	Rf = 9%, Rm = 14%
Stock A	Expected Return A = 9% + 1.75 × (14% - 9%) = 17.75%
Stock B	Expected Return B = 9% + 0.90 × (14% - 9%) = 13.5%
Stock C	Expected Return C = 9% + 0.65 × (14% - 9%) = 12.25%
Expected Vs. Actual Return	
Stock A	Actual: 19%, CAPM: 17.75% → Undervalued
Stock B	Actual: 13.5%, CAPM: 13.5% → Correctly Valued
Stock C	Actual: 11%, CAPM: 12.25% → Overvalued
Strategy Recommendation	
Stock A	Buy: Undervalued, higher return potential.
Stock B	Hold: Correctly valued, maintain current position.
Stock C	Sell: Overvalued, lower return potential.

86. Illustration

Scenario: A Portfolio Manager (PM) has three mutual funds in his portfolio. The details of these mutual funds are as follows:

Particulars	Growth Fund	Balanced Fund	Regular Fund	Market
Average Return (%)	7.5	6.3	5.4	-
Variance	-	-	-	50.41
Sharpe Ratio	-0.15	-0.36	-0.48	-
Treynor's Ratio	-2	-3	-4.80	-

The yield on 182 days Treasury bill is 9% per annum.

Requirements:

Calculate the variance of the funds.

Determine the coefficient of determination for the funds

(Nov'24 QP 8 marks)

Solution:

Particulars	Growth Fund	Balanced Fund	Regular Fund
Rs (%)	7.5	6.3	5.4
Rf (%)	9.0	9.0	9.0
Sharpe Ratio = (Rs-Rf)/ SD	(7.5-9)/SDg	(6.3-9)/SDb	(5.4-9)/SDr
Sharpe Ratio	-0.15	-0.36	-0.48
SD	(7.5-9)/-0.15	(6.3-9)/-0.36	(5.4-9)/-0.48
SD	-1.5/-.15	-2.7/-.36	-3.6/-.48
SD (%)	10 %	7.5%	7.5%

Treynor Ratio = (Rs-Rf)/ Beta	(7.5-9)/BETA g	(6.3-9)/BETA b	(5.4-9)/BETA r
Treynor Ratio	-2	-3	-4.8
Beta	(7.5-9)/-2	(6.3-9)/-3	(5.4-9)/-4.8
Beta	0.75	0.90	0.75
Variance = Square of SD	100 ($\%^2$)or (1%)	56.25 ($\%^2$)or (0.5625%)	56.25 ($\%^2$)or (0.5625%)
Co efficient of Determination = r^2			
Beta = r* SD of Security / SD of Mkt	0.75= r *10/7.1	0.90= r *7.5/7.1	0.75= r *7.5/7.1
r	0.5325	0.852	0.71
r^2	0.2835	0.726	0.5041

87. Illustration

Based on given information, Mr. XLY wants to create a portfolio equally as risky as the market and is having ₹ 20,00,000 to invest.

Assets	Investment	Beta
Stock A	₹ 4,00,000	0.70
Stock B	₹ 5,00,000	1.10
Stock C	?	1.60
Debenture (D)	?	0

How do you recommend and interpret the risk scenario and investment in all the securities?

(Model Test Paper – 10 (Feb 25) Q 3 (a))

Solution:

In order to have the same risk as the market, the beta of portfolio should be equal to 1

Total Portfolio Value in Rs. Lacs = 20

Weight of Stock A = Rs.4 lacs / Rs. 20 Lacs = 20%

Weight of Stock B = Rs.5 lacs / Rs. 20 Lacs = 25%

Wa + Wb + Wc + Wd = 1

20% + 25% + Wc + Wd = 100%

⇨ Wc + Wd = 55% ----- Eq 1

Portfolio Beta = 1 implies Weighted average beta = 1

Wa x Beta a + Wb x Beta b + Wc x Beta c + W d x Beta d = 1

20% x 0.7 + 25% x 1.1 + wc x 1.6 + wd x 0 = 1

⇨ 0.14 + 0.275 + 1.6 Wc = 1

⇨ 1.6 Wc = 1- 0.415

⇨ Wc = 0.415 / 1.6 = 0.365625 ---- Eq 2

Substituting Eq 2 in Eq 1

Wd + 0.365635 = 0.55

⇨ Wd = 0.184375

Investment in Stock C = Rs.20 Lacs x 36.5625% = Rs.7,31, 250

Investment in Debentute D = Rs.20 lacs x 18.4375% = Rs.3,68,750

88. Illustration

Mr. Tamarind Intends to invest in the equity shares of a Company, the value of which depends on various parameters as below:

Factor	Beta	Expected value in %	Actual value in %
GNP	1.20	7.70	7.70
Inflation	1.75	5.50	7.00
Interest rate	1.30	7.75	9.00
Stock market index	1.70	10.00	12.00
Industrial production	1.00	7.00	7.50

If the risk-free rate of interest is 9.25%, how much is the return of the share under Arbitrage Pricing Theory?

(ICAI SM)

Solution:

Expected return under Arbitrage Pricing theory :

$$E_R = R_f + \lambda_1\beta_1 + \lambda_2\beta_2 \ldots \lambda_n\beta_n$$

$$E_R = R_f + (AV_1 - EV_1)\beta_1 + (AV_2 - EV_2)\,\beta_2 \ldots \ldots (AV_n - EV_n)\beta_n$$

Where ʎ is Risk premium for the factors like GDP, inflation, interest rate, etc and

$AV_n - EV_n$

Is the Surprise Factor due to change in Value of Factor

Factor	Actual value in %	Expected value in %	Difference	Beta	Diff. x Beta
GNP	7.70	7.70	0.00	1.20	0.00
Inflation	7.00	5.50	1.50	1.75	2.63
Interest rate	9.00	7.75	1.25	1.30	1.63
Stock index	12.00	10.00	2.00	1.70	3.40
Ind. Production	7.50	7.00	0.50	1.00	0.50
				Sum	**8.16**
Risk free rate in %					**9.25**
Return under APT					**17.41**

89. Illustration

On 01/04/2020 Mr. K invested in the following companies to make his portfolio:

Name of Company	No. of Equity Share Purchase	Face Value per Equity Share	Purchase Price per Equity Share
PK Ltd.	2000	₹ 10	₹ 210
KD Ltd.	1000	₹ 10	₹ 290

Mr. K expects that:

- Dividend for the financial year 2020-21 of PK Ltd. & KD Ltd. will be 40% & 50% respectively.
- Probabilities of the Market Price as on 31/03/2021 are as under:

Probability Factor	Market Value per Equity Share of PK Ltd.	Market Value per Equity Share of KD Ltd.
0.4	₹ 200	₹ 300
0.4	₹ 240	₹ 320
0.2	₹ 260	₹ 350

You are required to:

(i) Calculate the Expected Market Price of Equity Shares of both the Companies as on 31/03/2021.

(ii) Calculate the Expected Average Return of the Portfolio for the year 2020-21.

(8 Marks – Q 3a Dec'21)

Solution:

Part 1

Expected market price on 31/3/2021 = Sum of (Probability x price)

Expected MP of PK Ltd = 200 x 0.4 + 240 x 0.4 + 260 x 0.2 = 80 + 96 + 52 = 228

Expected MP of KD Ltd = 300 x 0.4 + 320 x 0.4 + 350 x 0.2 = 120 + 128 + 70 = 318

Part 2

Return % = (Closing – Opening + Dividend)/ Opening

	PK	KD
Expected Closing Price – 32/3/21	228	318
Add: Dividend	4	5
Less: Opening price	210	290
Per share absolute return	22	33
Return %	22/210 = 10.476%	33/290 = 11.379%

Portfolio value

Name of Company	No. of Equity Share Purchase	Purchase Price per Equity Share	Total Value in ₹	Ratio
PK Ltd.	2000	₹ 210	210 x 2000 = 4,20,000	42
KD Ltd.	1000	₹ 290	290 x 1,000 = 2,90,000	29

Total			7,10,000	71

Return of portfolio = $W_{pk} \times R_{pk} + W_{kd} \times R_{kd}$

= 42/71 x 10.476% + 29/71 x 11.379%

=10.845%

90. Illustration

Following are risk and return estimates for two stocks

Stock	Expected returns (%)	Beta	Specific SD of expected return (%)
A	14	0.8	35
B	18	1.2	45

The market index has a Standard Deviation (SD) of 25% and risk free rate on Treasury Bills is 6%.

You are required to calculate:

(i) The standard deviation of expected returns on A and B.

(ii) Suppose a portfolio is to be constructed with the proportions of 25%, 40% and 35% in stock A, B and Treasury Bills respectively, what would be the expected return, standard deviation of expected return of the portfolio?

(8 Marks Q 6a Nov'19)

Solution:

In this question, the word Specific SD refers to Unsystematic Risk & The question is based on Sharpe single index model

Part 1

SD of Expected Returns of A & B.

Stock	A	B
Systematic variance = $Beta^2$ x Market Variance	$0.8^2 \times 25^2$ (Ignore % at time of computation and add it to final solution) => 0.64 x 625 = 400	= $1.2^2 \times 25^2$ = 625 x 1.44 = 900
Unsystematic or Specific Variance (SD already provided in Question & Variance = SD^2)	35^2 = 1225	45^2 = 2025
Total Variance = Systematic Variance + Unsystematic Variance	1225+400 = 1625	900 + 2025 = 2925
SD	Square root of 1625 = 40.31%	Square root of 2925 = 54.08%

Part 2

Expected Portfolio Return

Stock	Weight	Return	Weight x Return
A	25%	14	14% x 0.25 = 3.5%

B	40%	18	18% x 0.40 = 7.2%
C	35%	6	6% x 0.35 = 2.1%
Total	**100%**		**12.80%**

Expected Portfolio SD

Stock	Weight	Beta	Systematic Variance	Unsystematic Risk
A	25%	0.80	400	1225
B	40%	1.20	900	2025
C	35%	0.00		
Total	**100%**			

Portfolio Beta = 0.25 x 0.80 + 0.40 x 1.20 = 0.68

Portfolio Systematic Variance = Portfolio Beta ^{2} x Market Variance

= $0.68^2 \times 25^2$

=0.4624 x 625

=289

Portfolio Unsystematic Variance = Sum of (weight^2 x unsystematic Risk ^{2})

= (0.25^2 x 1225 + 0.40^2 x 2025 + 0.35^2 x 0)

= (0.0625x1225 + 0.16 x 2025+0)

= 400.56

Total portfolio Variance = 289 + 400.56 = 689.56

Portfolio SD = Square Root of (689.56) = 26.26

91. Illustration

Security	Standard Deviation (σ)	Weights (w)
R	20%	0.8
S	50%	0.2

Calculation upto two decimal places.

From the information given above, you are required to calculate Portfolio Standard Deviations, if-

(i) Securities returns are independent.

(ii) Securities returns are perfectly negatively correlated.

(iii) Securities returns are perfectly positively correlated.

(May'25 2(a) – 6 Marks)

Solution:

Variance of a portfolio = $(W_1SD_1)^2 + (W_2SD_2)^2 + 2W_1SD_1W_2SD_2\,r$

Scenario	1	2	3
Correlation	0	-1	1
Variance	$(0.8 \times 20)^2 + (0.2 \times 50)^2 + 2 \times 0.8 \times 0.2 \times 20 \times 50 \times 0$	$(0.8 \times 20)^2 + (0.2 \times 50)^2 + 2 \times 0.8 \times 0.2 \times 20 \times 50 \times -1$	$(0.8 \times 20)^2 + (0.2 \times 50)^2 + 2 \times 0.8 \times 0.2 \times 20 \times 50 \times +1$
Variance	256+100+0	256+100-320	256+100+320

Variance	356	36	676
SD^2 = Variance So. SD =	18.87%	6%	26%

92. Illustration

Mrs. SRS is your HNI Client and wants to invest in stock market. She has got the following information about individual securities and wants to select the securities to form an optimal portfolio from amongst these securities:

Security	Expected Return (%)	Unsystematic Risk (%)	Beta
A	5	25	0.5
B	25	20	2.5
C	15	10	1.0
D	10	10	1.5
E	**20**	**18**	**1.8**

Market Index Variance is 25% and the Risk-Free Rate of Return is 7%. Required:
Based on this information, help Mrs. SRS to:

(i) Prepare ranked table using Treynor's Ratio.
(ii) Calculate Cut-off Point.
(iii) Identify the securities to be included in optimal portfolio.

(Q 5a Jan 26 8 Marks)

Solution:
Steps:
1. Compute Treynor Ratio
2. Rank stocks from highest to Lowest
3. Compute Ci for all stocks cumulative from 1st stock till that stock

Formula

$$C_i = \frac{\sigma_m^2 \sum_{i=1}^{N} \frac{(R_i - R_f)\beta_i}{\sigma_{ei}^2}}{1 + \sigma_m^2 \sum_{i=1}^{N} \frac{\beta_i^2}{\sigma_{ei}^2}}$$

4. Highest C1 is cutoff point
5. Stock till cut off are part of Optimal portfolio
6. Stock below cutoff are ignored
7.Compute weight of each stock in optimal portfolio Xi0
8. No Short sales (-ve weights) are permitted. Weights

$$X_i^0 = \frac{Z_i}{\sum_{j=1}^{N} Z_i}$$

where

$$Z_i = \frac{B_i}{\sigma_{ei}^2}\left(\frac{R_i - R_0}{B_i} - C^*\right)$$

Treynor Ratio

Security	Ri	bi	Ri - Rf	Ri - Rf bi	Ranking
A	5	0.5	-2	-4	5
B	25	2.50	18	7.20	3
C	15	1.00	8	8.00	1
D	10	1.50	3	2.00	4
E	20	1.80	13	7.22	2

Security	$R_i - R_f$	β_i	σ^2_{ei}	$\frac{(R_i - R_f) \times \beta_i}{\sigma^2_{ei}}$	$\sum_{e=i}^{N} \frac{(R_i - R_f) \times \beta_i}{\sigma^2_{ei}}$	$\frac{\beta_i^2}{\sigma^2_{ei}}$	$\sum_{e=i}^{N} \frac{\beta_i^2}{\sigma^2_{ei}}$	C_i
C	8	1.0	10	0.80	0.80	0.10	0.10	5.71
E	13	1.80	18	1.30	2.10	0.18	0.28	6.56
B	18	2.50	20	2.25	4.35	0.313	0.593	6.87
D	3	1.50	10	0.45	4.80	0.225	0.818	5.59
A	-2	0.50	25	-0.04	4.76	0.01	0.828	5.48

CC = 25 x 0.80 / [1 + (25 x 0.10)] = 5.71
CE = 25 x 2.10 / [1 + (25 x 0.28)] = 6.562
CB = 25 x 4.35 / [1 + (25 x 0.593)] = 6.87747
CD = 25 x 4.80 / [1 + (25 x 0.818)] = 5.5976
CA = 25 x 4.76 / [1 + (25 x 0.828)] = 5.487
Cut-off Point: 6.87 747

Stocks C, E & B are selected

Weights – Additional point solved by faculty

Z_x = (β_i^)/(σ_ei^2)*[(R_i-R_f)/β_i - C^*]
Z_C = (1/10) x (8- 6.87747) = 0.112253
Z_E= (1.8/18)x (7.22- 6.87747) = 0.034253
Z_B= (2.5/20)x (7.2- 6.87747) = 0.04031625
Z_C+Z_E+Z_B= 0.1868225
Weights
ZC = 60.09% (0.112253/0.1868225)
ZE=18.33% (0.034253/0.1868225)
ZB = 21.58%(0.04031625/0.1868225)

SECURITIZATION (1Q)

Illustration 1:

Grow More Ltd. an NBFC is in the need of funds and hence it sold its receivables to MAC Financial Corporation (MFC) for ₹ 100 million. MFC created a trust for this purpose called General Investment Trust (GIT) through which it issued securities carrying a different level of risk and return to the investors. Further, this structure also permits the GIT to reinvest surplus funds for short term as per their requirement.
MFC also appointed a third party, Safeguard Pvt. Ltd. (SPL) to collect the payment due from obligor(s) and passes it to GIT. It will also follow up with defaulting obligor and if required initiate appropriate legal action against them.

(RTP May'24)

Based on above scenario, answer the following questions:

I. The securitized instrument issued for ₹ 100 million by the GIT falls under category of

(a) Pass Through certificate (PTCs)
(b) **Pay Through Security (PTS)**
(c) Stripped Security
(d) Debt Fund.

A PTS is one in which surplus money if any, is invested in order to even out cash flows. In a PTC, cash flows are passed on to the investors as received.

II. In the above scenario, the Originator is…………………
(a) Grow More Ltd.
(b) **MAC Financial Corporation (MFC)**
(c) General Investment Trust (GIT)
(d) Safeguard Pvt. Ltd.

Since MAC has created the a trust, they are the originator and not Grow More

III. In the above scenario, the General Investment Trust (GIT) is a/an…………………
(a) Obligor
(b) Originator
(c) **Special Purpose Vehicle (SPV)**
(d) Receiving and Paying Agent (RPA)

IV. In the above scenario, the Safeguard Pvt. Ltd. (SPL) is a/an………………

(a) Obligor
(b) Originator
(c) Special Purpose Vehicle (SPV)
(d) **Receiving and Paying Agent (RPA)**

V. Which of the following statement holds true?

(a) When Yield to Maturity in market rises, prices of Principal Only (PO) Securities tend to rise.
(b) When Yield to Maturity in market rises, prices of Principal Only (PO) Securities tend to fall.

(c) When Yield to Maturity in market falls, prices of Principal Only (PO) Securities tend to fall.
(d) When Yield to Maturity in market falls, prices of Principal Only (PO) Securities remain the same.

The holder of IO securities receives only interest while PO security holder receives only principal.

Interest rate in market	**Value of IO's securities**	**Value of PO's securities**
Raises	Raises	Falls (Borrower prefers to postpone the payment on cheaper loans)
Falls	Falls	Raises (Borrower tends to repay the loans as they prefer to borrow fresh loan at lower rate of interest)

1. Illustration

A mutual fund that had a net asset value of ₹16 at the beginning of a month, made capital gain and income distribution of ₹0.04 and ₹0.03 respectively per unit during the month, and then ended the month with a net asset value of ₹16.08. Calculate the monthly and annual returns.

(Similar ICAI SM, Old PM, MTP May'20 Old)

Solution:

Particulars	₹	%
Opening NAV	16	
CG Distribution	0.04	
Income Distribution	0.03	
Closing NAV	16.08	
Return		
= $\frac{(Closing\ NAV - Opening\ NAV + CG + ID)}{Opening\ NAV}$		
= $\frac{16.08-16+0.04+0.03}{16}$ = $\frac{0.15}{16}$ = 09375		0.94%
Monthly Return		0.9375%
Annualized Return		11.25%
Annualised Return Based on Compounding		
$(1+0.9375\%)^{12}-1$		**11.85%**

2. Illustration

A Mutual Fund having 300 units has shown its NAV of ₹8.75 and ₹9.45 at the beginning and at the end of the year, respectively.

The Mutual Fund has given two options:
(i) Pay ₹0.75 per unit as dividend and ₹0.60 per unit as a capital gain, or
(ii) These distributions are to be reinvested at an average NAV of ₹8.65 per unit.

What difference it would make in terms of return available and which option is preferable?

(ICAI SM, Sep-25 4M Similar, Nov'18 QP 8 marks, MTP Sept'23, MTP Sept'22, MTP Mar'18, Old PM)

Solution:

Particulars	₹	%
Opening Units	300	
Opening NAV	8.75	
Closing NAV	9.45	
Dividend	0.75	
CG	0.60	
Reinvestment NAV	8.65	
Option 1:		
Return		
= $\frac{(Closing\ NAV - Opening\ NAV + CG + ID)}{Opening\ NAV}$ = $\frac{2.05}{8.75}$		23.43%
Option 2:		
= $\frac{Amount\ Reinvested}{Units}$	1.35	
Total Amount Reinvested	405	
Reinvestment NAV	8.65	
No of units allotted on account of Reinvestment (Units)	46.82	

Closing Units	346.82	
Return		
$= \frac{Closing\ Value\ of\ Investment - Opening\ Value\ of\ Investment}{Opening\ Value\ of\ Investment}$ $= \frac{652.46}{2625}$		**24.86%**

Option 2 is preferrable

3. Illustration

Mr. X, an investor had purchased 200 units of ABC Mutual Fund at the rate of ₹8.50 per unit, one year ago. Over the year Mr. X received ₹0.90 as dividend and had received a capital gains distribution of ₹0.75 per unit.

You are required to find out:

(a)Mr. X's holding period return, assuming that this no-load fund has a NAV of ₹9.10 as on today.

(b)Mr. X's holding period return, assuming all the dividends and capital gains distributions are reinvested into additional units at average price of ₹8.75 per unit.

(Similar MTP Oct'19)

Solution:

Particulars	**Units**	**Rs.**	**%**
Opening NAV	200	8.50	
Closing NAV		9.10	
Dividend		0.90	
CG		0.75	
Reinvestment NAV		8.75	
Option 1			
Return			
$= \frac{(Closing\ NAV - Opening\ NAV + CG + ID)}{Opening\ NAV} = \frac{2.25}{8.5}$			26.47%
Option 2:			
Dividend + CG		1.65	
Total Dividend + CG paid		330.0	
Reinvestment NAV		8.75	
No. of Units received on Re investment	37.71		
Closing Units	237.71		
$= \frac{Closing\ Value\ of\ Investment - Opening\ Value\ of\ Investment}{Opening\ Value\ of\ Investment}$			
$= \frac{463.2}{1700}$			**27.25%**

Mr X has to go for option 2 i.e reinvestment option as it provides higher return

4. Illustration

The NAV of per unit of XYZ Mutual Fund (a Close Ended Fund) on 1.1.2014 was ₹28. The value of 31.12.2014 comes to ₹28.80. On the same date unit was trading in market at premium of 3% though on 1.1.2014 same was trading at a discount at 5%. On 31.12.2014, XYZ distributed a sum of ₹2.80 as incomes and capital gains.

You are required to compute rate of return to the investors during the year.

Solution:

Particulars	**Rs.**	**%**
Opening NAV	28.00	
Closing NAV	28.80	
Opening Discount		5.00%

Closing Premium		3.00%
Opening Market price	26.60	
Closing Market Price	29.66	
CG + Divided	2.80	
Return		
$= \frac{(Closing\ NAV - Opening\ NAV + CG + ID)}{Opening\ NAV} = \frac{5.86}{26.6}$		**22.05%**

5. Illustration

The unit price of Equity Linked Savings Scheme (ELSS) of a mutual fund is ₹10. The public offer price (POP) of the unit is ₹10.204 and the redemption price is ₹9.80. Calculate:

(i) Front end Load

(ii) Back end Load

[ICAI SM, May'18 QP (Old), Old PM]

Solution:

Particulars	Rs.	%
FV	10.000	
NFO	10.204	
Redemption price	9.800	
Entry Load = $\frac{10.204-10}{10}$		2.04%
Exit Load = $\frac{10-9.8}{10}$		2.00%

6. Illustration

During the year 2017 an investor invested in a mutual fund. The capital gain and dividend for the year was ₹3 per unit, which were re-invested at the year-end NAV of ₹23.75. The investor had total units of 26,750 as at the end of the year. The NAV had appreciated by 18.75% during the year and there was an entry load of ₹0.05 at the time when the investment was made.

The investor lost his records and wants to find out the amount of investment made and the entry load in the mutual fund.

[Nov'18 QP (Old), Sep-25 Similar 7 M]

Solution:

Particulars	**Units**	**₹**	**%**
Opening NAV		20.000	
Closing NAV	26750	23.750	18.75%
CG + Dividend		3.00	
Opening Units	x		
Reinvested CG + Dividend	x	3.00	
Total reinvestment value		3 * x	
Reinvested NV		23.75	
$x + \frac{3*x}{23.75} = 26750$			
$\frac{23.75x + 3x}{23.75} = 26750$			
$26.75x = 23.75 * 26750$			
$x = \frac{26750 * 23.75}{26.75}$	23750.00		
Opening	23750	20.00	
Investment value		4,75,000.00	
Entry load	23750	0.05	
Total load		1,187.50	
Total outflow for investor		**4,76,187.50**	

6a. Illustration

In 2024, Mr. Raj, an investor made a lump sum investment in an equity mutual fund that had an entry load of ₹ 0.05 per unit. By the end of the year, the NAV appreciated by 13.60%. Additionally, the fund declared a total capital gain and dividend of ₹ 5.00 per unit, which were reinvested at a year-end NAV of ₹ 25. As a result, the investor held 15,000 units at year-end. The fund also charges an exit load of 1% if redeemed within 1 year. The investor is in the 20% tax bracket. Inflation rate during the year is 4.50%.

You are to required to

(i) Calculate the number of units purchased by Mr. Raj at the beginning of the investment.

(ii) Calculate the NAV per unit and the total investment amount made by Mr. Raj at the beginning of the year.

(iii) Appraise the return percentage and the real return percentage, if Mr. Raj decided to exit the investment at the end of the year. Calculation up to 2 decimal points. *(Sep25- 7 Marks)*

Solution:

(1) Closing Unit count 15,000
Dividend & CG per unit 5
Reinvestment NAV 25

Let Opening Units be X & Opening NAV be Y

X + X*5/25 = 15000
6X/5 = 15000
X =12,500

(2) 1.136Y = 25
Y = 25/1.136
Y = 22.01

Initial Investment = X * (Y+0.05)
= 12500 * 22.06
= 275,750

(3)

Details	Amount in ₹
Closing Value of Investment (25 x 15000)	375000
Less: Exit Load 1%	-3750
Net Proceeds A	**371250**
Initial Investment B	275750
Return in ₹ C = A - B	95500
Less Taxes @ 20% D = C x 20%	19100
Net Return E = C-D	76400

Nominal Return = 76400/275750 = 27.71%
Real Return = (1+Nominal Return)/(1+ Inflation) – 1= 1.2771/1.045-1 = 22.21%

7. Illustration

Mr. A can earn a return of 16 per cent by investing in equity shares on his own. Now he is considering a recently announced equity based mutual fund scheme in which initial expenses are 5.5 per cent and annual recurring expenses are 1.5 per cent.
How much should the mutual fund earn to provide Mr. A return of 16 per cent?

(ICAI SM, Old PM)

Solution:

Return = $\frac{Investor\ Return}{1 - Initial\ Expenses\ \%} + Recurring\ Expenses$

Particulars	₹ Per Unit	%
Required Return		16.00%
Initial expenses		5.50%
Annual recurring expenses		1.50%
Let MF return = x		
$x = \frac{1.5\% + 16\%}{1 - 5.5\%}$		**18.43%**
		-1.50%
	9.45	16.9312%
	1.60	
	10.00	**16.00%**

8. Illustration

Mr. X earns 10% on his investments in equity shares. He is considering a recently floated scheme of a Mutual Fund where the initial expenses are 6% and annual recurring expenses are expected to be 2%. How much should the Mutual Fund scheme earn to provide a return of 10% to Mr. X?

(Old PM)

Solution:

Particulars	%
Expected return	10.00%
Initial expenses	6.00%
Recurring expenses	2.00%
Required return = = $\frac{Investor\ Return\ (Desired)}{1 - Initial\ Expenses\ \%} + Annual\ Expenses$	**12.64%**

9. Illustration

Mr. Alex, a practicing Chartered Accountant, can earn a return of 15 percent by investing in equity shares on his own. He is considering a recently announced equity based mutual fund scheme in which initial expenses are 6 percent and annual recurring expenses are 2 percent.

(i) How much should the mutual fund earn to provide Mr. Alex a return of 15 percent per annum?

(ii) Mr. Alex's current Annual Professional Income is ₹40 Lakhs. His portfolio value is ₹50 Lakhs and now he is spending 10% of his time to manage his portfolio. If he spends this time on profession, his professional income will go up in same proportion.

He is thinking to invest his entire portfolio into a Multicap Fund, assuming the fund's NAV will grow at 13% per annum (including dividend).
You are requested to advise Mr. Alex, whether he can invest the portfolio into Multicap Funds? If so, what is the net financial benefit?

[Nov'19 QP (Old)]

Solution:

Particulars	Total Rs.	%
Part 1		
MF Return % = $\frac{Desired\ Return}{1-Initial\ Expenses\ \%}$ $Recurring\ Expenses$		17.96% (15/(1-6%) +2
Part 2		
Annual Income	40,00,000.00	
Portfolio value	50,00,000.00	
% time in managing portfolio		10.00%
Present:		
Income from Profession	40,00,000.00	
Return on portfolio (50,00,000 *15%)	7,50,000.00	15.00%
Total income for the year	**47,50,000.00**	
Invest in MF	50,00,000.00	
Return on MF investment – 1	6,50,000.00	13.00%
Income from profession – 2	40,00,000.00	
Income from additional time spent – 3	4,00,000.00	10.00%
Total Income for the year (1+2+3)	**50,50,000.00**	

It is advisable to invest in multicap funds where income is higher.

10. Illustration

Mr. P has invested in three Mutual Fund Schemes:

Particulars	MF X	MF Y	MF Z
Date of investment	01.11.2016	01.02.2017	01.03.2017
Amount of investment (₹)	1,00,000	2,00,000	2,00,000
Net Asset Value (NAV) at entry date (₹)	10.30	10	10.10
Dividend received up to 31-3-2017 (₹)	2,850	4,500	NIL
NAV as on 31-3-2017 (₹)	10.25	10.15	10

Assume a year has 365 days. Show the amount of rupees up to two decimal points.
You are required to find out the effective yield (up to 3 decimal points) on per annum basis in respect of each of the three schemes to Mr. P up to 31.03.2017?

(Similar July'21 QP 8 marks, MTP Mar'23, Old PM)

Solution:

Particulars	Scheme X	Scheme Y	Scheme Z
Investment value	1,00,000	2,00,000	2,00,000
Date of Investment	01-Nov-16	01-Feb-17	01-Mar-17
Closing Date	31-Mar-17	31-Mar-17	31-Mar-17
Invested NAV	10.30	10.00	10.10
Closing NAV	10.25	10.15	10.00
Invested units (investment / Op NAV)	9,708.74	20,000.00	19,801.98
Closing investment value	99,514.56	2,03,000.00	1,98,019.80
Dividend	2,850.00	4,500.00	-
Absolute return in Rs.(Cl-OP + Div)	2,364.56	7,500.00	(1,980.20)
Absolute Return % (Abs return / Op invest)	2.365%	3.750%	-0.990%
No. of days	151	59.00	31.00
Effective annual yield (Abs return *365/ days)	**5.716%**	**23.199%**	**-11.658%**

11.Illustration

Mr. Y has invested in the three mutual funds (MF) as per the following details:

Particulars	MF X	MF Y	MF Z
Amount of investment (₹)	2,00,000	4,00,000	2,00,000
Net Assets Value (NAV) at the time of purchase (₹)	10.30	10.10	10
Dividend received up to 31-3-2018 (₹)	6,000	0	5,000
NAV as on 31-3-2018 (₹)	10.25	10	10.20
Effective Yield per annum as on 31.03.2018 (%)	9.66	-11.66	24.15

Assume 1 Year = 365 days

Mr. Y has misplaced the documents of his investment. Help him in finding the date of his original investment after ascertaining the following:

(i) Number of units in each scheme
(ii) Total NAV
(iii) Total Yield; and
(iv) Number of days investment held

[ICAI SM, MTP Oct'20, May'18 QP (Old), Old PM]

Solution:

Particulars	Scheme X	Scheme Y	Scheme Z
Amount of Investment	2,00,000.00	4,00,000.00	2,00,000.00
Purchase NAV	10.30	10.10	10.00
Dividend Received up to 31/3/2018	6,000.00	-	5,000.00
Closing NAV as on 31/3/2018	10.25	10.00	10.20
Effective Yield	9.66%	-11.66%	24.15%
Units in each scheme - invested	19,417.48	39,603.96	20,000.00
Total NAV / Closing investment value - 31/3/18	1,99,029.13	3,96,039.60	2,04,000.00
Total closing Investment	7,99,068.73		
CG + DI	6,000.00	-	5,000.00
Total return in Rs. Terms (closing +div-opening)	5,029.13	(3,960.40)	9,000.00
Total return from all 3 investments	10,068.73		
Annual Return in ₹ on original investment assuming investment was for full year	19,320.00	(46,640.00)	48,300.00
Actual Return	5,029.13	(3,960.40)	9,000.00
% of time (of a year) investments held (Actual return / Annual return)	0.26	0.08	0.19
No of days in a year	365	365.00	365.00
Period for which investment held	95.01	30.99	68.01
Reporting date	31-Mar-18	31-Mar-18	31-Mar-18
Investment date	26-Dec-18	28-Feb-18	22-Jan-18

12. Illustration

A Mutual Fund Company introduces two schemes – Dividend Plan and Bonus Plan. The face value of the Unit is ₹10 on 1-4-2014. Mr. R invested ₹5 lakh in Dividend Plan and ₹10 lakh in Bonus Plan. The NAV of Dividend Plan is ₹46 and NAV of Bonus Plan is ₹42. Both the plans matured on 31-03-2019. The particulars of Dividend and Bonus declared over the period are as follows:

Date	Dividend %	Bonus Ratio	NAV of Dividend Plan	NAV of Bonus Plan
31-12-2014	12%	-	47.0	42.0
30-09-2015	-	1:4	48.0	43.0
31-03-2016	15%	-	49.5	41.5
30-09-2017	-	1:6	50.0	44.0
31-03-2018	10%	-	48.0	43.5
31-03-2019	-	-	49.0	44.0

You are required to calculate the effective yield per annum in respect of the above two plans.

(Similar ICAI SM, May'19 QP 8 marks, MTP Apr'24, Old PM)

Solution:

Particulars	Units	Dividend	Reinvestment Units
Investment in Dividend – ₹ 5,00,000	10,869.57		
Investment in Bonus – ₹ 10,00,000	23,809.52		
Dividend Plan			
Opening	10,869.57		
31-Dec-14		13,043.48	277.52
Closing Units as on 31 Mar 2014	11,147.09		
31-Mar-16		16,720.63	337.79
Closing units as on 31 Mar 2016	11,484.88		
31-Mar-18		11,484.88	239.27
Closing units as on 31 Mar 2018	11,724.14		
Units matured	11,724.14		
NAV on date of maturity	49.00		
Total amount Realized	5,74,483.09		
Amount Invested	5,00,000.00		
Total Gain	74,483.09		
Absolute Return in %	14.90%		
Return p a %	**2.98%**		
Opening	23809.52		
bonus on 30 Sep 2015 1:4	5,952.38		
Closing Unit	29,761.90		
Bonus on 30 Sep 2018 - 1:6	4,960.32		
Closing units	34,722.22		
Maturity date NAV	44.00		
Maturity Value	15,27,777.78		
Investment value	10,00,000.00		
Absolute Gain	5,27,777.78		
Absolute gain %	52.78%		
Yearly return	**10.56%**		

13 Illustration

A mutual fund has two schemes i.e., Dividend plan (Plan-A) and Bonus plan (Plan-B). The face value of the unit is ₹10. On 01/04/2016 Mr. Anand invested ₹5,00,000 each in Plan-A and Plan-B when the NAV was ₹46 and ₹43.50 respectively, Both the Plans matured on 31/03/2019.
Particulars of dividend and bonus declared over the period are as follows:

Date	Dividend %	Bonus Ratio	Net Assets Value (₹)	
			Plan-A	Plan-B
30-06-2016	15%		46.80	44
31-08-2016		1:6	47.20	45.40
31-03-2017	10%		48	46.60
17-09-2017		1:8	48.40	47
21-11-2017	14%		49.60	47.20
25-02-2018	15%		50	47.80
31-03-2018		1:10	50.50	48.80
30-06-2018	12%		51.80	49
31-03-2019			52.40	50

You are required to calculate the Effective Yield Per annum in respect of the above two plans.

[May'19 QP (Old)]

Solution:

Particulars	Units	Dividend	Reinvestment Units
Investment in Plan A - Dividend 1/4/16	10,869.57		
30 June 2016 15% NAV 46.80		16,304.35	348.38
Closing units	11,217.95		
31 Mar 2017 10% NAV48		11,217.95	233.71
Closing units	11,451.66		
21 Nov 2017 14% NAV 49.60		16,032.32	323.23
Closing units	11,774.89		
25 Feb 2018 15% NAV 50		17,662.33	353.25
Closing Units	12,128.13		
30 June 2018 15% NAV 51.80		14,553.76	280.96
Closing units	12,409.10		
Maturity 31 Mar 2019 - NAV	52.40		
Maturity Value	6,50,236.60		
Investment Value	5,00,000.00		
Absolute return	1,50,236.60		
Absolute Return %	30.05%		
Return per annum	**10.02%**		

Outflow (01-04-2016)	**-5,00,000.00**
Inflow (31-03-2019)	6,50,236.60
IRR taken @ 10%	1.41936
PV of Inflow = $\frac{6,50,236.60}{(1.1)^3}$	488532.39
NPV	(11,467.61)
IRR taken at 9%	
PV of inflow= $\frac{6,50,236.60}{(1.09)^3}$	5,02,101.96

NPV	2,101.96
$5,02,101.96 - 488532.39$	13,569.58
$2,101.96 * \left(\frac{1\%}{13,569.58}\right)$	0.15%
IRR is 9% + 0.15%	9.15%
IRR taken at 9.15% = $\frac{6,50,236.60}{(1.0915)^3}$	5,00,034.76
Investment @NAV 43.5= $\frac{500000}{43.5}$	11,494.25
Add: 31 Aug 2016 Bonus 1:6 - NAV 45.4 = $\frac{11494.25}{6}$	1,915.71
= Closing units	13,409.96
Add: 17 Sep 2017 B: 1:8 - NAV 47 = $\frac{13,409.96}{8}$	1,676.25
= Closing units	15,086.21
Add: 31 Mar 2018 Bonus - 1:10 @ NAV 48.80 = $\frac{15,086.21}{10}$	1,508.62
= Closing Units	16,594.83
Maturity NAV	50
Maturity Value = $16,594.83 * 50$	8,29,741.38
Less: Investment Value	5,00,000.00
= Absolute Gain	3,29,741.38
Absolute Gain %	65.95%
Annual Return / gain %	21.98%
Investment value	(5,00,000.00)
Maturity Inflow	8,29,741.38

Investment value	**(5,00,000.00)**
Maturity Inflow	8,29,741.38
IRR @ 22%	
PV of inflow = $\frac{8,29,741.38}{(1.22)^3}$	4,56,944.29
PV of outflow	(5,00,000.00)
NPV	(43,055.71)
IRR @18% = $\frac{8,29,741.38}{(1.18)^3}$	
PV of inflow	5,05,006.22
PV of outflow	(5,00,000.00)
NPV	5,006.22
IRR @19% = $\frac{8,29,741.38}{(1.19)^3}$	
PV of inflow	4,92,381.66
PV of outflow	(5,00,000.00)
NPV	(7,618.34)
$5,006.22 - (7,618.34)$	12,624.56
$5,006.22 * \left(\frac{1\%}{12,624.56}\right)$	0.40%
IRR	18% + 0.4%
Annualised return	**18.40%**

14. Illustration

Sun Moon Mutual Fund (Approved Mutual Fund) sponsored open-ended equity-oriented scheme Chanakya Opportunity Fund.
There were three plans viz.

A – Dividend Reinvestment Plan,
B – Bonus Plan &
C – Growth Plan

At the time of Initial Public Offer on 1.4.2009, Mr. Anand, Mr. Bacchan & Mrs Charu, three investors invested ₹1,00,000 each & chose B, C & A Plan respectively. The History of the Fund is as follows:

Date	Dividend %	Bonus Ratio	Net Asset Value per unit (F.V. ₹10)		
			Plan A	Plan B	Plan C
28.07.2013	20	-	30.70	31.40	33.42
31.03.2014	70	5:4	58.42	31.05	70.05
31.10.2017	40	-	42.18	25.02	56.15
15.03.2018	25	-	46.45	29.10	64.28
31.03.2018		1:3	42.18	20.05	60.12
24.03.2019	40	1:4	48.10	19.95	72.40
31.07.2019			53.75	22.98	82.07

On 31st July, 2019 all three investors redeemed all the balance units. Calculate:

(i) Annual rate of return of Mrs Charu who has invested in A – Dividend Re-investment Plan
(ii) Annual rate of return of Mr. Anand who has invested in B – Bonus Plan
(iii) Annual rate of return of Mr. Bacchan who has invested in C – Growth Plan

Assumptions

1. Long-term Capital Gain is exempt from Income tax.
2. Short-term Capital Gain is subject to 10% Income tax.
3. Security Transaction Tax 0.2 per cent only on sale/redemption of units.
4. Ignore Education Cess

(ICAI SM, Nov'19 QP 10 marks, RTP May'18, Old PM)

Solution:

Particulars	₹	Units	Closing units
Mr. Charu Dividend Reinvestment Plan			
Investment on 1 St April 2009	1,00,000	10,000	
28 July 2013 - 20% Dividend - NAV 30.70	20,000	651.47	10,651.47
31 Mar 2014 - 70% Dividend - NAV 58.42	74,560.26	1,276.28	11,927.75
31 oct 2017 - 40% Dividend - NAV 42.18	47,710.98	1,131.13	13,058.87
15-03-2018 - 25% Dividend - NAV 46.45	32,647.18	702.85	13,761.72
24-03-2019 - 40% Dividend - NAV 48.10	55,046.88	1,144.43	14,906.15
31 July 2019 Redemption NAV 53.75	8,01,205.30		
STT @ 0.2%	1,602.41		
STCG @ 10% on 1144.43 x (53.75-48.10)	646.60		
Net Amount realized	7,98,956.28		
Amount invested	1,00,000		

Return	6,98,956.28		
Return % - Absolute	6.99		
Annual Return	67.64%		

Plan B - Bonus - Mr. Anand			
Mr. Anand - Bonus Plan			
Original Investment @ NAV 10	1,00,000	10,000	
31 Mar 2014 - Bonus 5:4 @ NAV 31.05		12,500	22,500
31-03-2018 - Bonus 1:3 @ NAV 20.05		7,500	30,000
24 March 2019 - Bonus 1:4 @ 19.95		7,500	37,500
Redemption on 31 July 2019 @ 22.98	8,61,750		
STCG Computation			
Net realization / Net consideration	1,72,350		
Less: COA	-		
STCG	1,72,350		
Tax on STCG @ 10%	17,235		
Gross amount realized	8,61,750		
STT @ 0.2%	(1,723.50)		
Net amount realised	8,60,026.50		
Less: Tax paid	(17,235)		
Final amount realized	8,42,791.50		
Investment made	1,00,000		
Absolute return	7,42,791.50		
Absolute return %	743%		
Annualized return %	71.88%		

Mr. Bachchan Plan C – Growth			
Investment made	1,00,000	10,000	
Redemption value @ NAV 82.07	8,20,700		
Less: STT @ 0.2%	1,641.40		
Net Realization	8,19,058.60		
Absolute Return	7,19,058.60		
Absolute Return %	719%		
Annualized return %	69.59%		

15. Illustration

A mutual fund raised ₹150 lakhs on April 1, 2018 by issue of 15 lakh units at ₹10 per unit. The fund invested in several capital market instruments to build a portfolio of ₹140 lakhs; Initial expenses amounted to ₹8 lakhs.
During the month of April, the fund sold certain instruments costing ₹44.75 lakhs for ₹47 lakhs and used the proceeds to purchase certain other securities for ₹41.6 Lakhs. The fund management expenses for the month amounted to ₹6 lakhs of which ₹50,000 was in arrears The fund earned dividends amounting to ₹1.5 lakhs and it distributed 80% of the realized earnings. The market value of the portfolio on 30th April, 2018 was ₹147.85 Lakhs.

An investor subscribed to 1000 units on April 1 and disposed it off at closing NAV on 30th April. Determine his annual rate of earnings.

[MTP Mar'21 New & Old, MTP May'20, Nov'18 QP (Old)]

Solution:

Particulars	Units	NAV	Value
Investment Made @ 10 - 1000 units	1,000	10	10,000
Add: CG & Dividend			200
Redemption	1,000	9.85	9,850
Absolute Return			50
Absolute Return %			0.50%
Annualized Return %			**6%**

WN: 1 - Cash Balance

	Amount ₹ Lakh	Closing Cash ₹ Lakh
1 April 2018 NFO @ ₹10 / Unit - 15 Lac Units	150	150
Less: Initial Expenses	(8)	142
Less: Investment 1	(44.75)	97.25
Less: Investment 2	(95.25)	2
Add: Investment sold	47	49
Less: Investment 3	(41.60)	7.40
Less: Fund Management Exp	(5.50)	1.90
Add: Dividend Earned	1.50	3.40
Less: Realized Earning Distributed 1	(1.20)	2.20
Less: Realized Earning Distributed 2	(1.80)	0.40
Closing Cash Balance		**0.40**

WN 2: NAV Computation

Market value of Securities	147.85	
Less: Payable	0.50	
Add Cash Balance	0.40	
Total NAV	147.75	
NAV Per unit	**9.85**	

16. Illustration

On 1-4-2012 ABC Mutual Fund issued 20 lakh units at ₹10 per unit. Relevant initial expenses involved were ₹12 lakhs. It invested the fund so raised in capital market instruments to build a portfolio of ₹185 lakhs.

During the month of April 2012, it disposed of some of the instruments costing ₹60 lakhs for ₹63 lakhs and used the proceeds in purchasing securities for ₹56 lakhs. Fund management expenses for the month of April 2012 was ₹8 lakhs of which 10% was in arrears In April 2012 the fund earned dividends amounting to ₹2 lakhs and it distributed 80% of the realized earnings. On 30-4-2012 the market value of the portfolio was ₹198 lakhs. Mr. Akash, an investor, subscribed to 100 units on 1-4-2012 and disposed of the same at closing NAV on 30-4-2012. What was his annual rate of earning?

(ICAI SM, RTP Nov'22, RTP May'18 Old, RTP May'20 Old, MTP Mar'19 Old)

Solution:

Computation of Return of Mr. Akash	Units	NAV	₹
Initial Investment	100	10	1,000
Income received (CG+Div)			20
Redemption Value - Realized	100	9.90	990
Absolute return			10
Absolute Return % - for 1 Month			1%
Annualized return			12%

WN 1: Cash Balance		
	Amt ₹ Lacs	Cash Balance ₹ Lacs
NFO - 20 Lac Units @ ₹ 10	200	200
Initial Expenses	(12)	188
Less: Investment 1	(60)	128
Less: Investment 2	(125)	3
Add: Investment 1 realized	63	66
Less: Investment 3	(56)	10
Less: Fund Expenses	(7.20)	2.80
Add: Dividends	2	4.80
Less: Dividend Distributed	(1.60)	3.20
Less: CG Distributed	(2.40)	0.80
Closing Cash Balance		0.80

WN 2: NAV		
Closing value of Marketable Securities	198	
Ad: Cash Balance	0.80	
Less: Fund Expenses arrears	(0.80)	
Total NAV	198	
NAV per unit	9.90	

17. Illustration

On 1st April 2009 Fair Return Mutual Fund has the following assets and prices at 4 p.m.

Shares	No. of Shares	Market Price Per Share (₹)
A Ltd.	10000	19.70
B Ltd.	50000	482.60
C Ltd.	10000	264.40
D Ltd.	100000	674.90
E Ltd.	30000	25.90
No. of units of funds		8,00,000

Please calculate:

(a) NAV of the Fund on 1st April 2009

(b) Assuming that on 1st April 2009, Mr. X, a HNI, send a cheque of ₹50,00,000 to the Fund and Fund Manager immediately purchases 18000 shares of C Ltd. and balance is held in bank. Then what will be position of fund.

(c) Now suppose on 2 April 2009 at 4 p.m. the market price of shares is as follows:

Shares	₹
A Ltd.	20.30
B Ltd.	513.70
C Ltd.	290.80
D Ltd.	671.90
E Ltd.	44.20

Then what will be new NAV.

(ICAI SM, RTP Nov'19, Old PM)

Solution:

Particulars	Shares / Units	Price / NAV	Value / Total NAV
1. NAV Computation as on 1st April, 2009 at 4 PM			
A Ltd	10,000	19.70	1,97,000
B Ltd	50,000	482.60	2,41,30,000
C Ltd	10,000	264.40	26,44,000
D Ltd	1,00,000	674.90	6,74,90,000
E Ltd	30,000	25.90	7,77,000
Total Value of Investments			**9,52,38,000**
No of Units	8,00,000		
NAV per unit		**119.05**	
2. NAV post Investment by Mr. X			50,00,000
No. of Shares of C purchased	18,000	264.40	47,59,200
Balance held in cash			2,40,800
No. of Units allotted to Mr. X	42,000.04	119.05	50,00,000
Total Units in the scheme post investment by Mr. X	8,42,000.04		
NAV after investment by Mr. X			
Mkt value of investments			9,52,38,000
Add: New investments			50,00,000
Total value of investments			**10,02,38,000**
A Ltd	10,000	19.70	1,97,000
B Ltd	50,000	482.60	2,41,30,000
C Ltd	28,000	264.40	74,03,200
D Ltd	1,00,000	674.90	6,74,90,000
E Ltd	30,000	25.90	7,77,000
Total Value of Investments			**9,99,97,200**
Add: Cash balance			2,40,800
Total Market value	**8,42,000.04**	**119.05**	**10,02,38,000**
3. New NAV based on Shares prices as on 2nd April 2009 @ 4 PM			
A Ltd	10,000	20.30	2,03,000
B Ltd	50,000	513.70	2,56,85,000

C Ltd	28,000	290.80	81,42,400
D Ltd	1,00,000	671.90	6,71,90,000
E Ltd	30,000	44.20	13,26,000
Total Value of Investments			**10,25,46,400**
Add: Cash balance			2,40,800
Total Market value	**8,42,000.04**	**122.08**	**10,27,87,200**

18. Illustration

A mutual fund made an issue of 10,00,000 units of ₹10 each on January 01, 2008. No entry load was charged. It made the following investments:

Particulars	₹
50,000 Equity shares of ₹100 each @₹160	80,00,000
7% Government Securities	8,00,000
9% Debentures (Unlisted)	5,00,000
10% Debentures (Listed)	5,00,000
	98,00,000

During the year, dividends of ₹12,00,000 were received on equity shares. Interest on all types of debt securities was received as and when due. At the end of the year equity shares and 10% debentures are quoted at 175% and 90% respectively. Other investments are at par.

Find out the Net Asset Value (NAV) per unit given that operating expenses paid during the year amounted to ₹5,00,000. Also find out the NAV, if the Mutual fund had distributed a dividend of ₹0.80 per unit during the year to the unit holders.

(ICAI SM, Similar May'22 QP 8 marks, MTP Sept'23, RTP May'18 Old, Old PM)

Solution:

Particulars	Shares / Units	Nav / Price	Value
1. Computation of NAV			
Equity Shares	50,000	175	87,50,000
7% GOI Securities			8,00,000
9% Unlisted debentures			5,00,000
10% Debentures @ 90% of PAR			4,50,000
Total Value of investments as at year end			**1,05,00,000**
Income			
Dividend on Equity shares			12,00,000
Interest on 7% GOI Securities			56,000
Interest on 9% unlisted debentures			45,000
Interest on 10% listed Debentures			50,000
Less: Operating expenses			5,00,000
Net Cash Received			8,51,000
Add: Cash not deployed			2,00,000
Total Cash Balance			**10,51,000**
Total Assets of the scheme & NAV	**10,00,000**	**11.55**	**1,15,51,000**
Less: Dividend during the year		0.80	
Revised NAV post Dividend		**10.75**	

19.Illustration

Calculate the NAV of a regular income scheme on per unit basis of Red Bull mutual fund from the following information:

Particulars	₹ in crores
Listed shares at cost (ex-dividend)	30
Cash in hand	0.75
Bonds & Debentures at cost (ex-interest)	2.30
Of these, bonds not listed & not quoted	1.0
Other fixed interest securities at cost	2.50
Dividend accrued	0.8
Amount payable on shares	8.32
Expenditure accrued	1
Value of listed bonds & debentures at NAV value	10

Number of units (₹10 face value) 30 lakhs
Current realizable value of fixed
income securities of face value of ₹100 is 106.50
The listed shares were purchased when index was 7100 and
The Present Index is 9000
Unlisted bonds and debentures are at cost. Other fixed securities are also at cost.

(May'16 QP)

Solution:

Particulars	Cost ₹ Cr	Mkt value ₹ Cr
Equity Shares	30	38.03
Bonds listed	1.30	10
Bonds unlisted	1	1
Other Fixed interest Securities	2.50	2.50
Add: Cash Balance		0.75
Total Value of Investments		52.28
Add: Dividend Accrued		0.80
Less: Expenditure accrued		1
Less: Amount Payable on Shares		8.32
Total Value of Investments + Incomes - Expenses		43.76
Number of Units		0.30
NAV per unit		145.86

20 Illustration

The following particulars relating to Vishnu Fund Schemes:

Particulars	Value ₹ in Crores
1. Investment in Shares (at cost)	
a. pharmaceuticals companies	79
b. Construction Industries	31
c. Service Sector Companies	56
d. IT Companies	34
e. Real Estate Companies	10
2. Investment in Bonds (Fixed Income)	
a. Listed Bonds (8000, 14% Bonds of ₹15,000 each)	12
b. Unlisted Bonds	7
3. No. of Units outstanding (crores)	4.2

4. Expenses Payable	3.5
5. Cash and Cash equivalents	1.5
6. Market expectations on listed bonds	8.842%

Particulars relating to each sector are as follows:

Sector	Index on Purchase date	Index on Valuation date
Pharmaceutical companies	260	465
Construction Industries	210	450
Service Sector Companies	275	480
IT Companies	240	495
Real Estate Companies	255	410

The fund has incurred the following expenses:

Consultancy and Management fees ₹480 Lakhs
Office Expenses ₹150 Lakhs
Advertisement Expenses ₹38 Lakhs

You are required to calculate the following:

(i) Net Asset Value of the fund
(ii) Net Asset Value per unit
(iii) If the period of consideration is 2 years, and the fund has distributed ₹3 per unit per year as cash dividend, ascertain the Net return (Annualized)
(iv) Ascertain the Expenses ratio *[RTP May'21 New & Old, May'19 QP (Old)]*

Solution:

Particulars	Cost in ₹ Cr	Market Value working	Mkt Value in ₹ Cr
Shares			
- Pharma Companies	79	79 * 465/260	141.29
- Construction Industries	31	31 * 450/210	66.43
- Service Sector Companies	56	56 * 480/275	97.75
- IT Companies	34	34 * 495/240	70.13
- Real Estate Companies	10	10 * 410/255	16.08
Total value of shares	**210**		**391.67**
Bonds			
' - Listed 8000 14% FV 15000	12	12 * 14/8.842	19
Unlisted bonds	7		7
Total value of Bonds	**19**		**26**
Expenses			
1. Consultancy & Mgmt.			4.80
2. Office Expenses			1.50
3. Advertisement Expenses			0.38
Total expenses			**6.68**
Expenses payable			3.50
Cash			1.50

Part 1- Fund NAV			**415.67**
No of Units			4.20
Part 2- NAV per unit			**98.97**
			Part 3 - Net return Annualized
Closing Value of Investments			417.67
Dividends paid		₹ 6 / unit x 4.2	25.20
Original Investment value			229
Expenses & Cash			(2)
Total return over 2-year period			211.87
Absolute Return %			92.52%
Annualized return			**46.26%**
Closing NAV			98.97
Opening NAV	229		54.52
Dividends			6
Absolute Return			50.44
Absolute Return %			92.52%
Annualized return %			**46.26%**
Part 4- Expense ratio (**Expense** Per Unit / Closing NAV per Unit)		**₹6.68 CR / ₹4.2 CR)/ ₹98.97 x 100**	**1.607%**

21.Illustration

Mr. X on 1.7.2015, during the initial offer of some Mutual Fund invested in 10,000 units having face value of ₹10 for each unit. On 31.3.2016, the dividend paid by the M.F. was 10% and Mr. X found that his annualized yield was 153.33%. On 31.3.2017, 20% dividend was given. On 31.3.2018, Mr. X redeemed all his balance of 11,296.11 units when his annualized yield was 73.52%.
What are the NAVs as on 31.3.2016, 31.3.2017 and 31.3.2018?

[ICAI SM, RTP Nov'24, Nov'17 QP (Old), RTP May'19 Old, Old PM]

Solution:

Particulars	**Units**	**Dividend -% & Value**	**Yield %**
Initial Investment - 1 July 2015 @ NAV 10	10,000		
Dividend Paid - 31 Mar 2016		10%	
Dividend Value		10,000	
Annualized yield 9 Months			153.33%
9-month Yield			115.0%
Total Return	1,15,000		
Less: Dividend	10,000		
Increase in NAV	1,05,000		
Increase in NAV per unit	10.50		
Closing NAV - 31 Mar 2016	**20.50**		
Dividend Units allotted	487.80		
Total Units as at 31 Mar 2016	10,487.80		

31 Mar 2017 Dividend		20%	
Dividend Value		20,975.61	
NAV on Dividend Reinvestment date - 31 Mar 2017	**25.95**		
Units Reinvested	808.31		
31 Mar 2018 Total Units	11,296.11		
NAV	?		
Annualized yield over a 33-month period			73.52%
Absolute Return over 33 months %			202.2%
Absolute return Value over 33 months			2,02,180
Closing Value of Investment			3,02,180
Closing NAV on 31 Mar 2018			**26.75**

22. Illustration

M/S. Corpus an AMC, on 1.04.2015 has floated two schemes viz. Dividend Plan and Bonus Plan. Mr. X, an investor has invested in both the schemes. The following details (except the issue price) are available:

Date	Dividend (%)	Bonus Ratio	NAV Dividend Plan	NAV Bonus Plan
1.04.2015			?	?
31.12.2016		1 :4 (One unit on 4 units held)	47	40
31.03.2017	12		48	42
31.03.2018	10		50	39
31.12.2018		1 :5 (One unit on 5 units held)	46	43
31.03.2019	15		45	42
31.03.2020	-	-	49	44

Additional details

Investment (₹)	₹9,20,000	₹10,00,000
Average Profit (₹)	₹27, 748.60	
Average Yield (%)		6.40

You are required to calculate the issue price of both the schemes as on 1.04.2015.

[Nov'20 QP 10 marks, RTP May'23, Similar Dec'21 QP (Old)]

Solution:

Particulars	Units	NAV	Value	% return
Investment - Dividend Plan			9,20,000	
Investment Bonus Plan			10,00,000	
Average Profit Dividend Plan - 5 years			27,748.60	
Total Profit over 5 years			1,38,743	
Closing Value of Investment - Dividend Plan	21,607	49	10,58,743	
Bonus Plan - Average Yield				6.40%
5 Year Yield				32%
Return over 5 years			3,20,000	
Closing Value of Investment on 31/3/2020	30,000	44	13,20,000	

Bonus Plan	20,000	**50**	10,00,000	
31 Dec 2016 Bonus 1:4	20,000	40	8,00,000	
31 Dec 2018 Bonus 1:5	25,000	43	10,75,000	
31 mar 2020 - Closing	30,000	44	13,20,000	
Dividend Plan	20,000	**46**	9,20,000	
Dividend 31 Mar 2017	20,000	48	9,60,000	12%
Dividend 31 Mar 2018	20,500	50	10,25,000	10%
Dividend 31 Mar 2019	20,910	45	9,40,950	15%
31 Mar 2020 - Closing	21,607	49	10,58,743	

WN 1- Dividend for 31 Mar 2019

Opening Units	$= Y$
$\frac{Y + (Y \times 10 \times 15\%)}{45}$	$= 21607$
$45Y + 1.5Y$	$= 21607 * 45$
$46.5Y$	$= 21607 * 45$
Y	$= \frac{972315}{46.5}$
	$= 20{,}910$

23. Illustration

On 01-07-2016, Mr. X Invested ₹50,000/- at initial offer in Mutual Funds at a face value of ₹10 each per unit. On 31-03-2017, a dividend was paid @ 10% and annualized yield was 120%. On 31-03-2018, 20% dividend and capital gain of ₹0.60 per unit was given. Mr. X redeemed all his 6271.98 units when his annualized yield was 71.50% over the period of holding. Calculate NAV as on 31-03-2017, 31-03-2018 and 31-03-2019. For calculations consider a year of 12 months.

(RTP Nov'23, RTP May'20, Old PM)

Solution:

Particulars	Units	NAV	Value	% return
Investment 1/7/2016	5,000	10	50,000	
31 Mar 2017 Dividend			5,000	10%
Total Return - Annualized				120%
Absolute Return % over 9 months				90%
Absolute Return Value			45,000	
Less: Dividend			5,000	
Increase in Investment			40,000	
Closing Value of Investment	5,000	18	90,000	
Dividend	277.78	18	5,000	
Closing Units 31 /3/17	**5,277.78**	**18**	**95,000**	
31/3/18 Dividend			10,555.56	20%
31/3/18 CG			3,166.67	
Entire Amount Reinvested	**994.20**	**13.80**	**13,722.22**	
Redemption 31/3/19	6,271.98			
Annualized yield 33 months				71.50%
Absolute Yield over 33 Months				196.63%
Total Return over 33 months			98,312.50	
Closing Value of investment	**6,271.98**	**23.65**	**1,48,312.50**	
Assumption: CG is distributed in cash & is reinvested				

Particulars	Units	NAV	Value	% return
Investment 1/7/2016	5,000.00	10.00	50,000.00	
31 Mar 2017 Dividend			5,000.00	10.00%
Total Return - Annualized				120.00%
Absolute Return % over 9 months				90.00%
Absolute Return Value			45,000.00	
Less: Dividend			5,000.00	
Increase in Investment			40,000.00	
Closing Value of Investment	5,000.00	18.00	90,000.00	
Dividend	277.78	18.00	5,000.00	
Closing Units 31 /3/17	**5,277.78**	**18.00**	**95,000.00**	
31/3/18 CG	5,277.78	0.60	3,166.67	
31/3/ 18 / Dividend	**994.20**	**10.62**	**10,555.56**	
Redemption 31/3/19	6,271.98			
Annualized Yield 33 months				71.50%
Absolute yield 33 months				196.63%
Absolute return in ₹			98,312.50	
Less: CG			3,166.67	
Increase in Investment value			95,145.83	
Total closing value of investment	**6,271.98**	**23.14**	**1,45,145.83**	

Assumption: CG is distributed in cash & not reinvested

24. Illustration

ANP Plan, a hedge fund currently has assets of ₹20 crore. CA. X, the manager of fund charges fee of 0.10% of portfolio asset. In addition to it the charges incentive fee of 2%. The incentive will be linked to gross return each year in excess of the portfolio maximum value since the inception of fund. The maximum value the fund achieved so far since inception of fund about one and half year ago was ₹21 crores. You are required to compute the fee payable to CA. X, if return on the fund this year turns out to be (a) 29%, (b) 4.5%, (c) -1.8%. ***(ICAI SM, MTP Apr'24, RTP Nov'19 Old, Old PM)***

Solution:

Particulars	₹ Cr
Opening Fund value	20.00
Highest value since inception	21.00
Fund management charges	0.10%
Incentive	2.00%

Return in CY	**29.00%**	**4.50%**	**-1.80%**
Management fees	0.02	0.02	0.02
Incentive Computation			
Portfolio Return %	29.00%	4.50%	-1.80%
Portfolio Return absolute	5.80	0.90	(0.36)
Total Value of Portfolio	25.80	20.90	19.64
Less: Max value of portfolio	21.00	21.00	21.00

Return eligible for incentive	4.80	-	-
Incentive	0.10	-	-
Total Fee + Incentive - Cr	0.12	0.02	0.02
Total Fee + Incentive in ₹	**11,60,000.00**	**2,00,000.00**	**2,00,000.00**

25. Illustration

On 1.4.2016, a mutual fund scheme had 9,00,000 units of face value of ₹10 each. The scheme earned ₹81 lakh in 2016-17, out of which ₹45 lakh was earned in first half year. 1 lakh units were sold on 30.9.2016 at NAV of ₹60. Compute the dividend equalization reserve value.

Solution:

Date	**Particulars**		**Credit (₹)- In Lakhs**	**Debit (₹)- In Lakhs**
	Bank A/c	Dr	65	
	To Unit Capital A/c			10
	To Reserves			50
	To Dividend Equalization Reserve			5
	Dividend A/c	Dr	81	
	Dividend Equalization Reserve A/c	Dr	6	
	To Bank A/c			86

	₹ in Lakhs
Original Unitholders (9*8.6)	77.4
New Unitholders	8.6
Total	86
Dividend Equalization	(5)
Dividend	81

Investment = ₹ 60 lakh+ ₹ 5 lakh
Money Available = ₹ 86 lakh
Dividend = ₹ 86 lakh/ 10
= ₹ 8.6

26. Illustration

On 1st April, an open-ended scheme of mutual fund had 300 lakh units outstanding with Net Assets Value (NAV) of ₹18.75.
At the end of April, it issued 6 lakh units at opening NAV plus 2% load, adjusted for dividend equalization. At the end of May, 3 Lakh units were repurchased at opening NAV less 2% exit load adjusted for dividend equalization. At the end of June, 70% of its available income was distributed.
In respect of April-June quarter, the following additional information are available:

	₹ in lakh
Portfolio value appreciation	**425.47**
Income of April	**22.950**
Income for May	**34.425**
Income for June	**45.450**

You are required to calculate

(i) Income available for distribution;
(ii) Issue price at the end of April;
(iii) repurchase price at the end of May; and

(iv) net asset value (NAV) as on 30th June

[ICAI SM, May-25 Similar 7M, Jan'21 QP (Old), MTP Oct'24, MTP Apr'19, MTP Aug'18, RTP Nov'18 Old, RTP May'19 Old, RTP Nov'19 Old, Old PM]

Solution:

Particulars	Lakh Unit	Per Unit	₹ Lacs
Part 1- Income Available for Distribution			
' - Income for April	300.0000	0.0765	22.9500
New Investment	6.0000	0.0765	0.4590
	306.0000	**0.0765**	**23.4090**
' - Income for May	306.0000	0.1125	34.4250
Less: Units redeemed	(3.0000)	0.1890	(0.5670)
Income available at may End	303.0000	0.1890	57.2670
Income for June	303.0000	0.1500	45.4500
Income available at June end	**303.0000**	**0.3390**	**102.7170**
Less: Income Distributed	303.0000	(0.2373)	(71.9019)
Closing money available	**303.0000**	**0.1017**	**30.8151**
Part 2 - Issue price at end of April			
1st April Data	300.0000	18.7500	5,625.0000
Issue price on 30th April			
- Opening NAV		18.7500	
- 2% Entry Load		0.3750	
- Dividend Equivalization		0.0765	
Issue price on 30th April		**19.2015**	
Part 3 - Repurchase price at end of May			
- Opening NAV		18.7500	
- Less: Load 2%		(0.3750)	
Add: Dividend Equalization - May		0.1125	
Add: Dividend Equalization - April		0.0765	
Repurchase price		**18.5640**	
Part 4- NAV as on June 30			
Opening Units - 1st April	300.0000	18.7500	5,625.0000
Portfolio Appreciation			425.4700
Income - April			22.9500
Income - May			34.4250
Income June			45.4500
Less: Dividend			-71.9019
Investment in April	6.0000	19.2015	115.2090
Repurchase in May	-3.0000	18.5640	-55.6920
Total Value of Assets	**303.0000**	**20.2670**	**6,140.9101**

27. Illustration

On 1st January 2020, an open-ended scheme of mutual fund had outstanding units of 300 lakhs with a NAV of ₹20.25. At the end of January 2020, it had issued 5 lakh units at an opening NAV plus a load of 2%, adjusted for dividend equalisation. At the end of February 2020, it had repurchased 2.5 lakh units at an opening NAV less 2% exit load adjusted for dividend equalisation. At the end of March

2020, it had distributed 70 per cent of its available income. In respect of January – March quarter, the following additional information is available:

Value appreciation of the Portfolio	₹460 lakhs
Income for January	₹24 lakhs
Income for February	₹36 lakhs
Income for March	₹47 lakhs

You are required to calculate:

(i) Income available for distribution
(ii) Issue price at the end of January
(iii) Repurchase price at the end of February
(iv) Closing value of Net Assets at the end of March

(Jan'21 QP, MTP Mar'23)

Solution:

Particulars	Lakh Unit	Per Unit	₹ Lacs
Part 1 - Income for Distribution			
Opening Units	300	20.25	6075
Income for Jan	300	0.08	24
New units issued	5	0.08	0.40
Income at end of Jan	305	0.08	24.40
Income for Feb	305	0.12	36
Less: Issued repurchased	2.50	0.20	0.50
Income at end of Feb	302.50	0.20	59.90
Income for March	302.50	0.16	47
Total Income (Jan - Mar)	302.50	0.35	106.90
Dividend Distribution @ 70%	302.50	0.25	74.83
Balance amount left	302.50	0.11	32.07
Part 2 - Issue price Jan			
Opening NAV	5	20.25	101.25
Add: Entry Load @ 2%	5	0.41	2.03
Add: Dividend Equalization	5	0.08	0.40
Issue Price	5	20.74	103.68
Part 3 - Repurchase price Feb			
Opening NAV	2.50	20.25	50.63
Less: Exit Load @ 2%			1.01
Add: Dividend Equivalization	2.50	0.20	0.50
Repurchase price	2.50	20.04	50.11
Part 4 - Closing value of Net Assets			
Opening Units	300	20.25	6075
Add: Portfolio Appreciation			460
Add: New issue	5		103.68
Less: Repurchase	2.50		50.11
Add: Incomes (Jan - Mar)			107
Less: Dividend Distributed			74.83
Total	**302.50**	**21.89**	**6620.73**

28. Illustration

Based on the following data, estimate the Net Asset Value (NAV) on 1st July 2016 on per unit basis of a Debt Fund:

Security	Face Value (₹)	Purchase Price (₹)	Maturity Date	No. of Securities	Coupon Date(s)	Duration of Bonds
10.71% GOI 2028	100	104.78	31/03/28	100000	Mar 31	7.3494
10% GOI 2023	100	100	31/03/23	50000	Mar 31 & Sep 30	5.086
9.5% GOI 2021	100	97.93	31/12/21	40000	June 30 & Dec 31	4.3949
8.5% SGL 2025	100	91.36	30/06/25	20000	June 30	6.5205

Number of Units (₹ 10 face value each): 100000.
All securities were purchased at a time when applicable Yield to Maturity (YTM) was 10%.
On NAV date, the required yield increased by 75 basis point and Cash in hand and accrued expenses were ₹ 6,72,800 and ₹ 2,37,400 respectively.

Solution:

	Security	Purchase Price	No:	Purchase value (in lakhs)
1	10.71% GOI 2028	104.78	100000	104.78
2	10% GOI 2023	100	50000	50
3	9.5% GOI 2021	97.93	40000	39.172
4	8.5% SGL 2025	91.36	20000	18.272
				212.224

	Portfolio Value	Macaulay Duration	Modified Duration	Δ Price	Revised/ Closing
1	10478000	7.3494	6.6813	-525048	9952952
2	5000000	5.086	4.8438	-181643	4818357
3	3917200	4.3949	4.1856	-122969	3794231
4	1827200	6.5205	5.9277	-81234	1745966
	21222400			**-910894**	**20311506**

Closing Bond Value	**₹ 20311506**
Add: Cash in Hand	₹ 672800
Less: Accrued Expense	₹ 237400
Add: Interest Accrued (WN 1)	₹ 392750
Value of Net Assets	₹ 21139656
Total Units	100000
NAV per Unit	**₹ 211.3965**

Modified Duration = $\frac{Macaulay\ Duration}{1+\frac{y}{n}}$

-Δ price = Δ yield* modified duration* price

Working Note: 1

Security		
10.71% GOI 2028	100* 100000* 10.71%* 3/12	267750
10% GOI 2023	100* 50000* 5%* 3/6	125000
		392750

29. Illustration

On 1st April, an open-ended scheme of mutual fund had 400 lakh units outstanding with Net Assets Value (NAV) of ₹ 19. At the end of April, it issued 5 lakh units at an opening NAV plus 2% load, adjusted for dividend equalization. At the end of May, 4 Lakh units were repurchased at the opening NAV less 2% exit load adjusted for dividend equalisation. At the end of June, 60% of its available income was distributed.

In respect of April-June quarter, the following additional information is available:

Particulars	**₹ in Lakhs**
Portfolio value appreciation	**515.67**
Income of April	**31.960**
Income of May	**46.125**
Income for June	**58.470**

You are required to calculate:

(i) Income available for distribution

(ii) Issue price at the end of April

(iii) Repurchase price at the end of May; and

(iv) Net Asset Value (NAV) as on 30th June.

(RTP May'22)

Solution:

Income Available for Distribution

Particulars	**Units in Lakhs**	**Per Unit**	**₹ Lacks Income**
Income for April	400	0.0799	31.96
Units Issued	5	0.0799	0.3995
End of April	**405**	**0.0799**	**32.3595**
Income for May	405	0.113888889	46.125
Closing Before Repurchase	405	0.193788889	78.4845
Repurchase Units	-4	0.193788889	-0.775155556
End of May	**401**	**0.193788889**	**77.70934444**
Income for June	401	0.145810474	58.47
End of June	**401**	**0.339599363**	**136.1793444**
Distribution at 60%	401	-0.2038	-81.7076
	401	**0.1358**	**54.4717**

Issue Price at April End

Particulars	**₹ per Unit**
Opening NAV (5 lakh units)	₹ 19
Add: Entry Load at 2%	₹ 0.38
Add: Dividend Equalization	₹ 0.0799
	₹ 19.4599

Repurchase Price at May End

Particulars	₹ per Unit
Opening NAV (4 lakh units)	₹ 19
Less: Exit Load at 2%	₹ -0.38
Add: Dividend Equalization	₹ 0.1938
	₹ 18.8138

Closing NAV at June End

Particulars		**Amount**
Opening NAV (400 lakh units)	₹ 19	₹ 7,600
Add: Incomes		
April	₹ 31.9600	
May	₹ 46.1250	
June	₹ 58.47	₹ 136.5550
Less: Dividend		₹ -81.7076
Add: Portfolio Appreciation		₹ 515.670
Add: Units Issued 5* 19.4599		₹ 97.2995
Less: Units Repurchased 4* 18.8138		₹ -75.2552
401 Lakh Units		**₹ 8,192.5617**
NAV	8192.5617/ 401	**₹ 20.4303**

30. Illustration

Mr. D had invested in three mutual funds (MF) as per the following details:

Particulars	MF 'A'	MF 'B'	MF 'C'
Amount of Investment	2,00,000	5,00,000	4,00,000
NAV at the time of purchase	10.00	25.00	20.00
Dividend Yield up to 31.03.2022	3%	5%	4%
NAV as on 31.03.2022	10.50	22.80	20.80
Annualized Yield as on 31.03.2022	9.733%	- 11.185%	15%

Assume 1 Year = 365 Days.

Mr. D has misplaced the documents regarding his investments. You are required to help Mr. D to find out the following:

(i) Number of units allotted in each scheme,

(ii) Value of his investments as on 31.03.2022,

(iii) Holding period of his investments in number of days as on 31.03.2022

(iv) Dates of original investments

(v) Total Return on investments

(vi) Assuming past performance of all three schemes will continue for next one year, what action the investor should take? What will be the expected return for the next one year after the above action?

(vii) Will your answer as above of point (vi) change is the MF charges exit load of 5% for investments redeemed within a year? If so advise the investor what and when should be taken to optimize the return.

(May'22 QP 8 marks)

Solution

i) Number of units allotted in each scheme

	Investment ₹ (A)	NAV (₹/unit) (B)	Units (A)/(B)
A	2,00,000	10	**20,000**
B	5,00,000	25	**20,000**
C	4,00,000	20	**20,000**

ii) Value of his investments as on 31.03.2022,

Units (A)	Closing NAV (₹/unit) (B)	Units (A)*(B)
20,000	10.5	2,10,000
20,000	22.8	4,56,000
20,000	20.8	4,16,000
		10,82,000

iii) Holding Period:

Total return % $= \frac{CG + Dividend}{Original\ Investment}$

	Investment		CG	Dividend Yield	Dividend Value	Total Value	Return %
	Closing (A)	Opening (B)	(C) = (A)- (B)	(D)	(E) = (B)*(D)/100	(F) = (C) + (E)	(F)*(B)/100
A	2,10,000	2,00,000	10,000	3%	6,000	16,000	8%
B	4,56,000	5,00,000	-44,000	5%	25,000	-19,000	-3.8%
C	4,16,000	4,00,000	16,000	4%	16,000	32,000	8%
			18,000		47,000	29,000	

MF	Absolute Return % (X)	Annualized Return % (Y)	No. of Days: $\frac{X}{Y} * 365$
A	8%	9.733%	**300**
B	-3.8%	-11.185%	**124**
C	8%	15%	**195**

iv) Dates of Original Investments:

Mutual Fund A: 65 days (365-300)
Mutual Fund B: 241 days (365-124)
Mutual Fund C: 170 days (365-195)

	A	B	C
April	30		30
May	31		31
June	**4**		30
July			31
August			31
September			**17**
October			
November		3	

December		31	
January		31	
February		28	
March		31	
	65	241	170

A: = 4^{th} June Evening or 5^{th} June Morning
B: = 27^{th} November Evening
C: = 17^{th} September Evening

v) Total Return on Investments:

$$\text{Total return \%} = \frac{(CG + Dividend) * 100}{Original\ Investment}$$

$$= \frac{29{,}000 * 100}{11{,}00{,}000} = \mathbf{2.636\%}$$

vi) Redeem all investments in Mutual Funds a and B and reinvest it in Mutual Fund C. The expected return for next 1 year of this Portfolio will be 15%

vii) Redeem Mutual Fund B right away and invest in C as if it has to be redeemed after 1 Year. This implies additional 240 days it has to be invested during which loss of 8% (11.85*241/365) has to be incurred which is greater than 5% exit load.

31. Illustration

Mr. S has invested in 3 different Mutual Fund Schemes. The following are the details of the same:

Particulars	Scheme A	Scheme B	Scheme C
Date of Investment	01-06-2022	01-07-2022	01-08-2022
Net Asset Value at Entry Date	₹ 11.00	₹ 10.50	₹ 12.00
Dividend received upto 31-03-23 (₹)	12,500.00	17,000.00	4,000.00
Unit NAV at 31-03-23 (₹)	11.25	11.48	10.80
Increase / (Decrease) in NAV (₹)	22,727.27	93,333.33	(50,000.00)
Effective Rate of Yield per annum	4.2296%	14.6978%	(-) 13.8190%

Ignore Entry/Exit load expenditure.

Assume 365 days in a year. Round off the investment to nearest ₹100. You are required to calculate:

- **The amount of investments made initially by Mr. S in these schemes.**
- **Number of units invested in the three schemes by Mr. S.**

Advise also whether he can continue to hold this investment or can he redeem now.

(Nov 23'QP 8 marks)

Solution

Method 1:

Particulars	Scheme A	Scheme B	Scheme C
NAV Opening / Investment Date	11.00	10.50	12.00
NAV Closing	11.25	11.48	10.80
Increase / (Decrease) in NAV (₹ per unit	0.25	0.98	-1.20

Increase / (Decrease) in NAV (₹)	22,727.27	93,333.33	(50,000.00)
Units = Inc in Scheme Value / Inc in NAV	22,727.27 / 0.25	93,333.33/0.98	(50,000.00) / -1.20
Units	90909.08	95238.09	41666.67
= > Investment value = Units X Investment NAV	11 x 90909.08	11.48 x 95238.09	12 x 41666.67
Investment Value	Rs.9,99,999.88	Rs.9,99,999.945	Rs.5,00,000.04
Investment value rounded off	Rs.10 lacs	Rs.10 Lacs	Rs.5 Lacs

Method 2:

Particulars	Scheme A	Scheme B	Scheme C
Date of Investment	1st June 2022	1st July 2022	01 August 2022
Days of Investment till 31st march 2023	304	274	243
Dividend	12500	17000	4000
Increase / (Decrease) in NAV (₹)	22,727.27	93,333.33	(50,000.00)
Total Return Div + Inc in NAV - A	35227.27	110333.33	-46000
Yield Per Annum	4.2296%	14.6978%	-13.8190%
Yield of days of investment - B	4.2296% x 304 / 365 = 3.5227%	14.6978%x 274 / 365 = 11.0334%	-13.8190% X 243 /365 = -9.2000%
Investment Value C = A / B	Rs.10,00,007.66	Rs.10,00,000	Rs.5,00,000
Investment value rounded off	Rs.10 lacs	Rs.10 Lacs	Rs.5 Lacs
Units of Investment made = C / Opening NAV	Rs.10 Lacs / 11	Rs.10 Lacs / 10.5	Rs.5 Lacs / 12
Units of investment made	90909.08	95238.09	41666.67

The investor should redeem investments in Schemes A & C and invest the money in Scheme B as it has higher return

32. Illustration

Mr. Kar a as per details below :

	MFX	MFY	MFZ
Amount of investment (Rs.)	5,50,000	4,20,000	1,00,000
Dividend received up to 31.03.2023 (Rs.)	10,000	6,000	Nil
NAV as on 31.03.2023 (Rs.)	11.50	11.00	9.50
Effective yield p.a. as on 31.03.2023	19.345	22.59%	-36.50%
Holding period	120	100	50

You are required to calculate Net Asset Value (NAV) at the time of purchase assuming 365 days in a year.

(May'24 QP)

Solution:

	MFX	MFY	MFZ
Amount of investment (Rs.) - A	5,50,000	4,20,000	1,00,000
Dividend received up to 31.03.2023 (Rs.)	10,000	6,000	Nil

NAV as on 31.03.2023 (Rs.)	11.50	11.00	9.50
Effective yield p.a. as on 31.03.2023	19.345	22.59%	-36.50%
Holding period	120	100	50 days
Yield for Holding Period	19.345% x 120 / 365	22.59% x 100/365	-36.50% x 50 / 365
Yield for Holding Period - B	6.36%	6.189%	-5%
Absolute yield generated in INR C = A x B /	34980	25994	(5000)
Less: Dividend - D	10,000	6,000	Nil
Increase in investment value in INR: E = C – D	24980	19994	(5000)
Closing value of Investment F = A + E	574980	439994	95000
Closing NAV – G	11.50	11.00	9.50
Closing Units H = F / G	49998.26	39999.45	10000
Opening NAV I = A / H	~11	~10.5	~10

33. Illustration

Mr. A, has invested in the Growrich Mutual Fund's Scheme. The details of the Mutual Fund Scheme are given below :

Asset Value at the beginning of the month	**₹78.50**
Annualized Return	**16%**
Distribution made in the nature of Income and Capital Gain (per unit respectively)	**₹0.40& ₹ 0.30**

You are required to :

(i) Calculate the month end Net Asset Value of the Growrich Mutual Fund Scheme (Round off to 2 decimals)

(ii) Comment briefly on the Month end NAV.

(May'24 QP, Similar ICAI SM)

Solution

Return = (Closing Value – Opening Value + Capital Gain Distributed + Dividend Distributed) / Opening Value

Annualised return = 16%

Monthly Return = 16/12 = 1.33%

1.33% = (Closing – 78.50+0.40 + 0.30) / 78.50

1.33% = (Closing – 77.80)/ 78.50

78.5 x 1.33% = Closing – 77.80

1.0467 = closing -77.80

Closing Value = 77.80 + 1.0467

Closing Value = 78.8467 = Rs.78.85

34. Illustration

Mr. X invested ₹1,00,000 on face value of ₹10 per unit in a dividend reinvestment plan in a mutual fund during its initial offer on 1st July, 2022. On 31st March, 2023, the fund declared a dividend of 10%. At that time Mr. X calculated his holding period return to be in the range of 115%.

On 31st March, 2024, the mutual fund declared a dividend of 20% and Mr. X redeemed all his investment and calculated his holding period return to be 193.134%.
You are required to calculate:
(i) The NAVs as on 31.03.2023 and 31.03.2024.
(ii) Calculate the total units redeemed.

(Similar ICAI SM, Nov'24 QP 6 marks)

Solution:

Amount Invested in ₹ Lacs	**100000**
Face Value per Unit in ₹	10
Units Invested = 100000/10	10000
Holding period Return 1 July 2022 to 31st March 2023	115%
Holding period Return = (Closing Value – Opening Value)/Opening Value	115%
(Closing – ₹100000)/₹100000 =	115%
Closing	115% x ₹100000 + ₹100000
Closing value	₹ 2,15,000
Less: Dividend Declared per WN 1 Below	₹ 10,000
Closing Value of AUM	₹ 2,05,000
Closing NAV = ₹205000 / 10000 units	₹ 20.50
Dividend is reinvested in MF @NAV of ₹20.50	
Units Allotted ₹10000/₹20.50	487.8
Closing Units = Opening Units + Dividend Reinvested Units	10000 + 487.80
31/3/2023 - Closing Units	10487.8
Holding period Return 1 /07/22 to 31/03/2024	193.13%
(Closing – ₹100000)/₹100000 =	193.13%
Closing Value of Amount Invested	₹193134+₹100000
Closing Value of Amount Invested	₹ 2,93,134
Dividend Declared = 10487.80 x 2	₹ 20,976
Closing Value of investment without Dividend	₹293134- ₹20976= 272158
NAV per unit of investment value without Dividend	₹272158 / 10487.80 Units
NAV per unit of investment value without Dividend	₹25.95 per Unit
Dividend Amount Reinvested	₹ 20,976
Units allotted for Dividend Amount	₹20976 / ₹25.95 per Unit = 808.32 Units
Total Unit count	10487.80 + 808.32 = 11296.12 Units
NAV	₹ 25.95

(Cross Check 25.95 x 11296.12 = 293134)

Conclusion

	31 Mar 2023	**31 Mar 2024**
NAV	₹20.50	₹25.95

Units Held	10487.80	11296.12
Total Units Redeemed	**11296.12**	

WN 1
Dividend Declared = 10%
Dividend Declared in ₹ = ₹100000 x 10% = ₹10000
Assumption – Entire Dividend is Invested Back in Units at same NAV

35. Illustration

Mr. X has excess cash of ₹ 50 lakhs which he wants to invest in 4 short-term marketable securities.
Expenditure relating to this investment will be ₹ 1,25,000 and the securities invested will have an annual yield of 9%.
Mr. X seeks your advice –
(i) as to the period of investment so as to earn a pre-tax income of 5%.
(ii) as to the minimum period to break-even his investment expenditure.

(May'25 3(b) – 4 Marks)

Solution

(i) Period of Investment for pre tax income of 5%

Assuming Expenditure is not incurred initially but towards the end

If X is the Number of months of Investment, then

(₹50 Lakhs * 9% * X/ 12)- ₹1.25 lakhs = ₹50 Lakhs x 5%

- ⇨ (₹4.5Lakhs X / 12) - ₹1.25 Lakhs = ₹2.5 Lakhs
- ⇨ (₹4.5 Lakhs X – 15 Lakhs)/12 = ₹2.5 Lakhs
- ⇨ ₹4.5 Lakhs X - ₹15 Lakhs = 12 *₹2.5 Lakhs
- ⇨ ₹4.5 Lakhs X - ₹15 Lakhs = ₹30 Lakhs
- ⇨ ₹4.5 Lakhs X = ₹45 Lakhs
- ⇨ X = ₹ 45 Lakhs/ ₹4.5 Lakhs = 10 months

Assuming Expenditure is incurred initially, then

(₹50 Lakhs -₹1.25 Lakhs) * 9% * X/ 12 -₹1.25 Lacs = ₹50 Lakhs x 5%

- ⇨ (₹48.75Lakhs *9%*X/ 12) – ₹1.25 = ₹2.5 Lakhs
- ⇨ 36562.5X = ₹3.75 Lakhs
- ⇨ X = ₹ 3.75 Lakhs/ ₹36562.5
- ⇨ X = 10.25 months

(ii) Break Even period for investment

Assuming Expenditure is not incurred initially but towards the end

If X is the Number of months of Investment, then

(₹50 Lakhs * 9% * X/ 12)- ₹1.25 lakhs = 0
₹4.5Lakhs X / 12 = ₹1.25 Lakhs
X = ₹1.25 Lakhs / ₹4.5 Lakhs * 12
X = 3.33 months

Assuming Expenditure is incurred initially

If X is the Number of months of Investment, then

((₹50 Lakhs-₹1.25 Lakhs) * 9% * X/ 12)- ₹1.25 lakhs = 0

₹48.75Lakhs * 9% *X / 12 = ₹1.25 Lakhs

₹48.75 Lacs * 9% *X = ₹15 Lakhs

X = ₹15 Lakhs / ₹4.3875 Lakhs

X = 3.4188 months

36. Illustration

On 1st April, an open-ended scheme of Progressive Mutual Fund had 800 Lakh units outstanding with Net Asset Value (NAV) of ₹ 44.30 per unit. At the end of April, it issued 16 Lakh units at opening NAV plus 2% load, adjusted for dividend equalization. At the end of May, 8 Lakh units were repurchased at opening NAV less 2% exit load adjusted for dividend equalization. At the end of June, 60% of its available income was distributed as dividend. In respect of April-June quarter, the following additional information is available

Particulars	₹ in Lakh
Portfolio Value Appreciation	965.280
Income of April	49.440
Income for May	75.070
Income for June	93.710

Calculation up to 4 decimal place.

You are required to calculate-

(i) Income available after distribution of dividend

(ii) Issue price at the end of April

(iii) Repurchased price at the end of May

(iv) Net Asset Value (NAV) per unit as on 30th June

(May'25 5(a) – 7 Marks)

Solution:

Computation of Income available for Distribution

Total Income	**218.22**
Add Div Equalisation received	0.9888
Less: Div equalisation paid	-1.2304
Grand Total	**217.9784**
Distributed @60%	130.7870
Balance	**87.1914**

April End issue price

NAV	44.3618
Add: Entry loan @ 2% of opening NAV	0.8860
Net price	**45.2478**
Units	16.0000
Total Inflow	723.9648

Re purchase price May

NAV	44.4538
Less: Exit Load @%	0.8860
Net price	**43.5678**
Units	8.0000
Total Outflow	**348.5424**

30th June NAV	
Portfolio Opening	35,440.0000
Add: Appreciation	965.2800
Incomes	218.2200
Less: Dividend paid	- 130.7871
New units issued 16 L	723.9648
Repurchase 8 lacs	- 348.5424
Asset total	**36,868.1354**
Units	808.0000
NAV	45.6289

Working Notes:

Date	Description	Units in Lacs	NAV	Value
01-Apr	Opening	800	44.3	35440.0000
30-Apr	Income	800	0.0618	49.4400
30-Apr	**Total**	**800**	**44.3618**	**35489.4400**
30-Apr	New units issued	16	44.3618	709.7888
30-Apr	**Closing**	**816**	**44.3618**	**36199.2288**
01-May	Opening	816	44.3618	36199.2288
31-May	Income	816	0.091998	75.0700
31-May	**Total**	**816**	**44.4538**	**36274.3008**
31-May	Repurchase	-8	44.4538	-355.6304
31-May	**Closing**	**808**	**44.4538**	**35918.6704**
01-Jun	Opening	808	44.4538	35918.6704
30-Jun	Income	808	0.115978	93.7100
30-Jun	**Closing**	**808**	**44.56978**	**36012.3804**
30-Jun	Less: Income distributed	808	-0.16187	-130.7870
30-Jun	**Closing**	**808**	**44.40791**	**35881.5934**

37. Illustration

Mr. DK has 1,000 units of AM Mutual Fund. NAV of it is ₹ 17.50 per unit and ₹ 18.90 per unit at the beginning and at the end of the year respectively. The Mutual Fund has given two options:

Option I: Pay ₹ 1.50 per unit as dividend and ₹ 1.20 per unit as a capital gain, or

Option II: Reinvest these distributions at an average NAV of ₹ 17.30 per unit.

You are required to compute the holding period return percentage for both the options and select which option is preferable.

(Sep-25– 4 Marks)

Solution:

NAV Beginning of the Period = ₹17.50

NAV End of the Period = ₹18.90

Option 1:

HPR = (Closing – Opening + Div + CG)/ Opening

= (18.90 -17.50+1.50+1.20)/17.50

= 4.1/17.5

= 23.43%

Option 2:

Total Distributions CG + Div = 1000units (1.5 +1.2) = ₹2700
Units allotted = ₹2700/₹17.3 per unit = 156.0694

Total units closing = 1156.0694

HPR = (11556.0684 x 18.90 – 17.50 x 1000)/(17.50 x 1000)
= (21849.70 – 17500)/17500
=24.855%

Option 2 is preferrable as the Holding period Return is higher in this case

38.Illustration

M/s. Wealth Builders, an Asset Management Company (AMC), launched a dividend bonus scheme on 1st April 2019. The fund demonstrated strong performance over the years.

Key events are as follows:

- **On 30th September 2021, the fund declared a bonus of 1:4 (one bonus unit for every four existing units held).**
- **On 30th September 2023, a second bonus of 2: 5 (two bonus units for every five existing units held) was declared.**

Ms. Investor made a lump-sum investment of ₹ 25 lakhs in the scheme at its inception and remained invested throughout. As of 31st March 2025, her investment has generated an average annual yield of 16.8%.

The Net Asset Value (NAV) of the scheme on various dates is provided below:

Particulars	30.09.2021	30.09.2023	31.03.2025
NAV (in ₹)	78	88	110

Required: Determine the opening NAV per unit as on 1st April 2019 for Ms. Investor's holding. ("Round off all intermediate and final calculations to two decimal places.")

(Jan 26 – Q 1(a) 6 Marks)

Solution:
Given:
Issue Date 1/04/19

Original NAV	?	
30/9/21	78	Bonus 1 for 4
30/9/23	88	Bonus 2 for 5
31/3/2025	110	
Investment	₹ 25 Lacs	

Average Annual yield for 6 years = 16.8%
=>Holding period Yield = 16.8 x 6 = 100.8%
Let opening NAV be Y and opening units be X then XY = ₹25 Lacs
Bonus on 30/9/21 - 1 for 4

⇨ For X units X/4 units are allotted as bonus: Revised units = X + X/4 = 5X/4 units

Bonus on 30/9/23 – 2 for 5

⇨ Revised units = 5X/4 + [5X/4] x 2/5

= 2X/4 + 5X/4

Revised units =7X/4

Closing Investment Value = ₹110 x 7X/4 Units = ₹192.5 X------(1)

Opening Investment Value = ₹25 Lacs = XY ------------(2)

Closing investment based on yield = ₹25 x (1+100.8%) = ₹50.2 Lacs --------(3)

Equating (1) and (3) ₹192.5X = ₹50.2 Lacs

⇨ X = ₹50.2 lacs /₹192.5

⇨ X = 26077.92 Units

⇨ Substituting value of X in (2)

⇨ Y = ₹25 lacs / 26077.92 Units

⇨ Y = ₹95.87/Unit

Opening Units = 26077.92 & Opening NAV = ₹95.87/unit

39. Illustration

Stork Capital, a SEBI Registered Mutual Fund, launched its first New Fund Offer (NFO) on June 1, 2024, with a face value of ₹ 10 per unit. The fund received subscriptions for 180 lakh units.

An underwriting agreement was in place with Griffin Securities Ltd., which agreed to underwrite the entire issue of 200 lakh units for a commission of 2.0%. The fund's financial activities are summarized below:

- **Initial investments in various capital market instruments amounted to ₹ 1,780 lakhs.**
- **Marketing expenses for the NFO were ₹ 25 lakhs.**
- **During the financial year ended March 31, 2025, the fund sold securities with a cost of ₹ 250 lakhs for ₹ 280 lakhs.**
- **The fund subsequently purchased new securities for ₹ 265 lakhs.**
- **Management expenses are regulated by SEBI and cannot exceed 0.50% of the average funds invested during the year. The actual management expenses incurred were ₹ 5.50 lakhs, of which ₹ 50,000 was outstanding at year-end.**
- **Dividends earned on investments amounted to ₹ 5.0 lakhs, of which ₹ 40,000 was yet to be collected.**
- **The fund's policy is to distribute 80% of all realized earnings (capital gains and dividends).**
- **The market value of the investment portfolio as of March 31, 2025, was ₹ 2,150.50 lakhs.**

Required: Determine the closing per unit Net Asset Value (NAV) of the fund as on March 31, 2025. Show all necessary workings. (Q 6a Jan 26 - 6 Marks Similar to FR Nov 2011)

(Note: Round off all intermediate and final calculations to two decimal places.)

Solution:

(1) Computation of Opening Cash Balance

Particulars	Amount (₹ in Lakhs)
Proceeds from NFO (200 Lakh units @ ₹ 10)	2000.00

Less: Initial Purchase of Securities	(1780.00)
Less: Underwriting Commission (2% of 2000)	(40.00)
Less: Meeting Expenses	(25.00)
Opening Cash Balance	**155.00**

(2) Computation of Closing Balance

Particulars	**Amount (₹ in Lakhs)**
Opening Cash Balance	155.00
Add: Cash Inflows	
Proceeds from Sale of Securities	280.00
Dividend Received (₹ 5.00 Lakh Earned – ₹ 0.40 Lakh uncollected)	4.60
Less: Cash Outflows	
Cost of New Securities Purchased	(265.00)
Management Expenses	(5.00)
Capital Gains Distributed [80% of (₹ 280 sale – ₹ 250 cost)]	(24.00)
Dividend Distributed (80% of ₹ 5.00 Lakh total earned)	(4.00)
Closing Cash Balance	141.60

(3) Management Expenses

		Amount (₹ in Lakhs)
A.	Actual Expenses Incurred	5.50
B.	SEBI Limit (0.50% of Average Funds)	
	- Opening Investment	1780.00
	- Closing Investment (1780 – 250 + 265)	1795.00
	- Average Fund Invested	1787.50
	- SEBI Limit (0.50% of ₹ 1787.50)	8.94
Chargeable Expenses (Lower of A or B)		5.50
Less: Amount unpaid (Outstanding)		(0.50)
Management Expenses Paid		**5.00**

(4) Net Asset Value (NAV) as on March 31, 2025

Particulars	**Amount (₹ in Lakhs)**
Assets:	
Closing Cash Balance	141.60
Closing Market Value of Investment	2150.50
Accrued Dividends (Receivable)	0.40
Total Assets	2292.50
Less: Liabilities	
Outstanding Management Expenses	(0.50)
Closing Net Assets (A)	2292.00
Total Units Outstanding (in Lakhs) (B)	200.00
NAV per Unit (A/B)	**₹ 11.46**

1. Illustration

The 6-months forward price of a security is ₹208.18. The borrowing rate is 8% per annum payable with monthly rests. What should be the spot price?

(ICAI SM, Old PM)

Solution:

Particulars	₹	%	Period
Forward Rate	208.18		
Interest rate - PA - monthly rests		8%	
Contract tenure			6 months
Spot	?		
Forward Price = $Spot * (1 + r/n) \wedge (nt)$			
$F = S * (1 + 8\%/12) \wedge (12 * 0.5)$			
$208.18 = S * (1 + 0.67\%) \wedge 6$			
$208.18 = S * 1.0409$			
$S = 208.18/1.0409$	200		
Spot	200		

2. Illustration

The following data relate to Anand Ltd.'s share price:

- Current price per share ₹. 1,800
- 6 months future's price/share ₹. 1,950

Assuming it is possible to borrow money in the market for transactions in securities at 12% per annum, you are required:

(i) to calculate the theoretical minimum price of a 6-months forward purchase; and
(ii) to explain arbitrate opportunity.

(ICAI SM, RTP May'21, MTP Sept'22, RTP Nov'18 Old, Old PM)

Solution:

Particulars	₹	%	Period
Spot	1800		
Future price	1950		
Int rate %		0.12	
Contract Tenure			6 months
$Forward = Spot\ x\ (1 + Coc\%)$			
$Coc\% = 12\%\ x\ 6/12$			
$Coc\% = 6\%$			
$Forward = Spot\ X\ (1 + 6\%)$			
$F = 1800\ X\ (1.06)$	1908		
On day 0			
Borrow money for 6 months @ 12%	1800	0.12	6 months
Invest in Stock of Anand Limited	-1800		

Short futures @	1950		
After 6 months			
Deliver Share	1950		
Repay loan 1800+ 12% interest	-1908		
Net gain	**42**		

3. Illustration

On 31-8-2011, the value of stock Index was ₹. 2,200. The risk-free rate of return has been 8% per annum. The dividend yield on this Stock Index is as under

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Dividend p.a (%)	3	4	3	3	4	3	3	4	3	3	4	3

Assuming that interest is continuously compounded daily, find out the future price of contract deliverable on 31-12-2011. Given: $e^{0.01583}$= 1.01593.

(ICAI SM, Old PM)

Solution:

Particulars	₹	%	Period
Value of Index		8%	
Rf Rate			31-Dec-11
Maturity date of future contract			
Dividend Accrued in 4 months- 3%,3%,4%,3%		3.25%	
Average Dividend per month			
$F = Spot * e^{\wedge}((r - d) * t)$			
$F = Spot * e^{\wedge}((8\% - 3.255) * 4/12)$			
$F = 2200 * e^{\wedge}(1.5833\%)$			
$F = 2200 * e^{\wedge}(0.015833)$			
$F = 2200 * 1.01593$	**2235.046**		

4. Illustration

Calculate the price of 3 months PQR futures, if PQR (Face Value is ₹.10) quotes ₹. 220 on NSE and the three months future price quotes at ₹. 230 and the per annum borrowing rate is given as 15 percent and the expected annual dividend is 25 percent per annum payable before expiry. Also examine arbitrage opportunities.

(ICAI SM, MTP Sept'23, RTP May'18 Old, RTP Nov'19 Old, MTP Mar'18 Old, Old PM)

Solution:

Particulars	₹	%
Spot of PQR (FV 10)	220	
3-month Future	230	
Borrowing Rate PA		0.15
Annual Expected Dividend	2.5	0.25
Theoretical price of Future		
$F = S + Coc - Div$		
$F = \frac{220 + 220 * 15\% * 3/12}{} - 2.5$		
$F =$	**225.75**	
Day 0		
Borrow 220 @ 15%	220	
Invest in Stock in spot	-220	

Sell Futures @ 230	0	
Total Cash flow	**0**	
Day 91		
Deliver stock against futures & take cash	230	
Repay Loan	-220	
Repay interest	-8.25	
Dividend inflow	2.5	
Total Cash flow	**4.25**	
* Assumption - Dividend is received on last day		
By borrowing & investing a stock and shorting futures on day zero & closing the positions on Day 90 / 91 by delivering the stock - one can make profit of ₹ 4.25.		

5. Illustration

BSE	5000
Value of portfolio	₹.1010000
Risk free interest rate	9% p.a.
Dividend yield on Index	6% p.a
Beta of portfolio	1.5

We assume that a future contract on the BSE index with four months maturity is used to hedge the value of portfolio over next three months.
One future contract is for delivery of 50 times the index. Based on the above information calculate:
(i) Price of future contract.
(ii) The gain on short futures position if index turns out to be 4,500 in three months.

(ICAI SM, MTP Sept'22, MTP Apr'18 Old, Old PM)

Solution:

Particulars	₹ / Index	%
BSE Spot	5000	
Portfolio Value	1010000	
R_f		9%
Y%		6%
Portfolio Beta	1.5	
BSE Futures - 4 months		
Hedging for 3 months		
Lot size - 50 units		
$F = S + Coc - D$		
$F = 5000 + 9\% * 4/12 * 5000 - 6\% * 4/12 * 5000$		
$F = 5000 + 5000 * 4/12 * (9\% - 6\%)$	5050	
Lot size	50	
Single contract	252500	
Assuming Beta is 1 No of contracts to be entered into for portfolio to be fully hedged by using Short futures position	4	
For Beta of 1.5, No. of Contracts is	4 x 1.5	
For Beta of 1.5, No. of Contracts is	**6**	
Spot Value of Index after 3 months	4500	
1 Month future after 3 months is?	4511.25	

Price of future at time of shorting	5050	
Price of future at time of shorting	5050	
Price of future after 3 months	4511.25	
No of contracts	6	
Lot size	50	
Total Gain	**161625**	

6. Illustration

The share of X Ltd. is currently selling for ₹ 300. Risk free interest rate is 0.8% per month. A three months futures contract is selling for ₹ 312. Develop an arbitrage strategy and show what your riskless profit will be 3 months hence assuming that X Ltd. will not pay any dividend in the next three months.

(ICAI SM, MTP Sept'22, Old PM)

Solution:

Particulars	₹ / Index	%
Spot price of X Ltd Share	300	
3 Month future Price of X Ltd share	312	
R_f interest rate pm		0.008
Dividend		0
$Theoretical\ future\ price = S + Coc$		
$= 300 + 300 * 0.8\% * 3 - Simple\ Interest$	307.2	
$= 300 * (1 + 0.8\%)^\wedge 3 - Compound\ Interest$	307.258	
$= 300X\ e^\wedge(0.8\% * 3) - Continous\ Compounding$	307.287	
Compound Interest based Futures price	307.258	
Current Futures price	312	
Transactions Day zero		
1. Short Future @312	0	
2. Buy Stock @ Spot - 300	-300	
3. Borrow money ₹300 @ 0.8% pm	300	
	0	
Transactions at maturity		
1. Return money	-307.258	
2. Deliver the stock against the future contract	0	
3. Take money on delivery of stock against future contract	312	
Net cash flow	**4.742**	
Difference between Futures price (theoretical - actual)	**-4.742**	

Additional Case:

Particulars	₹ / Index	%
Spot price of X Ltd Share	300	
3 Month future Price of X Ltd share	303	
R_f interest rate pm		0.008
Dividend		0
$Theoretical\ future\ price = S + COC$		

$= 300 + 300 * 0.8\% * 3$ − *Simple Interest*	307.2	
$= 300 * (1 + 0.8\%)\text{^}3$ − *Compound Interest*	307.258	
$= 300X\ e\ \text{^}(0.8\% * 3)$ − *Continous Compounding*	307.287	
Compound Interest based Futures price	307.258	
Current Futures price	303	
Transactions Day zero		
1. Buy Future @ 303	0	
2. Sell the stock - Shorting - 300	300	
3. Invest / lend cash @0.8% for 3 months	-300	
Transaction on maturity		
1. Cash returned to us	307.258	
2. Give cash & take stock delivery on futures contract	-303	
3. Hand over the stock to settle to the shorting of stock transaction	0	
Net cash flow	**4.258**	
Difference between Futures price (theoretical - actual)	**4.26**	

7. Illustration

A Mutual Fund is holding the following assets in ₹ Crores

Investments in diversified equity shares	90.00
Cash and Bank Balances	10.00
Total	100.00

The Beta of the equity shares portfolio is 1.1. The index future is selling at 4300 level. The Fund Manager apprehends that the index will fall at the most by 10%. How many index futures he should short for perfect hedging? One index future consists of 50 units. Substantiate your answer assuming the Fund Manager's apprehension will materialize.

(ICAI SM, MTP Mar'19, Old PM)

Solution:

Particulars	₹ / Index	%
Index Future Value	4300	
Lot Size - 50 units		
MF Investments in Cr		
- Cash	10	
- Diversified Equity Shares	90	
Total	100	
Beta of Equity shares	1.1	
1. No. Of Futures to be shorted for a perfect hedge		
Contract Value of 1 futures contract	215000	
Value of Equity shares	900000000	
No of Futures contracts to be entered into	4604.6512	

Round off futures contracts	4605	
Future contracts shorted	4605	
2. In case index falls by 10% then		
Loss on Equity shares	99000000	11%
Gain on Short futures		
No. Of futures	4605	
Lot size	50	
Gain per contract	430	
Total Gain on Futures	**99007500**	
The Loss on Equity shares portfolio of Rs 9.9 Cr is offset by gain on futures & hence the hedging transaction served its purpose.		

8. Illustration

A trader is having in its portfolio shares worth ₹ 85 lakhs at current price and cash ₹ 15 lakhs. The beta of share portfolio is 1.6. After 3 months the price of shares dropped by 3.2%. Determine:

(i) Current portfolio beta

(ii) Portfolio beta after 3 months if the trader on current date goes for long position on ₹100 lakhs Nifty futures.

(ICAI SM, July'21 QP 8 marks, MTP Mar'24, RTP May'20 Old, MTP Mar'19 Old, Old PM)

Solution:

Particulars	₹ Lacs / Index	Beta
Equity Portfolio	85	1.6
Cash	15	0
Total	100	1.36
Beta of portfolio = WA of Beta of respective components		
$= \frac{85*1.6 + 15*0}{85+100}$ =1.36		
Part 2		
Price of Equity shares dropped by 3.2% after 3 months		
Given Beta of portfolio is 1.6; the market index dropped by 2% in 3 months (i.e., 3.2%/1.6)		
Portfolio beta = 1.6		
Change in Portfolio = 3.2% drop		
=> Change in market = $3.2\%/1.6$ = 2%		

Equity portfolio	85	1.6	-3.2%	82.28
Cash	15	0	0	15
Nifty Futures long	100	1	-2%	98
Beta of portfolio		2.36		4.72

9. Illustration

Which position on the index future gives a speculator, a complete hedge against the following transactions:

(i) The share of Right Limited is going to rise. He has a long position on the cash market of ₹ 50 lakhs on the Right Limited. The beta of the Right Limited is 1.25.

(ii) The share of Wrong Limited is going to depreciate. He has a short position on the cash market of ₹ 25lakhs on the Wrong Limited. The beta of the Wrong Limited is 0.90.

(iii) The share of Fair Limited is going to stagnant. He has a short position on the cash market of ₹ 20 lakhs of the Fair Limited. The beta of the Fair Limited is 0.75.

(ICAI SM, Old PM)

Solution:

Company Name	View / Position	₹ Lacs Investment	Beta	₹ Lacs Hegde	Hedge Position
Right Ltd	Rise - long	50	1.25	-62.5	Short Index
Wrong Ltd	Dep - Short	25	0.9	22.5	Long Index
Fair Ltd	Stagnant - Short	20	0.75	15	Long Index
				-25	Short Index

10. Illustration

Ram buys 10,000 shares of X Ltd. at a price of ₹ 22 per share whose beta value is 1.5 and sells 5,000 shares of A Ltd. at a price of ₹ 40 per share having a beta value of 2.
He obtains a complete hedge by Nifty futures at ₹ 1,000 each. He closes out his position at the closing price of the next day when the share of X Ltd. dropped by 2%, share of A Ltd. appreciated by 3% and Nifty futures dropped by 1.5%.
What is the overall profit/loss to Ram?

(ICAI SM, Sep-25 Similar 4 M, Nov-24 Similar 4M, Jan'21 QP 4 marks, RTP May'23, RTP Nov'22, RTP Nov'18, MTP Oct'20 New & Old, MTP Apr'19, Old PM)

Solution:

Company Name	Position	Shares & Price	₹ Lacs Investment	Beta	
X Ltd	Long / Buy	10000 x 22	2.2	1.5	
A Ltd	Short / Sell	5000 x 40	-2	2	
			0.2	-3.5	-0.7
Hedge using Nifty	Long	1000 x 70	0.7	1	0.7

Total position as per existing transactions – ₹ Lacs	-0.7
Hedge should be – ₹ Lac	0.7
Contract value – ₹	1000
Number of contracts	70

Part 2

Company Name	Position	₹ Lacs Investment	Change %	₹ Lacs Value	Change ₹ Lacs
X Ltd	Long / Buy	2.2	-2%	2.156	-0.044

A Ltd	Short / Sell	-2	3%	-2.06	-0.06
Nifty	Long	0.7	-1.5%	0.6895	-0.0105
		0.2			-0.1145
Total Loss on portfolio is ₹ 11450					

11. Illustration

On January 1, 2013 an investor has a portfolio of 5 shares as given below.

Security	Price	Number of Shares	Beta
A	349.30	5,000	1.15
B	480.50	7,000	0.40
C	593.52	8,000	0.90
D	734.70	10,000	0.95
E	824.85	2,000	0.85

The cost of capital to the investor is 10.5% per annum. You are required to calculate

i. Beta of his portfolio
ii. The theoretical value of NIFTY futures for February 2013
iii. The number of contracts of NIFTY the investor needs to sell to get a full hedge until February for his portfolio if the current value of NIFTY is 5900 and NIFTY futures have a minimum trade lot requirement of 200 units. Assu/me that the futures are trading at their fair value.
iv. The number of future contracts the investor should trade if he desires to reduce the beta of his portfolios to 0.6. No. of days in a year be treated as 365.Given: Ln (1.105) = 0.0998 and $e^{(0.015858)}$= 1.01598

(ICAI SM, Nov-24 Similar 6M, MTP Oct'23, MTP Mar'19, RTP May'20, MTP Oct'19 Old, Old PM)

Solution:

Security	Price	# Shares	₹ Lacs value	Beta
A	349.3	5000	17.465	1.15
B	480.5	7000	33.635	0.4
C	593.52	8000	47.4816	0.9
D	734.7	10000	73.47	0.95
E	824.85	2000	16.497	0.85
			188.5486	0.84907
Part 1: Portfolio Beta				**0.84907**

NIFTY Spot	5900
Coc	0.105
# of days in year	365
NIFTY Future	$= Spot * e^{\wedge} (r * t)$
Part 2: Nifty Future value	**5999.27 (59 days)** **5994.28 (58 days)**

Short Nifty futures for Full hedge

Nifty Futures value	5999.27	5994.28
Lot size	200	200
Notional of 1 Nifty future contract	1199853	1198856

Part 3: Total Nifty future contracts to be shorted	188.5486 lacs x 0.84908 / 1199853 = 13.34	188.5486 lacs x 0.84908 / 1198856 = 13.35

	₹ Lacs value	Beta	
Portfolio Data	188.5486	0.84907	160.091
Short Nifty	-46.96198	1	- 46.962
Revised portfolio	188.5486	0.6	113.129
Notional short nifty position	-4696198	-4696198	
1 Contract notional	1199853	1198856	
Part 4: No. of contracts to be shorted	**3.914**	**3.917**	

12. Illustration

Details about portfolio of shares of an investor is as below:

Share	No. of Shares (Lakhs)	Price per share	Beta
A Ltd.	3.00	₹ 500	1.40
B Ltd.	4.00	₹ 750	1.20
C Ltd.	2.00	₹ 250	1.60

The investor thinks that the risk of portfolio is very high and wants to reduce the portfolio beta to 0.91. He is considering two below mentioned alternative strategies:

(i) Dispose off a part of his existing portfolio to acquire risk free securities or

(ii) Take appropriate position on Nifty Futures which are currently traded at ₹ 8125 and each NIFTY point is worth ₹ 200.

You are required to determine:

(1) portfolio beta,

(2) The value of risk-free securities to be acquired,

(3) The number of shares of each company to be disposed off,

(4) The number of Nifty contracts to be bought/sold; and

(5) The value of portfolio beta for 2% rise in Nifty

(ICAI SM, Nov'22 QP 8 marks, RTP Nov'21, MTP Apr'23, MTP Mar'22, MTP Oct'19, MTP Apr'19, RTP May'21 Old, Old PM)

Solution:

Part 1

Security	Price	# Shares L	₹ Lacs value	Beta	Beta *Value	Weightage
A Ltd	500	3	1500	1.4	2100	0.3
B Ltd	750	4	3000	1.2	3600	0.6
C Ltd	250	2	500	1.6	800	0.1
			5000	1.3	6500	

Portfolio Beta is 1.3		

Part 2	₹ Lacs value	Beta
Target portfolio beta	5000	0.91
Total proportion of RF securities	R_f	0
Total value of portfolio	5000	0.91
Value of Equity Portfolio	5000-R_f	1.3
	$(R_f * 0 + (1 - R_f) * 1.3)/1 = 0.91$	
	$(0 + 1.3 - 1.3R_f) = 0.91$	
	$1.3 - 0.91 = 1.3R_f$	
	$0.39 = 1.3\ RF$	
	$R_f = 0.39/1.3$	
	$R_f = 0.3$	
	$R_f = 30\%$	
	Equity Portfolio = 70%	
Value of Risk-Free securities acquired	**1500**	

Part 3

Security	Price	# Shares L	₹ Lacs value	% Disposed	# of Shares Disposed
A Ltd	500	3	1500	0.3	0.9
B Ltd	750	4	3000	0.3	1.2
C Ltd	250	2	500	0.3	0.6
			5000	0.3	

Part 4	₹ Lacs value	Beta	Beta x Value
Reduce Beta to 0.91 by Entering to Nifty contracts			
Current portfolio	5000	1.3	6500
Short Nifty	1950	-1	-1950
Target portfolio	5000	0.91	4550
Total NIFTY Contract Value Short	1950		
Current Nifty Futures	8125		
Each NIFTY is worth	200		
Total contracts	120		

Part 5	Price	₹ Lacs value	Beta	Beta* Value
NIFTY Change	0.02			
Loss on NIFT Short	39			
Equity portfolio		5000	1.3	
Change in portfolio value		130		
Revised portfolio value		5130		
Less: Loss on Nifty		-39		
Total value of portfolio		5091		

Net change in total portfolio		91		
% change in total portfolio		1.82%		
% change in NIFTY		2%		
Beta		0.91		
Revised portfolio Beta				
Equity		5130	1.3	6669
Nifty Short		1911	-1	-1911
Total Beta		**5130**	**(4758/5130) = 0.92749**	**4758**

When there is a 2% Rise in NIFTY, the portfolio value will change by 1.82% because Beta of portfolio is 0.91

Once the portfolio weights change and no new transactions are entered into the BETA will change to 0.927

13. Illustration

On April 1, 2015, an investor has a portfolio consisting of eight securities as shown below:

Security	Market Price	No. of Shares	Beta Value
A	29.40	400	0.59
B	318.70	800	1.32
C	660.20	150	0.87
D	5.20	300	0.35
E	281.90	400	1.16
F	275.40	750	1.24
G	514.60	300	1.05
H	170.50	900	0.76

The cost of capital for the investor is 20% p.a. continuously compounded. The investor fears a fall in the prices of the shares in the near future. Accordingly, he approaches you for the advice to protect the interest of his portfolio.

You can make use of the following information:

(1) The current NIFTY value is 8500.
(2) NIFTY futures can be traded in units of 25 only.
(3) Futures for May are currently quoted at 8700 and Futures for June are being quoted at 8850.

You are required to calculate:

(i) The beta of his portfolio.
(ii) The theoretical value of the futures contract for contracts expiring in May and June.. Given ($e^{0.03}$ =1.03045, $e^{0.04}$ = 1.04081, $e^{0.05}$ =1.05127)
(iii) The number of NIFTY contracts that he would have to sell if he desires to hedge until June in each of the following cases:

(A) His total portfolio
(B) 50% of his portfolio
(C) 120% of his portfolio

(ICAI SM, MTP Mar'18 Old, Old PM)

Solution:

Part 1

Security	Price	# of shares	₹	Beta
A	29.4	400	11760	0.59
B	318.7	800	254960	1.32
C	660.2	150	99030	0.87
D	5.2	300	1560	0.35
E	281.9	400	112760	1.16
F	275.4	750	206550	1.24
G	514.6	300	154380	1.05
H	170.5	900	153450	0.76
Portfolio Beta			994450	**1.10195**

Part 2

Nifty Spot	8500
COC	20%
May Futures	8700
June Futures	8850
May Theoretical Futures Price	8788.18
June Theoretical Futures Price	8935.8

WN	t (days)	r	rt	$e^{(rt)}$	$F = S * e^{(rt)}$
t May	60	0.2	0.032876712	1.0339	8788.17833
t June	90	0.2	0.049315068	1.05127	8935.795

$e^{(0.03)}$	1.03045
$e^{(0.04)}$	1.04081
$e^{(0.01)}$	0.01036
$e^{(0.033)}$	1.0339

Part 3

Hedge 100% of portfolio with NIFTY	% coverage	Total Notional of NIFTY Short (Portfolio value * % coverage * Portfolio Beta)	June Futures price	Lot size	# of contracts = Total Notional / Futures price / Lot size
Total NIFTY Notional to be shorted	1	1095832.3	8850	25	4.952914
Total NIFTY Notional to be shorted	0.5	547916.15	8850	25	2.476457
Total NIFTY Notional to be shorted	1.2	1314998.76	8850	25	5.943497

14. Illustration

Sensex futures are traded at a multiple of 50. Consider the following quotations of Sensex futures in the 10 trading days during February 2009:

Day	High	Low	Closing
04-02-2009	3306.40	3290.00	3296.50
05-02-2009	3298.00	3262.50	3294.40
06-02-2009	3256.20	3227.00	3230.40
07-02-2009	3233.00	3201.50	3212.30
10-02-2009	3281.50	3256.00	3267.50
11-02-2009	3283.50	3260.00	3263.80
12-02-2009	3315.00	3286.30	3292.00
14-02-2009	3315.00	3257.10	3309.30
17-02-2009	3278.00	3249.50	3257.80
18-02-2009	3118.00	3091.40	3102.60

Abhishek bought one Sensex futures contract on February 04. The average daily absolute change in the value of contract is ₹ 10,000 and standard deviation of these changes is ₹ 2,000. The maintenance margin is 75% of initial margin. You are required to determine the daily balances in the margin account and payment on margin calls, if any.

(ICAI SM, May-25 Similar 7M, RTP Nov'19, MTP Oct'24, MTP April'22, RTP Nov'18 Old, Old PM)

Solution:

Average Change in Contract value	Mean	10000
1 SD in average contract value	SD	2000
based on the principles of VAR, Margin is Mean + 3 SD		16000
Initial Margin		16000
Maintenance Margin	0.75	12000

Day	Closing	MTM Computation	MTM	Total Margin	Margin call	Closing margin
4th Feb	3296.5	0		16000	0	
5th Feb	3294.4	50 * (3294.4-3296.5)	-105	15895	0	
6th Feb	3230.4	50 * (3230.4-3294.4)	- 3200	12695	0	
7th Feb	3212.3	50 * (3212.3-3230.4)	-905	11790	4210	16000
10th Feb	3267.5	50 * (3267.5-3212.3)	2760	18760	0	
11th Feb	3263.8	50 * (3263.8-3267.5)	-185	18575	0	
12th Feb	3292	50* (3292-3263.8)	1410	19985	0	
14th Feb	3309.3	50 * (3309.3-3292)	865	20850	0	
17th Feb	3257.8	50 * (3257.8-3309.3)	- 2575	18275	0	
18th Feb	3102.6	50 * (3102.6-3257.8)	- 7760	10515	5485	16000
Total Gain			- 9695		9695	
Total Gain	(3102.6-3296.5) * 50	-9695				
				3296.5		

* Assumption purchase is made on Feb 4th at closing price						
Contracts	1					
Lot size	50					
Initial margin	16000					

15. Illustration

Suppose that there is a future contract on a share presently trading at ₹ 1000. The life of future contract is 90 days and during this time the company will pay dividends of ₹ 7.50 in 30 days, ₹ 8.50 in 60 days and ₹ 9.00 in 90 days.
Assuming that the Compounded Continuously Risk-free Rate of Interest (CCRRI) is 12% p.a. you are required to find out:

(a) Fair Value of the contract if no arbitrage opportunity exists.

(b) Value of Cost to Carry.

[Given $e^{-0.01}$ = 0.9905, $e^{-0.02}$ = 0.9802, $e^{-0.03}$ = 0.97045 and $e^{0.03}$ = 1.03045]

Solution:

Spot		1000
Dividend	30	7.5
	60	8.5
	90	9
CCROI		12% PA

Given	
$e^{(-0.01)}$	0.9905
$e^{(-0.02)}$	0.9802
$e^{(-0.03)}$	0.9705
$e^{(0.03)}$	1.03045

PV of Div inflows	30	7.5	$7.5 * e^{((-30/360) * 12\%)}$	$7.5 \times e^{(-0.01)}$	7.42875
	60	8.5	$8.5 * e^{((-60/360) * 12\%)}$	$8.5 \times e^{(-0.02)}$	8.3317
	90	9	$9.0 * e^{((-90/360) * 12\%)}$	$9.0 \times e^{(-0.03)}$	8.7345
					24.49495

Futures price	$(1000 - 24.49495) * e^{(90/360 \times 12\%)}$	
	$(1000 - 24.49495) * e^{(0.03)}$	
	1005.21	
Cost of carry	29.7041	
PV of Div	-24.495	
Net cost of carry	5.20918	
$e^{(-0.03)}$	0.97045	$= 1/(e^{(0.03)})$
$e^{(-0.01)}$	0.9905	
$e^{(0.03)}$	1.03045	
	0.02005	
	0.01003	
$e^{(-0.02)}$	0.98048	

16. Illustration

A futures contract is available on R Ltd that pays annual dividend of ₹ 4 whose stock is currently priced at ₹ 125. Each future contract calls for the delivery of 1000 shares to stock in one year. Corporate treasury bill is ₹ 8%.
Given the above information what should be the price of one future contract if the company's stock price decreases by 6%.
And as a result of the company's stock price decrease will the investor long position in one futures contract realise the gain or loss, what would be amount of gain or loss.
Ignore margin and taxation if any.

(Nov'19 QP 8 marks, MTP Mar'21)

Solution:

R Ltd Spot price		**125**
Annual Dividend		4
Futures contract Duration		1 year
Futures lot size		1000
ROI		0.08
Price of Futures contract		$S + COC - D$
COC		10
Futures price of a single share		131
Dividend is paid on the last day and dividend is directly deducted from cost of carry		
Futures price of a contract of 1000 shares		131000
Spot		125
Less: Reduction	0.06	7.5
		117.5
1 Futures contract price		$1000\ x\ (\ 117.5(1+8\%) - 4$
		122900
Because of reduction in value of the stock, the Investor will Lose ₹8100		**8100**

17. Illustration

Mr. V decides to sell short 10,000 shares of ABC plc, when it was selling at yearly high of 5.60 pounds. His broker requested him to deposit a margin of 45% and commission of 1,550 pounds. While Mr. V was short the share, ABC plc. Paid a dividend of 0.25 pounds. At the end of one year, Mr. V buys 10,000 shares of ABC plc. at 4.50 pounds to close out the position and was charged a commission of 1,450 pounds.
You are required to calculate return on investment of Mr. V.

(RTP Nov'11)

Solution:

# of shares of ABC Pls Short sold		10000	
Price at which short sold		5.6	
Margin Requirement	45%	25200	
Dividend paid during the year	25%	2500	
Year-end position close at		4.5	

Initial commission		1550	
Closure commission		1450	
ROI		*Profit / Investment*	0.2183
Profit		Reduction in price	11000
		Less: Commissions	-3000
		Less: Dividend	-2500
		Net return	**5500**
		Investment	25200

0	1
-29250	34750
10% NPV	2340.91
20% NPV	-291.667
19% NPV	-48.3193
0.01	-243.347
	0.00199
ROI	19%-0.2%
	0.188

18. Illustration

Suppose current price of an INDEX is ₹ 13,800 and yield on INDEX is 4.8% per annum. 6 Month future contract of an INDEX is trading at ₹ 14,340.

Assuming the Risk-free rate of interest is 12%, show that Mr. X, an arbitrager can earn an abnormal rate of return irrespective of outcome after 6 months.

You can assume that after six months; INDEX closes at ₹ 10,200 or ₹ 15,600 and 50% of the stock including INDEX shall pay dividend in next 6 months.

Also calculate the implied Risk-free rate.

(RTP Nov'13)

Solution:

Spot Index price	**13800**
Index Yield p.a.	0.048
6-month futures price	14340
Rf	0.12
Index closing value after 6 months	10200
	15600
Index stock paying div in 6 months	50%
F = S+ COC - Div.	
$= 13800 + 13800 * 12\% * 6/12 - 4.8\% * 50\% * 13800$	14296.8
Theo Futures price	**14296.8**
Spot	13800
COC	828
Dividend	331.2

		Cash Flow (Day 0)
Short Index Futures	14340	0
Buy Index spot (portfolio or ETF)	13800	-13800
Borrow Money @ 12%	13800	13800

Cash flow after 6 months	
Dividend	331.2
Add: Cash	14340
Less: Loan	-13800
Less: Interest	-828
Profit	43.2

	Index 10200		Index 15600
Dividend	331.2	Dividend	331.2
Profit on Futures shorted	4140	Loss on Futures	-1260
Loss on Shares	-3600	Gain on shares	1800
Less: Interest	-828	Less: Interest	-828
Net profit / loss	43.2	Net profit / loss	43.2

At Implied risk-free rate	$\mathbf{14340 = 13800 + 13800 * x\% * 6/12 - 4.8\% * 50\% * 13800}$
	$14340 - 13800 = 13800 * x\% * .5 - 331.2$
	$540 + 331.2 = 13800 * x\% * .5$
	$871.2 = 13800 * x\% * 1/2$
	$871.2 * 2/13800 = x\%$
Implied risk-free rate	$\mathbf{0.12626}$

19. Illustration

The NSE-50 Index futures are traded with rupee value being ₹ 100 per index point. On 15th September, the index closed at 1195, and December futures (last trading day December 15) were trading at 1225.

The historical dividend yield on the index has been 3% per annum and the borrowing rate was 9.5% per annum.

(i) Determine whether on September 15, the December futures were underpriced or overpriced?

(ii) What arbitrage transaction is possible to gain out this mispricing?

(iii) Calculate the gains and losses if the index on 15th December closes at(a) 1260 (b) 1175.

Assume 365 days in a year for your calculations.

[Nov'19 QP (Old)]

Solution:

Spot on 15th Sep	1195
15th Dec Future price	1225
Y	0.03
R	0.095
Lot size	100
Part 1	
Theoretical Futures price per 1 unit of index	$= S + S * (r - y) * t$
Futures price per contract	1214.37

December futures are overpriced on the stock exchange

Part 2

Since Futures are overpriced. We will sell them. Secondly, since it is an arbitrage transaction with no risk, one will borrow money @9.5% and invest it in the spot value of Index in order to cover the short futures position. On the Maturity date, all positions will be closed out and profit will be booked.

Part 3	(a)	(b)
Gain / Loss if index value on Dec 15th is	1260	1175
Day 0 Cash flow		
Short Futures 100 contracts @ 1225	0	0
Borrow money @ 9.5%	119500	119500
Buy Spot Index 100 units @ 1195	-119500	-119500
Total CF on Day 0	0	0
Day 91 CF		
(Loss)/ Profit on Futures	-3500	5000
Sell the index spot	126000	117500
Dividend @ 3%	893.7945205	893.7945
Repay Loan	-119500	-119500
Repay Interest @ 9.5%	-2830.349315	-2830.35
Total Cash flow	**1063.445205**	**1063.445**
Irrespective of the Index value the arbitrager can lock in profit of ₹1063.45 per lot of futures		

20. Illustration

On 1st April 2015, Sunidhi was holding a portfolio of 10 securities whose value was ₹ 9,94,450. The weighted average of beta of 9 securities was 1.10.
Since she was expecting a fall in the prices of the shares in near future to hedge her portfolio, she sold 5 contracts of NIFTY Futures (Multiplier of 25) expiring in May 2015, which was trading at 8767.07 on 1st April.

i. Calculate the beta of the 10th security.
ii. Reconcile the reasons in spite of 2% fall in the market as per Sunidhi's apprehension if she would have earned some profit on her cash position.

(MTP Mar'18)

Solution:

Portfolio Value	994450
Beta of 9 Securities	1.1
Nifty Contracts Shorted	5
Lot size	25
Futures price	8767.07
Total Notional value Shorted	1095883.75
Beta of 10th Security	
First find out Total portfolio Beta	
Total futures value = Portfolio x beta	

Beta = Total Futures value /Portfolio value	
Beta of full portfolio of 10 securities =	1.10199985

Assuming weights of 9 securities & the 10th security are in same proportion	

9	1.1
1	B
10	1.102
10 X 1.102 = 9 x 1.1 + 1 x B	
B = 10 x 1.102-9.9	1.12
	1.12
Part 1 Beta of 10th Security is	1.12
Part 2	
Even though portfolio beta is 1.1, & market fell by 2%, if Sunidhi has made a profit on her portfolio, it means some of her stocks may not have followed the overall market and as a result, the gains from those stocks, offset the negative impact of overall fall in the market	

21. Illustration

Miss K holds 10,000 shares of IBS Bank @ 2,738.70 when one month INDEX Future was trading at 6,086 and the share has a Beta of 1.2. How many INDEX Futures should she short to perfectly hedge her position? A single INDEX Future is a lot of 50 indices. Justify your result in the following cases:

i. **When the INDEX zooms by 1%**
ii. **When the INDEX plummets by 2%**

(RTP May'17)

Solution:

Data Provided		
No of Shares of IBIS held by Miss K	10000	
Price per share	2738.7	
Total portfolio value	27387000	
Beta	1.2	
Futures price	6086	
Futures lot size	50	
Notional contract value of 1 Future	304300	
Part 1		
Number of futures contracts to be shorted	$= (portfolio\ value / notional\ contract\ value\ of\ 1\ future) * Beta$	
Number of futures contracts to be shorted	108	
Part 2		

Value of IBS Shares + Future Short when Index changes by	0.01	-0.02
IBS Shares	$beta * index\ gain * IBS\ value$	
Future position Short	$(Index\ Future - Spot) * contracts * lot\ size * Future\ price$	
IBS Shares gain / (loss)	328644	-657288
Short futures gain / (loss)	-328644	657288
Total Gain / (loss)	0	0
Because Miss K has fully hedged shares of IBS bank with Index futures, the movement in market will not affect her portfolio value		

22. Illustration

BSE	30,000
Value of Portfolio	₹ 60,60,000
Risk free interest rate	9% p.a.
Dividend yield on INDEX	6% p.a.
Beta of Portfolio	1.5

We assume that a future contract on the BSE index with four months maturity is used to hedge the value of portfolio over next three months. One future contract is for delivery of 50 times the index. Based on the above information calculate:

(i) Price of future contract.

(ii) The gain on short futures position if index turns out to be 27,000 in three months.

(RTP May'19 Old)

Solution:

BSE Spot	**30000**
Portfolio value	6060000
r	0.09
y	0.06
Beta of portfolio	1.50
Lot size of futures	50.00
Part 1	
Futures price - 4 months	**30300**
Part 2	
Index position in 3 months	27000
Total Futures Shorted to hedge the portfolio	90,90,000.00
No of futures contracts shorted	6.00
1 Month future price after 3 months when spot is 27000	27067.5
Gain on single unit of futures is	3232.5
Total gain on futures	**969750**

23. Illustration

On April 1, 2019, Kasi has a portfolio consisting of four securities as shown below

Security	A	K	S	P
Market Price	₹ 48.50	₹ 332.68	₹ 13.99	₹ 292.82
No. of Shares	673	480	721	358
Beta Value	0.74	1.28	0.54	0.46

Cost of Capital is 16% p.a. compounded continuously. Kasi fears a fall in prices of shares in future. Accordingly, he approaches you for the advice to protect the interest of his Portfolio.
You can make use of the following information:

(i) The current NIFTY Value is 9380.
(ii) NIFTY Futures can be traded in units of 25 only.
(iii) Futures for September are currently quoted at 9540 and Futures for October are being quoted at 9820.

You are required to calculate:
The Beta of his Portfolio.
Theoretical Value of Futures for contracts expiring in September & October.
Given ($e^{0.067}$ = 1.0693, $e^{0.08}$ = 1.0833, $e^{0.093}$ = 1.0975)
The number of NIFTY Contract that he would have to sell, if he desires to hedge 150% of the Portfolio until October.

[May'19 QP (Old)]

Solution:

Part 1

Security	Price	Shares	Total Value	Beta	B X value
A	48.50	673.00	32640.50	0.74	24153.97
K	332.68	480.00	159686.40	1.28	204398.59
S	13.99	721.00	10086.79	0.54	5446.87
P	292.82	358.00	104829.56	0.46	48221.60
			307243.25	0.92	282221.03
Portfolio Beta is				**0.92**	

Part 2

NIFTY Spot	9380.00	
COC	16%	
Futures lot size	25.00	
Futures value theoretical	$F = S * e^{\wedge}(rt)$	Market quote
- Sep	10161.35	9540
- Oct	10294.55	9820

WN 1

Cost of carry for Sep	$e^{\wedge}(16\% * 6/12)$	$e^{\wedge}(0.08)$	1.0833
Cost of carry for Oct	$e^{\wedge}(16\% * 7/12)$	$e^{\wedge}(0.093)$	1.0975

Part 3

No. of contracts for hedging 150% of the portfolio till October	
Portfolio value	307243.25
Beta	0.92
October futures price	9820.00
Lot size	25.00
Total number of contracts for 100% hedge	1.15
For 150% hedge	1.73

Kasi has to short 1.73 or 2 contracts of NIFTY October futures to hedge 150% of his portfolio

24. Illustration

Mr. A has a portfolio of ₹ 5 crores consisting of equity shares of X Ltd. and Y Ltd. with beta of 1.15.

Other information is as follows: Value of Index Future = 21000

Multiplier = 150

You are requested to reduce beta of portfolio to 0.85 and increase beta to 1.45 by using:

(a) Change in composition through Risk Free securities

(b) Index futures

(RTP Nov'14)

Solution:

	in ₹	in ₹ Cr
Portfolio value	50000000	5
Portfolio Beta		1.15
Index futures current price	21000	
Lot size	150	

Reduce portfolio beta to 0.85 using Risk free securities.

Let proportion of Risk-free securities be R_f		
$0.85 = (1 - R_f) * 1.15 + R_f * 0$		In Cr
$0.85 = 1.15 - 1.15RF$		
$1.15R_f = 0.30$		
$R_f = 0.30/1.15$	0.260869	1.304348

26.09% of portfolio i.e., Rs.1.30 Cr should be taken out of X & Y Ltd.'s in their respective weights & invested in Risk free securities to reduce portfolio beta to 0.85

Increase Beta to 1.45 by Changing composition using Risk free securities

	in ₹	in ₹ Cr
$1.45 = (1 - R_f) * 1.15 + R_f * 0$		
$1.45 = 1.15 - 1.15R_f$		
$1.45 - 1.15 = -1.15R_f$		
$0.30 = -1.15R_f$		
$R_f = -0.30/1.15$	-0.26086957	-1.304348

₹ 1.30 Cr should be borrowed by shorting risk-free securities and invested in X & Y Ltd.'s in their respective weights in order to increase portfolio beta to 1.45

Changing beta by using Futures

Reducing Beta to 0.85	
	$0.85 * 5 = 5 * 1.15 - F * 1$
	$4.25 = 5.75 - F$
	$F = 1.5$
Notional value of Futures contract to be shorted	15000000
Futures value current	21000
Lot size	150
No of futures contracts shorted	4.761904762

For increasing Beta to 1.45	Beta	₹ Cr
Portfolio	1.45	5
Equity shares	1.15	5
Index long	1	F
	$1.45x\,5 = 1.15\,x\,5 + 1 * F$	
	$7.25 - 5.75 = F$	
	$F = 1.5$	

Notional contract value of Futures to be purchased	15000000
Current futures price	21000
Lot size	150
No of Future contracts gone long	4.761904762

25. Illustration

Mr. X, is a Senior Portfolio Manager at ABC Asset Management Company. He expects to purchase a portfolio of shares in 90 days. However, he is worried about the expected price increase in shares in coming day and to hedge against this potential price increase he decides to take a position on a 90-day forward contract on the Index.

The index is currently trading at 2290. Assuming that the continuously compounded dividend yield is 1.75% and risk-free rate of interest is 4.16%, you are required to determine:

Calculate the justified forward price on this contract.

Suppose after 28 days of the purchase of the contract the index value stands at 2450 then determine gain/ loss on the above long position.

If at expiration of 90 days, the Index Value is 2470 then what will be gain on long position.

Note: Take 365 days in a year and value of $e^{0.005942}$ = 1.005960, $e^{0.001849}$ = 1.001851.

(RTP May'15)

Solution:

Given data	
Spot on Day 0	2290
Forward contract tenure in days (t)	90
Dividend Yield (y)	0.0175
Risk Free rate (r)	0.0416
(a)	
$F = S * e\,\hat{}((r-y) * t/365)$	$2290 * e\,\hat{}(4.16\% - 1.75\%) * 90/365$
	$2290 * e\,\hat{}(2.41\%\,x\,90/365)$
	$2290 * e\,\hat{}\,(0.0059425)$
90 Day forward price	2303.6484

(b)	
Current Spot	2450
Original spot	2290
28 Day forward price for original spot	$2290\ x\ e\ \hat{}(2.41\% * 28/365)$
Forward price	2294.23879
Gain / Loss	**155.76121**

(c)	
90-day spot	2470
90-day that day maturity forward price	2470
Original forward contract price	2303.6484
Gain / loss	**166.3516**

26. Illustration

A is an investor and having in its Portfolio Shares worth ₹. 1,20,00,000 at current price and Cash ₹ 10,00,000.

The Beta of Share Portfolio is 1.4. After four months the price of shares dropped by 1.8%.

You are required to determine:

(i) Current Portfolio Beta and

(ii) Portfolio Beta after four months-if A on current date goes for long position on ₹ 1,30,00,000 Nifty futures.

(iii) Show calculations in ₹. Lakhs with four decimal points.

Solution:

Share portfolio in Rs. Lacs	120
Cash (₹ Lacs)	10
Beta	1.4
Reduction in portfolio prices in 4 months	-1.8%
Nifty Futures Long (Rs. Lacs)	130

(a)

	Rs. Lacs	Beta	Beta x Rs. L
Share portfolio	120	1.4	168
Cash	10	0	0
Total	130	1.292308	168
Portfolio Beta is		**1.292308**	

(b)

Change in Value of portfolio	-0.018
Change in NIFTY	$change\ in\ portfolio\ /portfolio\ beta$
Change in NIFTY	-0.01285714

Current date portfolio beta with NIFTY	₹ Lacs	Beta	Beta * ₹ L
Share portfolio	120	1.4	168
Cash	10	0	0
NIFTY Futures	130	1	130
Total	130	2.292308	298
Portfolio Beta is		2.292308	

	₹ Lacs	Change	Value after 4 months	Change
Change in Value of Equity Shares	120	-0.018	117.84	2.16
Change in NIFTY contract	130	-0.012857	128.3286	1.671429
Cash	10	0	10	0
Total	130	0.029473		3.831429
Portfolio Beta	$Change\ in\ portfolio\ (\%) / Change\ in\ index\ (\%)$			
Portfolio Beta	2.292307692			

27. Illustration

The price of March Nifty Futures Contract on a particular day was 9170. The minimum trading lot on Nifty Futures is 50. The initial margin is 8% and the maintenance margin is 6%. The index closed at the following levels on next five days:

Day	1	2	3	4	5
Settlement Price	9380	9520	9100	8960	9140

You are required to calculate:

(i) Mark to market cash flows and daily closing balances on account of

(a) An investor who has taken a long position at 9170

(b) An investor who has taken a short position at 9170

(ii)Net profit/ loss on each of the contracts

(Jan'21 QP, MTP Mar'23)

Solution:

	9,170.00
Lot size	50
Day	Price ₹
1	9,380.00
2	9,520.00
3	9,100.00
4	8,960.00
5	9,140.00

Maintenance Margin is fixed at 6% of initial contract value = 9170 x 50 x 6% = Rs.27510

Initial margin is fixed at 8% of initial contract value = 9170 x 50 x 8% = Rs.36680

Scenario A - Long NIFTY @ 9170

Day	Settlement price	Index movement	size	MTM Gain /loss	OP Margin A/C	Add margin	Closing margin, A/C	Main margin req @ 6%	Margin shortfall recouped
0	9,170.00	0.00	50.00	0		36680	36680	27510	
1	9,380.00	210.00	50.00	10,500	36680		47180	27510	
2	9,520.00	140	50.00	7,000	47180		54180	27510	
3	9,100.00	-420.00	50.00	-21,000	54180		33180	27510	
4	8,960.00	-140.00	50.00	-7,000	33180	10500	36680	27510	10500
5	9,140.00	180.00	50.00	9,000	36680	0	45680	27510	
Total				-1,500					
Gain / loss	-1,500.00								
Change in margin A/c							-1500		

Scenario A - Short NIFTY @ 9170

Day	Settlement price	Index movement	Lot size	MTM Gain/ loss	OP Margin A/C	Add margin	Closing margin, A/C	Main margin req @ 6%	Margin shortfall recouped
0.00	9,170.00	0.00	50.00	0		36680	36680	27510	
1	9,380.00	-210.00	50.00	-10,500	36680	10500	36680	27510	10500
2	9,520.00	-140.00	50.00	-7,000	36680		29680	27510	
3	9,100.00	420.00	50.00	21,000	29680		50680	27510	
4	8,960.00	140.00	50.00	7,000	50680		57680	27510	
5	9,140.00	-180.00	50.00	-9,000	57680		48680	27510	
Gain / (loss)	1,500.00			1,500					
Margin Change							1500		

28. Illustration

Following information is available for consideration;

BSE Index	25,000
Value of portfolio	₹ 50,50,000
Risk free interest rate	9% p.a.
Dividend yield on Index	6% p.a.
Beta of portfolio	1.5

We assume that a future contract on the BSE index with 4 months maturity is used to hedge the value of portfolio over next 3 months. One future contract is for delivery of 50 times the index. Based on the above information calculate:

(i) Price of future contract.

(ii) Gain on short futures position if index turns out to be 22,500 in 3 months.

Note: Daily compounding (exponential) formula is not required to be used.

[July'21 QP 8 marks, RTP May'22, Nov'20 QP (Old)]

Solution:

f = spot+ cost of carry-on spot- dividend on spot = $s * e^{(r-y)t}$

(i) Price of Future Contract;

f = 25000+ 25000* $(\frac{9\%*4}{12})$ - 25000* $(\frac{6\%*4}{12})$

= 25000+ 25000* $(\frac{3\%*4}{12})$ = 25000+ 250 = **25250**

One future contract has lot size of 50 times the index value

So, future price = 50* 25250 = ₹ **1262500**

(ii) Gain on Short future position if index turns out to be 22500 in 3 months;

Index value after 3 months = 22500

4 months futures value after 3 months

= 22500+ 22500* $(\frac{9\%-6\%*1}{12})$ = 22500+ 56.25 = **22556.26**

No. of contracts hedged = $\frac{portfolio\ value*portfolio\ \beta}{price\ of\ future\ contract}$ = $\frac{5050000*1.5}{1262500}$ = **6 contracts**

Original futures value shorted = 1262500* 6 = **₹ 7575000**

At expiry, 6 futures contracts are worth = 50 lot size* future value* 6 no. of contracts

= 50* 22556.25* 6 = ₹ 6766875

Short Value = 7575000

Square up Value = 6766875

Gain = **₹ 808125**

29. Illustration

A company is long on 10 MT of copper @ ₹ 474 per kg (spot) and intends to remain so for the ensuing quarter. The standard deviation of changes of its spot and future prices are 4% and 6% respectively, having correlation coefficient of 0.75.

What is its hedge ratio? What is the amount of the copper future it should short to achieve a perfect hedge if initial margin is 8%?

(RTP Nov'23, RTP Nov'21, MTP Oct'20, MTP Apr'18 Old, Old PM)

Solution

σ (Spot) = 4%

σ (Future) = 6%

r = 0.75

Hedge Ratio, = $\rho * \frac{\sigma_s}{\sigma_f}$ = $0.75 * \frac{4\%}{6\%}$ = 0.75*0.67 = **0.5**

Hedge using 10 MT*0.5 = 5 MT of Futures of Copper (short) in order to Hedge the spot exposure.

Initial Margin = 5000 kg* 8% * 474/kg

= ₹ 1,89,600

30. Illustration

On August 1, 2023, an investor has a portfolio consisting of 5 securities as shown below:

Security	Market Price (₹)	No. of Shares	Beta
A	60.00	450	0.87
B	320.00	850	1.31
C	640.00	200	0.94
D	130.00	500	0.66
E	480.00	600	1.50

The cost of capital for the investor is 20% p.a. compounded. The current NIFTY value is 19,500. NIFTY futures are available with expiry for 3 months (Oct-23) and 4 months (Nov-23) & are currently quoted at 19,700 and 19,900 respectively. Each NIFTY futures can be traded in units of 50 only.You are required to calculate:

(iii) The beta of his portfolio;

(ii) Theoretical value of Futures contract for contracts expiring in Oct. and Nov. (Given e0.05 = 1.05127, e0.06=1.06184, e0.07=1.07251)

(iii) The number of contracts the NIFTY the investor needs to sell to get a full hedge until November for his portfolio.

(iv) The number of future contracts the investor should trade if he desires to reduce the beta of his portfolio to 0.25.

(Nov'23 QP 8 marks)

Solution:

(i) Calculation of Portfolio Beta

Security	Price of the Stock	No. of shares	Value	Weightage wi	BetaBi	Weighted Beta
A	60.00	450	27,000	0.0346	0.87	0.0301
B	320.00	850	2,72,000	0.3488	1.31	0.4569
C	640.00	200	1,28,000	0.1641	0.94	0.1543
D	130.00	500	65,000	0.0833	0.66	0.0550
E	480.00	600	2,88,000	0.3692	1.50	0.5538
			7,80,000			**1.2501**

(ii) Calculation of Theoretical Value of Future Contracts Cost of Capital = 20% p.a.

Theoretical futures price = Spot + Coc – Dividend

(1) For October contract, t = 3/12 = 0.25 Further $F = Se^{rt}$
$F = ₹\ 19{,}500e^{(0.20)(0.25)}$ $F = ₹\ 19{,}500e^{0.05}$
F = ₹ 19,500 X 1.05127 = ₹ 20,499.77

(2) For November contract, t = 4/12 = 0.3333 Further $F= Se^{rt}$
$F = ₹\ 19{,}500e^{(0.20)(0.3333)}$
$F = ₹\ 19{,}500e^{0.0667}$

$e^{0.0667}$ shall be computed using Interpolation Formula as follows:

$e^{0.07}$ = 1.07251

$e^{0.06}$ = 1.06184
$e^{0.01}$ = 0.01067
$e^{0.007}$ = 0.00747
$e^{0.0033}$ = 0.00352
$e^{0.0667}$ = 1.06184 + 2/3 x (0.01067)
= 1.06184 + 0.00711
= 1.068953
Accordingly, the price of the November Contract

19500 X 1.068953 = ₹ 20,844.59

(iii) When total portfolio is to be hedged:
= Value of Spot Position requiring hedging X Portfolio Beta /Value of Future Contract
= 7,80,000 × 1.25 / 19,900 x 50
= 0.9799 contracts say 1 contract

The investor has to short one lot of Nifty Contracts to fully hedge his / her portfolio of equity shares

(iv) When total portfolio beta is to be reduced to 0.25:

Number of Contracts to be sold =

Portfolio Value (Beta of Portfolio – Desired Beta of Portfolio) / (Contract price x Lot size)

October Contracts
= 7,80,000 × (1.25 - 0.25) / 19,700 X 50
= 0.79 contracts say 1 contract

Alternatively, if we use November Contracts
= 7,80,000 × (1.25 - 0.25) / 19,900x 50
= 0.78 contracts say 1 contract

31. Illustration – Nov'24 (6 Marks)

On January 1, 2023, an investor has a portfolio of 5 securities as given below:

Security	Price (₹)	No. of shares	Beta
A	612.65	3000	?
B	334.20	5000	1.15
C	454.45	6000	0.40
D	775.10	10000	0.95
E	781.05	3000	0.85

Portfolio beta is 0.859.

The cost of capital of the investor is 10.5% p.a.
You are required to calculate:
(i) Beta of Security A
(ii) The theoretical value of the Nifty futures for February 2023. Current value of Nifty is 6500.
(iii) The number of contracts of Nifty the investor needs to sell to get a full hedge against the until February for his portfolio if the current value of NIFTY is 6500 and Nifty Contracts have a minimum trade lot requirement of 200 units. Assume that Nifty futures are trading at their fair value.

(iv) What would be the new Beta if 4 future contracts are sold to the investors?
(Note: No. of days in a year to be treated as 365 days

Ln (1.105) = 0.0998 $e^{0.015858}$ = 1.01598 and $e^{0.01668}$ = 1.01682)

Solution:

Part 1

Security	Price (₹)	No. of shares	Investment Value ₹	Weights	Beta	Weighted Beta
A	612.65	3,000	18,37,950	11.2552%	Beta A	11.26% x Beta A
B	334.2	5,000	16,71,000	10.2328%	1.15	0.11768
C	454.45	6,000	27,26,700	16.6977%	0.4	0.06679
D	775.1	10,000	77,51,000	47.4654%	0.95	0.45092
E	781.05	3,000	23,43,150	14.3489%	0.85	0.12197
Total			1,63,29,800			0.75736

Weighted Beta = 0.859
0.75736 + 11.2552% x Beta A = 0.859
Beta A = (0.859 -0.75736) /11.2552%
Beta A = 0.9030

Part 2
Theoretical Value of Futures = Spot x e^{rt}
Method 1
r = 10.5% Continuously compounded rate
t = 59 days from 1Jan till 28th Feb (ICAI has considered 58 days in all its workings in ICAI SM)
Future Value = 6500 x $e^{(0.105 \times 58/365)}$
Futures Value = 6500 x $e^{0.016685}$ (If 59 days are considered value would be 6500 x $e^{0.016972}$)
Futures Value = 6500 x 1.01682
Futures Value = 6609.33

Method 2
r = 10.5% pa rate
Continuously compounded rate is ln (1.105) = 0.0998
t = 59 days from 1Jan till 28th Feb (ICAI has considered 58 days in all its workings in ICAI SM)
Future Value = 6500 x $e^{(0.0.0998 \times 58/365)}$
Futures Value = 6500 x $e^{0.015858}$
Futures Value = 6500 x 1.01598
Futures Value = 6603.87

Part 3
Futures to be sold in order to hedge the portfolio Fully
Portfolio Beta x Portfolio Value = Nifty Beta x Nifty Lot size x No. of Contracts x Nifty Futures Value

0.859 x 1,63,29,800 = 1 x 200 x No. Of Contracts x 6609.33
1,40,27,298.2 = 200x 6609.33 x No. of Contracts
No of Contracts = 1,40,27,298.2/(200x 6609.33)
No of Contracts = 10.61
Since 10.61 contracts cannot be shorted, the company will short 11 Nifty Future Contracts

Part 4
Long 4 Nifty Futures contracts
Value of Nifty Futures gone long = 4 x 200 x 6609.33 = ₹52,87,464
Revised Portfolio beta = $\dfrac{\text{Old Portfolio Value x Beta + Nifty Contact Value Long x beta}}{\text{Value of Old Portfolio *}}$

	Amount	Beta	Portfolio Value x Beta
Portfolio	1,63,29,800	0.859	1,40,27,298.20
Nifty Futures	+13,21,866 x 4	1.000	+52,87,464.00
Total	1,63,29,800 (A)*	1.1828 (C = B/A)	1,93,14,762.20 (B)

Revised Portfolio beta = 1.1828
*When Nifty Futures are entered, on the date of the contract the value of the Derivative is zero hence they are ignored in Denominator (On initiation date Futures have a price not Value)

32. Illustration – Nov'24

Mohan buys 10,000 shares of X Ltd. @ ₹ 25 per share whose beta value is 1.5 and sells 5,000 shares of A Ltd. @ ₹ 40 per share having a beta value of 2. He obtains a complete hedge by buying 25 Nifty Futures. He closes out his position at the closing price of the next day when the share of X Ltd. has fallen by 4% and Nifty Futures has dropped by 2.50%. In the process he suffered a loss of ₹ 16,625.

You are required to determine:
(i) The value of the Nifty future
(ii) Initial cash outlay
(iii) Cash inflow at the close out
(iv) Percentage Gain/loss to Shares of A Ltd. at the time of closure (4 Marks)

Solution:
Part 1

Security	Count	Unit Price	Long / Short	Beta
X Ltd	10,000	25	Long	1.5
A Ltd	5000	40	Short	2.0
Nifty Futures	25	X	Long	1

Assuming Price per Nifty Future to be X. If the portfolio is full hedged the Beta of Portfolio will be 0
=> 10000*1.5*25-5000*40*2+25*X*1 = 0
=> 3,75,000-4,00,000+25X= 0
=> -25,000+25X = 0
=> X = 1000

Part 2: Initial Cash Outflow

Security	Amount ₹	Amount ₹
X Ltd – Buy **(Outflow)**	10,000 x 25	2,50,000
A Ltd – Sell **(Inflow)**	5,000x 40	+2,00,000
Nifty Futures – Buy		Nil
Net Cashflow		-50,000

Cash outflow Assuming No Outflow on Account of Nifty Futures = 50,000

If Nifty Futures also have an outflow (incorrect assumption by ICAI in certain old exams) then Cash Outflow will be -75,000 (i.e -50,000-25000)

Part 3:

Given Loss on Close out is ₹16,625 then Cash inflow at close out is:

If Outflow is 50,000 then inflow is 50,000-16,625 = 33,375

If Outflow is 75,000 then inflow is 75,000 – 16,625 = 58,375

Part 4:

Change in Price of Share A

Security	**Count**	**Org Price**	**Long / Short**	**Revised Price**	**Gain Loss**
X Ltd	10,000	25	Long	25 -4% = 24	-1 x 10000 = -10000
A Ltd	5000	40	Short	2.0	Loss on A (Bal Fig)
Nifty Futures	25	1000	Long	1000-2.5% = 975	-25 x25= -625
Total					-16625

Loss on shares of A limited 6000 (i.e. 16625-10000-625)

=> Opening Inflow -Closing outflow = -6000

Inflow = 200,000

Outflow = 2,06,000

Share price of A limited = 206000/5000 = 41.2

Original price = 40

Increase in price = 3% (1.2/40)

33. Illustration

On 1st July 2021 Mr. P has made the following investment:

Name of Company	**No. of Equity Share**	**Beta Value**	**Purchase Price per Equity Share**
ML Ltd	**1000**	**1.25**	**₹ 700**

He wants to hold the investment till end of September 2021 with an expectation of huge dividends to be announced in the AGM.

On the date of investment, September Nifty Futures are quoting at 17500 and tradeable with lot size of 50 for each contract.

You are the Investment advisor to Mr. P.

(i) Please advise Mr. P how to hedge his market exposure using the available data.

(ii) Calculate the profit or loss of Mr. P during the expiry of September 2021 futures in following situation:

(a) Nifty Future rises by 10%

(b) ML Ltd. falls by 5%

(iii) Is it possible stock as well as nifty to raise or fall at the same percentage? Please state the reason.

(8 Marks – Q 3b Dec'21)

Solution:

Part 1

Since Mr P is long the portfolio of stock ML Ltd, he has to short Nifty Futures in order to hedge the portfolio.

A portfolio is hedged perfectly if

Portfolio value x Portfolio Beta = NIFTY Contract value x Beta of Nifty (1)

NIFTY contract value = NIFTY Futures price 17500 x Lot size 50 x No. of contracts

Assume No of NIFTY contracts shorted is X then

⇨ 1000 x 700 x 1.25 = 17500 x 50 x X contracts x 1

⇨ 875000 = 875000 x X contracts

⇨ X = 1

So only one Lot of NIFTY is shorted

Part 2

NIFTY Future rises by 10% on Expiry

Loss on NIFTY Futures (Loss because Mr. P has shorted and NIFTY has risen)	8,75,000 x 10%	-87500
Gain on ML Stock with Beta of 1.25	7,00,000x 1.25 x 10%	87,500
Net Gain / Loss		Nil

ML Stock falls by 5% on Expiry

Loss on ML Stock	7,00,000x 5%	-35,000
Gain on NIFTY Futures (Gain because Mr. P has shorted and NIFTY has Fallen) Relationship between NIFTY & ML is beta of 1.25 If Nifty moves by 1% ML moves by 1.25% If ML moves down 5% then it means NIFTY would have moved by 5%/1.25 = 4%	8,75,000 x 4%	+35,000
Net Gain / Loss		Nil

Part 3

If the Systematic Risk Relation holds good, then it is not possible for NIFTY & ML stock to move by same % as ML ha a Beta of 1.25 & not 1. If ML had a beta of 1 then the stock would move in same *proportion as NIFTY. However, since it has Beta of 1.25, For a move of 1% in NIFTY, ML stock would move by 1.25%*

However, there can be a possibility that NIFTY & the stock both move by same percentage if there is an unsystematic element affecting the stock price.

34. Illustration

On 31st December, 2024, Mr. RS has taken a long position of 2 lots of Nifty Futures at price 25,400. One lot of Nifty Future is 50 units.

Margins:
(i) Initial margin required is 10% of contract value.
(ii) Maintenance margin required is 80% of initial margin.
The closing prices of Nifty Future for 5 days are given below:

Date	Closing price of Nifty Future
01 January, 2025	25,520
02 January, 2025	25,390
03 January, 2025	25,250
04 January, 2025	24,800
05 January, 2025	25,100

Evaluate the following:
(i) Daily balances in margin account and payment on margin calls, if any.
(ii) If contract squared off on 5th January 2025, gain or loss to Mr. RS.
(iii) If Mr. RS taken the short position, gain or loss to Mr. RS.

(May'25 6(a) – 7 Marks)

Solution:

Part 1

Initial Contract Value = 2 Lots x 50 Units per Lot x ₹25,400 per unit = ₹25,40,000

Initial Margin = 10% of Original Contract Value = ₹2,54,000

Maintenance Margin = 80% of Initial Margin = ₹2.54 Lakhs x 80% = ₹2,03,200

Daily Balances in ₹

Date	Opening NIFTY	Closing of NIFTY	Gain / Loss Nifty	Opening Margin	Margin Gain / Loss	Margin Funding	Closing Margin
31-Dec-24		25400					
01-Jan-25	25400	25520	120	254000	12000	0	266000
02-Jan-25	25520	25390	-130	266000	-13000		253000
03-Jan-25	25390	25250	-140	253000	-14000		239000
04-Jan-25	25250	24800	-450	239000	-45000	60000	254000
05-Jan-25	24800	25100	300	254000	30000		284000

Margin call Payment of ₹60,000 on 4th January as margin balance fell to ₹1,94,000 below maintenance requirement of ₹2,03,200. So the Margin balance has to be brought back to initial margin level of ₹2,54,000

Part 2: If contract is squared off on 5th January then gain / loss on the contract is computed as below

Margin Balance closing on Jan 5, 2025	₹2,84,000
Less: Original Funding in Margin Account	₹2,54,000
Less: Margin Funding	₹60,000
Net Loss	**- ₹ 30,000**

Method 2 for computation of loss : Change in Nifty in x no of units of NIFTY
= (25400 – 25100) x 100 = -300 x 100 = ₹30,000 Loss
Part 3: IF short position were taken by Mr. RS he would have made a profit of ₹ 30,000 computed as
= (25100 – 25400) x 100 = 300 x 100 = ₹30,000 Profit
As one makes a gain when we short a future and market price of future falls.

1. Illustration

Mr. A purchased a 3-month call option for 100 shares in XYZ Ltd. at a premium of Rs. 30 per share, with an exercise price of Rs.550. He also purchased a 3-month put option for 100 shares of the same company at a premium of Rs. 5 per share with an exercise price of Rs. 450. The market price of the share on the date of Mr. A's purchase of options, is Rs. 500. Calculate the profit or loss that Mr. A would make assuming that the market price falls to Rs. 350 at the end of 3 months.

(Old PM)

Solution:

	Call option	Put Option	Total
Spot	₹ 500	₹ 500	
X	₹ 550	₹ 450	
Type	Buy	Buy	
Premia	₹ 30	₹ 5	₹ 35
# of Shares/ Options	100	100	
Expiry in Months	3	3	
Closing Price	₹ 350	₹ 350	
Profit/ Loss	0	(450-350) *100	= ₹ 10,000
Less: Premia			₹ 3,500
Net Pay Off for 1 Put + 1 Call an option of share each respectively			₹ 65
Lot Size			100
Total Net Pay off for 1 Call& 1 Put with lot size of 100 shares each			₹ 6,500

2. Illustration

The market received rumour about ABC corporation's tie-up with a multinational company. This has induced the market price to move up. If the rumour is false, the ABC corporation stock price will probably fall dramatically.
To protect from this an investor has bought the call and put options.
He purchased one 3 months call with a striking price of Rs. 42 for Rs. 2 premium, and paid Rs.1 per share premium for 3 months put with a striking price of Rs. 40.
Determine the Investor's position if the tie up offers bids the price of ABC Corporation's stock up to Rs. 43 in 3 months.
Determine the Investor's ending position, if the tie up program fails, and the price of the stocks falls to Rs. 36 in 3 months.

(ICAI SM, May'24 QP, RTP May'19, RTP Nov'18 Old, Old PM)

Solution:

Type	Call option	Put Option	Total
Action	Buy	Buy	
X	₹ 42	₹ 40	
Premia	₹ 2	₹ 1	
Expiry in Months	3	3	
Case (a)			
Closing Price	₹ 43	₹ 43	
Gain/ Loss on option	₹ 1	₹ 0	₹ 1
Less: Premia Paid			₹ -3
Net Gain/ Loss			₹ -2
Case (b)			
Closing Price	₹ 36	₹ 36	
Gain/ Loss on option	₹ 0	₹ 4	₹ 4
Less: Premia Paid			₹ 3
Net Gain/ Loss			₹ 1
Assumption: Each Call/ Put Option is for 1 share of ABC Ltd.			

3. Illustration

Equity share of PQR Ltd. is presently quoted at Rs. 320. The Market Price of the share after 6 months has the following probability distribution:

Market Price in Rs.	Probability
180	0.1
260	0.2
280	0.5
320	0.1
400	0.1

A put option with a strike price of Rs. 300 can be written.
You are required to find out expected value of option at maturity (i.e., 6 months)

(ICAI SM, Old PM)

Solution:

Spot	₹ 320
X	₹ 300
Option Type	Put
Expiry in Months	6

Stock Price after 6 Months	₹ 180	₹ 260	₹ 280	₹ 320	₹ 400	
Probabilities	0.1	0.2	0.5	0.1	0.1	1
Expected Stock Price (Irrelevant)	18	52	140	32	40	282
Payoff from Buy Put	₹ 120	₹ 40	₹ 20	₹ 0	₹ 0	
Probabilities	0.1	0.2	0.5	0.1	0.1	1
Expected Pay Off of a Buy Put	₹ 12	₹ 8	₹ 10	₹ 0	₹ 0	₹ 30
Expected value of option at maturity i.e., after 6 months	**₹ 30**					

4. Illustration

You as an investor had purchased a 4-month call option on the equity shares of X Ltd. of Rs.10, of which the current market price is Rs. 132 and the exercise price Rs. 150. You expect the price to range between Rs. 120 to Rs. 190. The expected share price of X Ltd. and related probability is given below:

Expected Price	Rs. 120	Rs. 140	Rs. 160	Rs. 180	Rs. 190
Probability	0.05	0.20	0.50	0.10	0.15

Compute the following:

(i) Expected Share price at the end of 4 months.

(ii) Value of Call Option at the end of 4 months, if the exercise price prevails.

(iii) In case the option is held to its maturity, what will be the expected value of the call option?

[ICAI SM, MTP Apr'24, MTP Mar'22, MTP Apr'18, Nov'18 QP (Old), MTP Oct'19 Old, Old PM]

Solution:

Option Type	Call	Expected Price	Probability	Probability Weighted Expected Share Price	Value of Option	Probability Weighted Expected Value of Call Option	
Action	Buy	₹ 120	5%	6	₹ 0	-	*
Expiry	4 months	₹ 140	20%	28	₹ 0	-	
Premia	₹ 10	₹ 160	50%	₹ 80	₹ 10	₹ 5	
Spot	₹ 132	₹ 180	10%	18	₹ 30	₹ 3	
X	₹ 150	₹ 190	15%	28.5	₹ 40	₹ 6	
Price Range	₹ 120-190			**160.5**		**₹ 14**	

i) Expected Share Price after 4 months is ₹ 160.5

ii) If Exercise Price and Closing Price on Date of Expiry are the same @ ₹ 150, then the value of the call option is zero

iii) If the Option is held till maturity the expected value of the call option is ₹ 14

*- When Market Price is below the strike price, call options are not exercised and hence they do not have any value at that time.

5. Illustration

Mr. X established the following strategy on the Delta Corporation's stock:

- Purchased one 3-month call option with a premium of Rs. 30 and an exercise price of Rs. 550.
- Purchased one 3-month put option with a premium of Rs. 5 and an exercise price of Rs. 450.

Delta Corporation's stock is currently selling at Rs. 500. Determine profit or loss, if the price of Delta Corporation's:

i. Remains at Rs. 500 after 3 months.

ii. Falls at Rs. 350 after 3 months.

iii. Rises to Rs. 600.

Assume the size option is 100 shares of Delta Corporation.

(ICAI SM, MTP Mar'24, MTP Apr'22, MTP Oct'18, RTP May'18 Old, Old PM)

Solution:

Company Name	Delta Corp
Buy Put X	₹ 450
Buy Put Premia	₹ 5
Option Expiry (Call and Put)	+ 3 months

Stock Price	₹ 500	₹ 350	₹ 600
Pay off from Buy Call	₹ 0	₹ 0	₹ 50
Pay off from Buy Put	₹ 0	₹ 100	₹ 0
Total Premia Paid	- ₹ 35	-₹ 35	-₹ 35
Net Gain/ Loss - From Call and Put Options with Lot size of 1 share each	-₹ 35	₹ 65	₹ 15
Lot size for each Put/ Call Option	100	100	100
Net Pay Off from both Put and Call for Lot sizes of 100 shares for each option	-₹ 3500	₹ 6,500	₹ 1,500

6. Illustration

The equity share of VCC Ltd. is quoted at Rs. 210. A 3-month call option is available at a premium of Rs. 6 per share and a 3-month put option is available at a premium of Rs. 5 per share.

Ascertain the net payoffs to the option holder of a call option and a put option separately.

(i) the strike price in both cases in Rs. 220; and

(ii) the share price on the exercise day is Rs. 200,210,220,230,240.

Also indicate the price range at which the call and the put options may be gainfully exercised.

(ICAI SM, Nov'18 QP 8 marks, RTP Nov'24, MTP Oct'19, RTP Nov'19 Old, Old PM)

Solution:

Company	VCC Ltd
Spot	₹ 210.00
Buy Call X	₹ 220.00
Buy Call Premia	₹ 6.00
Buy Put X	₹ 220.00
Buy Put premia	₹ 5.00
Call & Put Expiry	3 months

Closing price on Expiry	₹ 200.00	₹ 210.00	₹ 220.00	₹ 230.00	₹ 240.00
Pay off From Call	Max (0, Closing price - X)				
Pay off from Call	₹ 0.00	₹ 0.00	₹ 0.00	₹ 10.00	₹ 20.00
Premia Paid	-₹ 6.00	-₹ 6.00	-₹ 6.00	-₹ 6.00	-₹ 6.00
Net Pay off call	-₹ 6.00	-₹ 6.00	-₹ 6.00	₹ 4.00	₹ 14.00
Pay off from Put	Max (0, X-Closing price)				
Pay off from Put	₹ 20.00	₹ 10.00	₹ 0.00	₹ 0.00	₹ 0.00
Premia Paid	-₹ 5.00	-₹ 5.00	-₹ 5.00	-₹ 5.00	-₹ 5.00
Net Pay off Put	₹ 15.00	₹ 5.00	-₹ 5.00	-₹ 5.00	-₹ 5.00

The Call option will be exercised at every level where the closing price is > X i.e., ₹220. However, the net pay off will be positive (the option will achieve breakeven) @ closing price >226
The Put option will be exercised at every level where the closing price is < X i.e., ₹220. However, the net pay off will be positive (the option will achieve breakeven) @ closing price <215

7. Illustration

Sumana wanted to buy shares of EIL which has a range of Rs. 411 to Rs. 592 a month later. The present price per share is Rs. 421. Her broker informs her that the price of this share can soar up to Rs. 522 within a month or so, so that she should buy a one-month CALL of EIL.
In order to be prudent in buying the call, the share price should be more than or at least Rs. 522 the assurance of which could not be given by her broker.
Though she understands the uncertainty of the market, she wants to know the probability of attaining the share price Rs. 592 so that buying of a one-month CALL of EIL at the execution price of Rs. 522 is justified. Advise her.
Take the risk-free interest for one month to be 3.60% and $e^{0.036}$ = 1.037.

(ICAI SM, MTP Aug'18, Old PM)

Solution:

Company Name	EIL
Price Range	411 -592
Spot	₹ 421.00
X	₹ 522.00
Option	Call
Action	Buy
S_{0u}	₹ 592.00
S_{0d}	₹ 411.00
u	1.406
d	0.976
$e\ (3.6\%)$	1.037
p	?
p	$\frac{(e^{rt}) - d}{u - d}$
p	**14.19%**

Probability of achieving a price of ₹592 is 14.19%

8. Illustration

Mr. Dayal is interested in purchasing equity shares of ABC Ltd. which are currently selling at Rs. 600 each. He expects that price of share may go up to Rs. 780 or may go down to Rs. 480 in three months. The chances of occurring such variations are 60% and 40% respectively. A call option on the shares of ABC Ltd. can be exercised at the end of three months with a strike price of Rs. 630.

i. **What combination of share and option should Mr. Dayal select if he wants a perfect hedge?**
ii. **What should be the value of option today (the risk-free rate is 10% p.a.)?**
iii. **What is the expected rate of return on the option?**

(ICAI SM, RTP Nov'22, MTP Oct'22, MTP Apr'21 New & Old, MTP Mar'18, MTP Aug'18 Old, Old PM)

Solution:

Company name	ABC Ltd	
S_0	₹600	
S_{0U}	₹ 780	60%
S_{0d}	₹ 480	40%
X	₹ 630	
Expiry	3months	
$R_f$pa	0.1	
u	1.3	
d	0.8	
Interest rate for 3 months	2.5%	
FV factor	1.025	
p	$\frac{(e^{rt}) - d}{u - d}$	
p	45%	
$1 - p$	55%	

Part 1				
Δ	$\frac{C_U - C_d}{S_U - S_d}$	=	$\frac{₹150 - 0}{₹780 - 480}$	0.5
C_U	₹ 150.00 (780-630)			
C_d	0			
Mr. Dayal Should Sell 1 Call & Buy 0.50 Shares or If he buys 1 Share, he has to sell 2 calls				
Part 2				
Expected value of the option	$\frac{(P * (C_U)) + (1 - P) * C_d}{FV\ Factor}$	$\frac{₹67.50}{1.025}$	₹ 65.85	
Assumption: Probabilities of 60% & 40% provided in the questions are not relevant as u / d/ & FV are already provided and p can be computed from there				

Part 3			
Expected rate of return of the option based on Mr. Dayal's expected variations of 60% & 40% = 60% x Max (0,780-630) + 40% x Max (0,480-630) =60% x 150 = 90			
PV	FV	Return for 3 months	PA Return
₹ 65.85	₹ 67.50	(90/65.85)-1 = 36.67%	36.67% x 4 = 146.67%

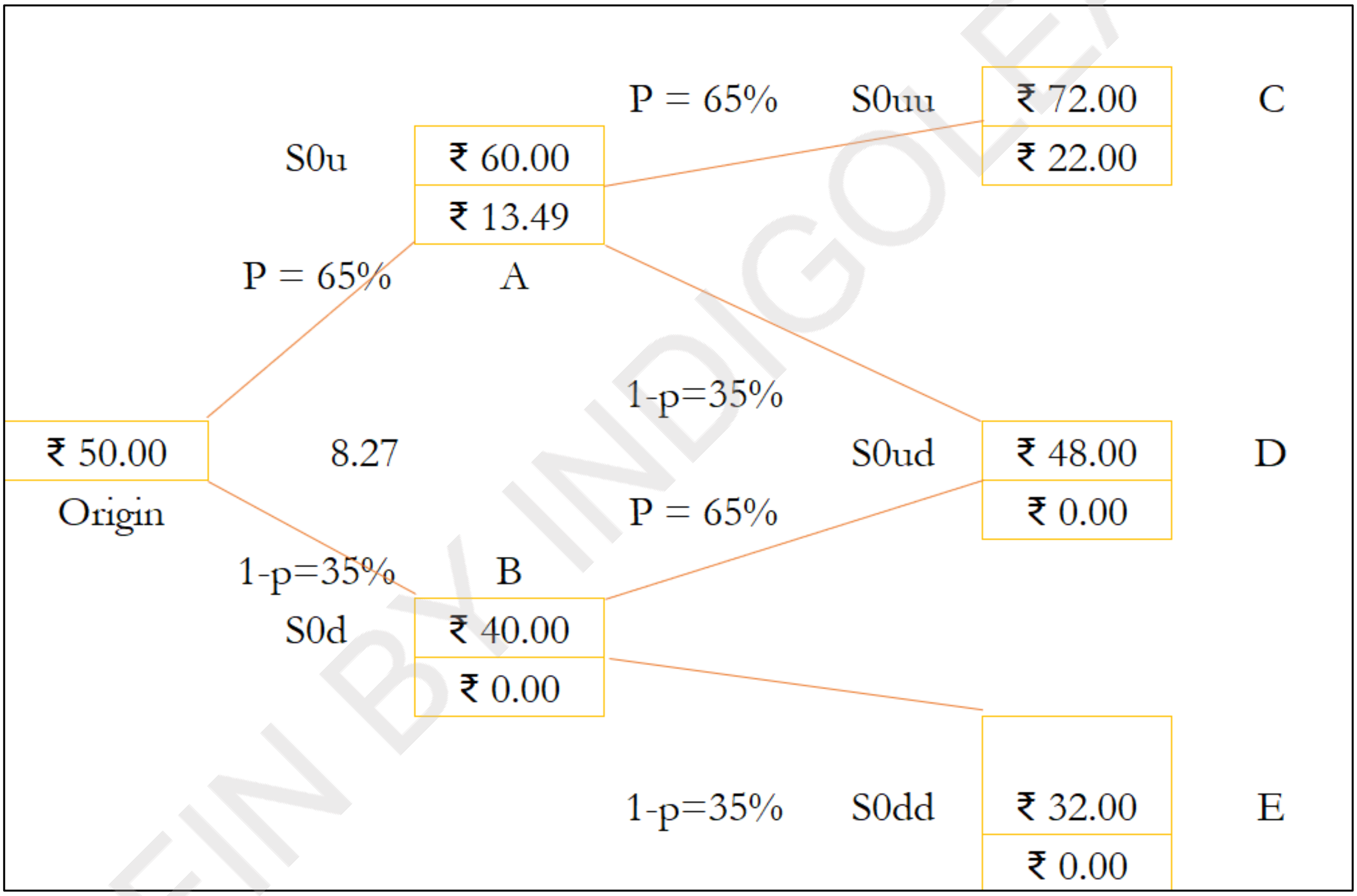
9. Illustration

Consider a two-year call option with a strike price of Rs. 50 on a stock the current price of which is also Rs. 50. Assume that there are two-time periods of one year and in each year the stock price can move up or down by equal percentage of 20%.
The risk-free interest rate is 6%. Using binominal option model, calculate the probability of price moving up and down. Also draw a two-step binomial tree showing prices and payoffs at each node.

(ICAI SM, RTP Nov'19, Old PM)

Solution:

Option Type	Call
Action	Buy
S_0	50
X	50
u	1.2
d	0.8
Periods	2
R_f	0.06
e^{rt}	R
R	1.06
A/ E	European
p	$\frac{R-d}{u-d}$
p	0.65



Node	Option Value	Option Value
A	((65%*22) + (35%*0))/1.06	13.4906
B		Nil
Origin	(65%*13.49+35%*0)/1.06	8.27217
Probability of price moving up is 65%		
Probability of price moving down is 35%		

10. Illustration

The current market price of an equity shares of Penchant Ltd is Rs. 420. Within a period of 3 months, the maximum and minimum price of it is expected to be Rs. 500 and Rs. 400 respectively. If the risk-free rate of interest be 8% p.a., what should be the value of a 3 months Call option under the "Risk Neutral" method at the strike rate of Rs. 450? Given $e^{0.02}$ = 1.0202.

(ICAI SM, MTP Sept'22, MTP Oct'18, Old PM)

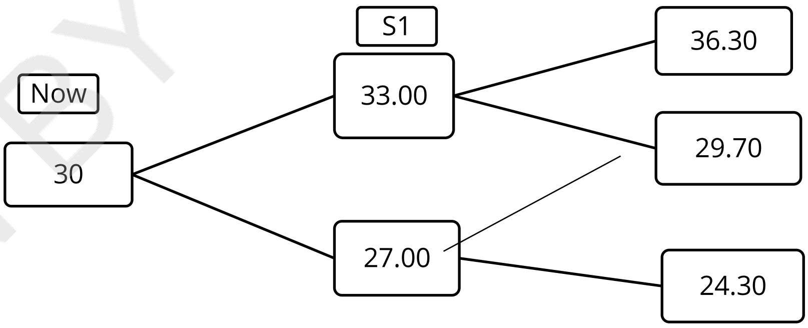
Solution:

Company	Penchant Ltd
S_0	420
Expiry	3 months
S_{0U}	500
S_{0D}	400
U	1.19047619
D	0.952380952
R_f	0.08
Option type	Call
X	450
Value of 3-month call	?
$e^{\wedge}(0.02)$	1.0202

P	$\frac{(e^{rt}) - d}{u - d}$	$\frac{0.067819}{0.238095}$	0.28484	
Option price	$\frac{\{(C_{0U}\ x\ P) + (C_{0D}\ x\ (1-p))\}}{e^{\wedge}rt}$	$\frac{28.48\%\ x\ 50 +\ 71.52x\ 0}{1.0202}$	$\frac{14.242}{1.0202}$	13.96

11. Illustration

Following is a two sub-periods tree of 6-months each for the share of CAB Ltd.:



Using the binomial model, calculate the current fair value of a regular call option on CAB Stock with the following characteristics: X = Rs. 28, Risk Free Rate = 5 percent p.a. You should also indicate the composition of the implied riskless hedge portfolio at the valuation date.

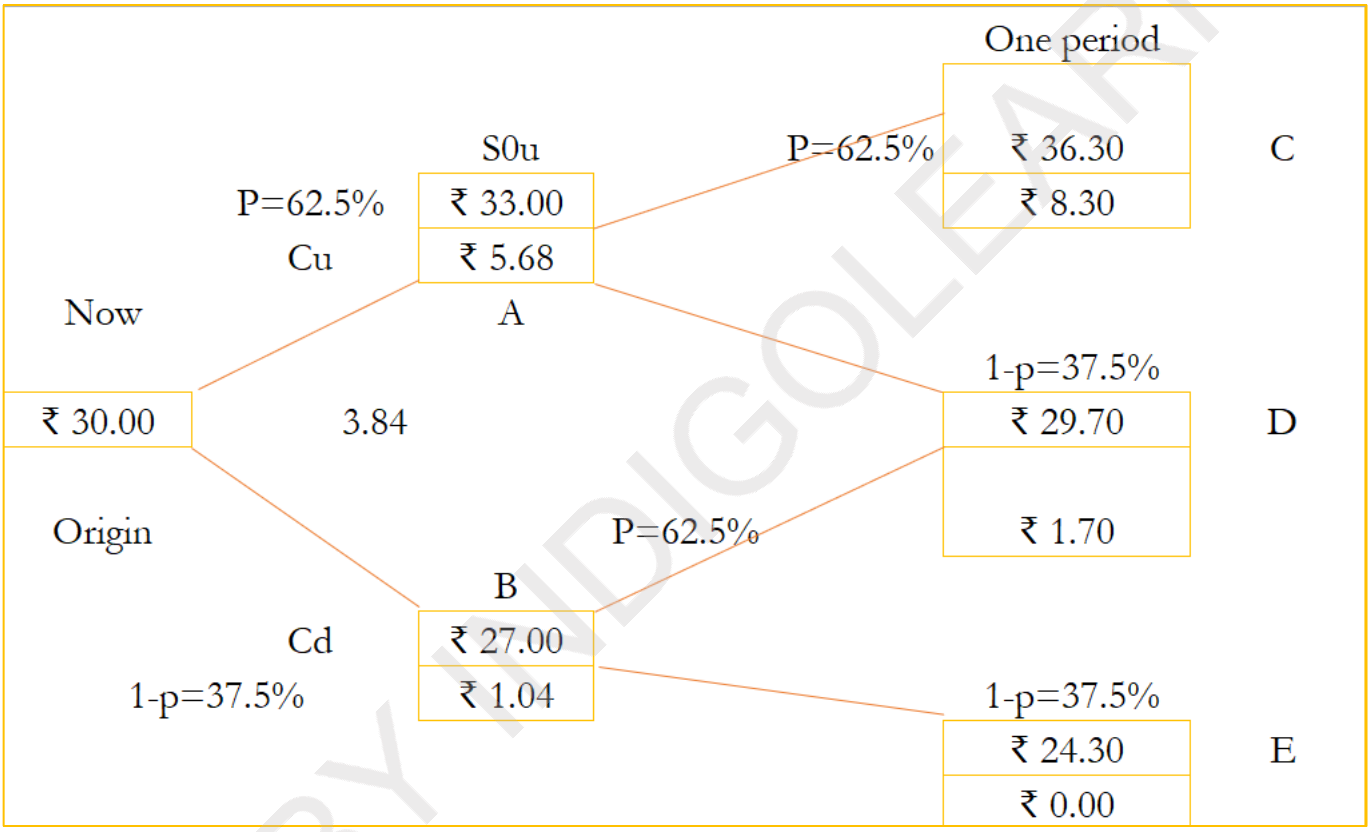
(ICAI SM)

Solution:

S_0	₹	30.00
S_{0U}	₹	33.00
S_{0D}	₹	27.00
U	1.1	
D	0.9	

X	28
Period	6 months
R_f	5%
R	1.025

R	1.025	Assumed that half year rate is 1.025	$(1.05)^\wedge\left(\frac{1}{2}\right)$	1.025
p	$\frac{R-d}{u-d}$	$\frac{1.025-0.90}{1.1-0.9}$	$\frac{0.125}{0.2}$	0.625
$1-p$				0.375
A/E	European			



Node	Option Value	Option Value		
A	(62.5% x 8.3+37.5% x 1.7)/1.0247	**5.68293**		
B	(62.5% x 1.7+37.5% x 0)/1.0247	**1.03659**		
Origin	(62.5% x 5.68+37.5% x 1.04)/ (1.0247)	**3.84444**		
Δ	$(C_U - C_d)/(S_U - S_D)$	$\frac{5-0}{33-27}$	$\frac{5}{6}$	0.83333

For every Call option sold - One should go along on 5/6 Shares **in order to create a riskless portfolio.**

12. Illustration

Mr. KK purchased a three-month call option for 100 shares in PQR limited at a premium of Rs. 40 per share with exercise price of Rs. 560. He also purchased three month put option for 100 shares of the same company at a premium of Rs. 10 per share with an exercise price of Rs. 460.
The market price of shares on the date of KK purchase of options is Rs. 500. Compute the profit or loss that KK would make assuming the market price falls to Rs. 360 at the end of three months.

(May'18 QP 4 marks)

Solution:

Company Name	PQR Ltd
Spot	500
Call expiry	3 months
XC	560
C_0	40
Lot size	100
Put expiry	3 months
P_0	10
XP	460
Lot size	100
Closing price 3 months	**360**

	Call option	Put Option	Total
Closing Spot	360	360	
X	560	460	
Pay off from Option	0	100	
Premia paid	-40	-10	
Net payoff per 1 share	-40	90	
Lot size	100	100	
Total Pay off	-4000	9000	**5000**

Mr. KK would make a net profit of ₹5000 at the expiry date from the call and put options, if the closing price on expiry date is ₹360

13. Illustration

Mr John established the following spread on TTK Ltd's stock.

1. Purchased one three month put option with a premium of Rs. 15 and an exercise price of Rs. 900
2. Purchased one three-month call option with a premium of Rs. 90 and an exercise price of Rs. 1,100

TTK Ltd stock is currently selling at Rs. 1,000. Calculate gain or loss if the price of stock of TTK Ltd

i. Remains at Rs. 1,000 after 3 months
ii. Falls to Rs. 700 after 3 months
iii. Raises to Rs. 1200 after 3 months

Assume the size of option is 200 shares of TTK Ltd.

(May'19 QP 8 marks)

Solution:

Company Name	TTK Ltd	
Investor	John	
Spot	1000	
	Call	Put
Expiry	3 months	3 months
X	1100	900
Premia	90	15
Lot size	200	200
Closing price A	1000	
Closing price B	700	
Closing price C	1200	

Closing Price	1000	700	1200
Call option			
X	1100	1100	1100
Pay off from Call	0	0	100
Call premia	90	90	90
Total Pay off per 1 share	-90	-90	10
Lot size	200	200	200
Total pay off for 200 shares	-18000	-18000	2000
Put option			
Closing Price	1000	700	1200
X	900	900	900
Pay off	0	200	0
Premia	15	15	15
Net Pay off	-15	185	-15
Lot Size	200	200	200
Total Pay off for 200 shares	-3000	37000	-3000
Total Gain or loss for Call + Put	**-21000**	**19000**	**-1000**

14. Illustration

Fresh Bakery Ltd.' s share price has suddenly started moving both upward and downward on a rumour that the company is going to have a collaboration agreement with a multinational company in bakery business. If the rumour turns to be true, then the stock price will go up but if the rumour turns to be false, then the market price of the share will crash.

To protect from this an investor has purchased the following call and put options:

i. One 3 months call with a striking price of Rs. 52 for Rs. 2 premium per share.

ii. One 3 months put with a striking price of Rs. 50 for Rs. 1 premium per share.

Assuming a lot size of 50 shares, determine the followings:

1. The investor's position, if the collaboration agreement pushes the share price to Rs. 53 in 3 months.
2. The investor's ending position, if the collaboration agreement fails and the price crashes to Rs. 46 in 3 months' time.

(May'16 QP 5 marks)

Solution:

Company Name	Fresh Bakery	
Option	Call	Put
Expiry	3 months	3 months
X	₹ 52.00	₹ 50.00
Premia	₹ 2.00	₹ 1.00
Lot size	50.00	50.00
Closing Price A	₹ 53.00	
Closing Price B	₹ 46.00	

	Case A	Case B
Closing Price	₹ 53.00	₹ 46.00
Call Option		
X	₹ 52.00	₹ 52.00
Pay off from call	₹ 1.00	₹ 0.00
Les: Premia	-₹ 2.00	-₹ 2.00
Net Pay off	-₹ 1.00	-₹ 2.00
Lot size	50.00	50.00
Total pay off from call	**-₹ 50.00**	**-₹ 100.00**
Put Option		
X	₹ 50.00	₹ 50.00
Closing Spot	₹ 53.00	₹ 46.00
Option Pay off	₹ 0.00	₹ 4.00
Less: Premia	-₹ 1.00	-₹ 1.00
Net payoff	-₹ 1.00	₹ 3.00
Lot size	50.00	50.00
Total pay off from Put	**-₹ 50.00**	**₹ 150.00**
Total pay off from Call + Put	**-₹ 100.00**	**₹ 50.00**

15. Illustration

A call and put exist on the same stock each of which is exercisable at Rs. 60. They now trade for:

Market price of Stock or stock index	**Rs. 55**
Market price of call	**Rs. 9**
Market price of put	**Rs. 1**

Calculate the expiration date cash flow, investment value, and net profit from:

(i) **Buy 1.0 call**
(ii) **Write 1.0 call**
(iii) **Buy 1.0 put**
(iv) **Write 1.0 put**

for expiration date stock prices of Rs. 50, Rs. 55, Rs. 60, Rs. 65, Rs. 70.

(Old PM)

Solution:

	Call	Put
Spot	₹ 55.00	₹ 55.00
Premia	₹ 9.00	₹ 1.00
X	₹ 60.00	₹ 60.00

Expiry date CF					
Closing prices	₹ 50.00	₹ 55.00	₹ 60.00	₹ 65.00	₹ 70.00
Buy Call	₹ 0.00	₹ 0.00	₹ 0.00	-₹ 60.00	-₹ 60.00
Sell Call	₹ 0.00	₹ 0.00	₹ 0.00	₹ 60.00	₹ 60.00
Buy Put	₹ 60.00	₹ 60.00	₹ 0.00	₹ 0.00	₹ 0.00
Sell Put	-₹ 60.00	-₹ 60.00	₹ 0.00	₹ 0.00	₹ 0.00

Expiry Date Investment Value					
Closing prices	₹ 50.00	₹ 55.00	₹ 60.00	₹ 65.00	₹ 70.00
Buy Call	₹ 0.00	₹ 0.00	₹ 0.00	₹ 5.00	₹ 10.00
Sell Call	₹ 0.00	₹ 0.00	₹ 0.00	-₹ 5.00	-₹ 10.00
Buy Put	₹ 10.00	₹ 5.00	₹ 0.00	₹ 0.00	₹ 0.00
Sell Put	-₹ 10.00	-₹ 5.00	₹ 0.00	₹ 0.00	₹ 0.00

Net Profit					
Closing prices	₹ 50.00	₹ 55.00	₹ 60.00	₹ 65.00	₹ 70.00
Buy Call	-₹ 9.00	-₹ 9.00	-₹ 9.00	-₹ 4.00	₹ 1.00
Sell Call	₹ 9.00	₹ 9.00	₹ 9.00	₹ 4.00	-₹ 1.00
Buy Put	₹ 9.00	₹ 4.00	-₹ 1.00	-₹ 1.00	-₹ 1.00
Sell Put	-₹ 9.00	-₹ 4.00	₹ 1.00	₹ 1.00	₹ 1.00

16. Illustration

Ram holding shares of Reliance Industries Ltd. which is currently selling at Rs. 1000. He is expecting that this price will further fall due to lower-than-expected level of profits to be announced after one month. As on following option contract are available in Reliance Share

Strike Price	Option	Premium
1030	Call	40
1010	Call	35
1000	Call	30
990	Put	35
970	Put	20
950	Put	8
930	Put	5

Ram is interested in selling his stock holding as he cannot afford to lose more than 5% of its value.

Recommend a hedging strategy with option and show how his position will be protected.

*(RTP Nov'18, MTP Apr'23, MTP Nov'21 New & Old)*Solution:

Solution:

Company	RIL
Investor	Ram
Shares Held	?
S0	₹ 1,000.00
Max loss	5%
Min sale price	₹ 950.00
Strategy	Buy Put
950 Put premia	₹ 8.00

Mr. Ram intends to protect his downside from reduction in share price - So he has to buy a Put. Since he wants a loss no more than 5%, he should but a put with X of S0-5% i.e., ₹1000-5% i.e., ₹950

Net payoff		
Scenario	A	B
Closing Spot	>950	<950
Pay off	ST / Closing Price	950.00
Premia Paid	(8.00)	(8.00)
Net Pay off	ST-8	942.00
Mr. Ram gets a minimum value of ₹942 from hedging his RIL shares by buying a put @K of 950 & Premia of ₹8		

17. Illustration

A call option has been entered into by Arnav for delivery of share of X Ltd. at Rs. 460. The expected future prices at the time of expiry of contract are as follows:

Price in Rs.	Probability
470	0.20
450	0.25
480	0.35
490	0.05
500	0.15

Determine the premium at which Arnav will break even.

Solution:

Company Name	X ltd
Investor	Arnav
Option Type	Call
Action	Buy
K	₹ 460.00

Price	₹ 470.00	₹ 450.00	₹ 480.00	₹ 490.00	₹ 500.00	Total
Probability	0.20	0.25	0.35	0.05	0.15	
Option payoff	₹ 10.00	₹ 0.00	₹ 20.00	₹ 30.00	₹ 40.00	
Expected option payoff	₹ 2.00	₹ 0.00	₹ 7.00	₹ 1.50	₹ 6.00	₹ 16.50
Break-even Premia	₹ 16.50					

18. Illustration

AB Ltd.'s equity shares are presently selling at a price of Rs. 500 each. An investor is interested in purchasing AB Ltd.'s shares. The investor expects that there is a 70% chance that the price will go up to Rs. 650 or a 30% chance that it will go down to Rs. 450, three months from now. There is a call option on the shares of the firm that can be exercised only at the end of three months at an exercise price of Rs. 550.

Calculate the following:

(i) If the investor wants a perfect hedge, what combination of the share and option should he select?

(ii) Explain how the investor will be able to maintain identical position regardless of the share price.

(iii) If the risk-free rate of return is 5% for the three months period, what is the value of the option at the beginning of the period?

(iv) What is the expected return on the option? *(Nov'19 QP 8 marks)*

Solution:

Company Name AB Ltd

Spot	₹ 500.00				
Investor expectation					
70%	₹ 650.00	u	1.30		
30%	₹ 450.00	d	0.90		
Option	Call				
Action	Sell				
X	₹ 550.00				
Expiry period	3 months				
Part 1					
Delta Δ	$\frac{(Change\ in\ Option\ price)}{\text{Change in Stock Price}}$	$\frac{C_U - C_d}{S_U - S_d}$	$\frac{100-0}{650-450}$	$\frac{100}{200}$	0.50

Give that Delta is 0.5, The investor should Buy 0.5 shares for every call sold in order to have a perfect hedge

Part 2

Position / Stock Price	₹ 650.00	₹ 450.00
Portfolio	0.5 Stock - 1 Call	0.5 Stock - 1 Call
Value of 0.5 Stock	₹ 325.00	₹ 225.00
Value of 1 call sold	-₹ 100.00	₹ 0.00
Total Portfolio value	₹ 225.00	₹ 225.00

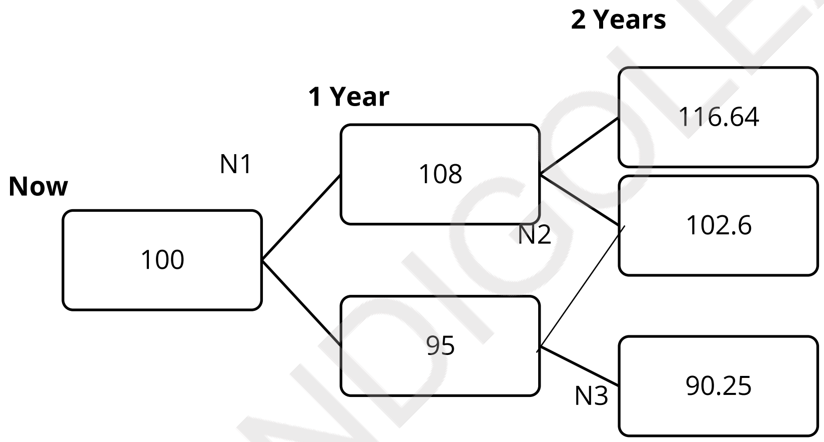
Part 3

3-month R_f	5%
(0.5 stock PV - 1 Call) * (1+R_f)	=FV of portfolio
$(250 - C) * (1 + 5\%)$	₹ 225.00
$\mathbf{250 - C}$	225/1.05
$\mathbf{250 - C}$	214.29
$\boldsymbol{C}$	250-214.29
$\boldsymbol{C}$	35.71
Using Risk Neutral Method	

P	$\frac{R-d}{u-d}$
P	(1.05-.9) / (1.3-.9)
P	37.50%
$1-P$	62.50%
FV of C	37.50
PV of C	35.71
Return =	$\frac{EV}{PV}-1$
EV	70.00
PV	35.71
Return	96.00%

19. Illustration

A two-year tree for a share of stock in ABC Ltd., is as follows:



Consider a two-year American call option on the stock of ABC Ltd., with a strike price of Rs. 98. The current price of the stock is Rs. 100. Risk free return is 5 per cent per annum with a continuous compounding and $e^{0.05}$ = 1.05127. Assume two time periods of one year each.

Using the Binomial Model, calculate:

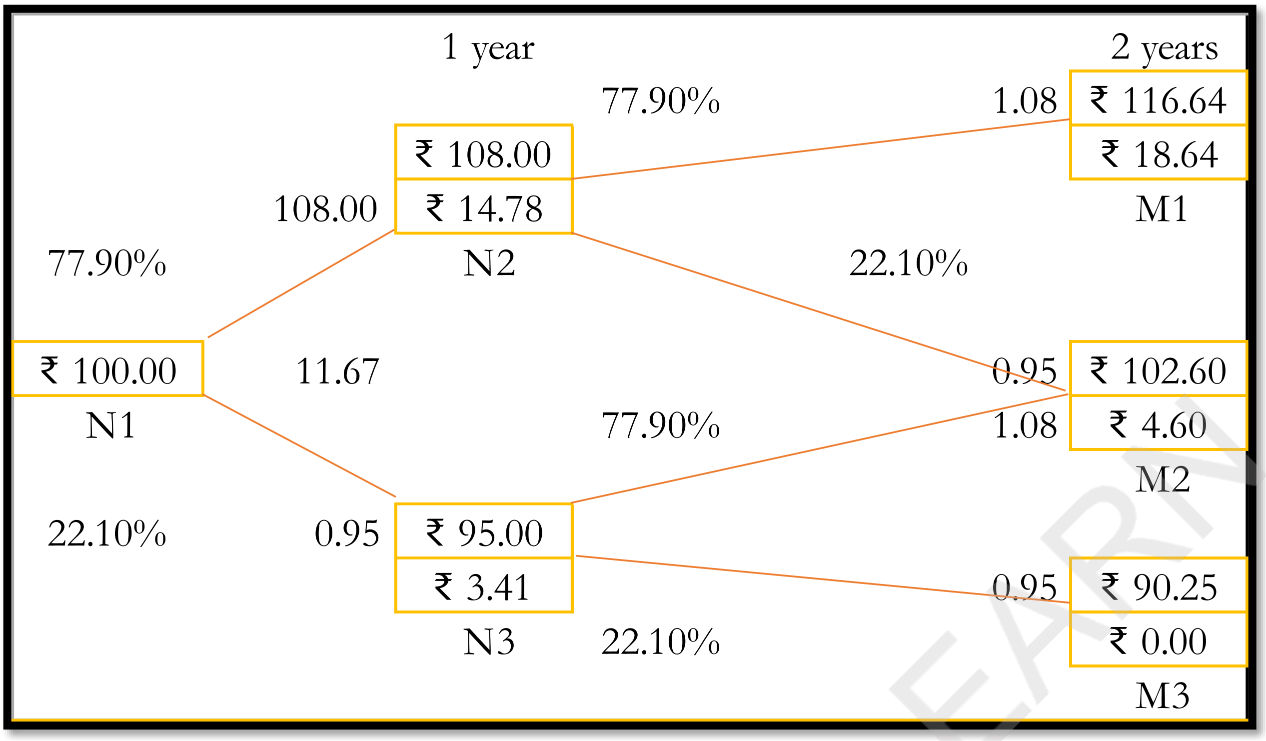
(i) The probability of price moving up and down;

(ii) Expected pay offs at each node i.e., N1, N2 and N3 (round off up to 2 decimal points).

(Nov'20 QP 8 marks, RTP May'23)

Solution:

S_0	₹ 100.00
X	₹ 98.00
$e^{\wedge}(0.05)$	1.05127
u	1.08000
d	0.95000
p	$\frac{R-d}{u-d}$
R	$e^{\wedge}(rt)$
R	1.05127
p	77.90%
$1-p$	22.10%



N1	11.67	(14.78*77.90%+3.41*22.10%)/1.05127
N2	14.78	(18.64*77.90%+4.60*22.10%)/1.05127
N3	3.41	(0*22.1%+4.60*77.90%)/1.05127

20. Illustration

Mr. P established the following spread on the Coastal Corporation's stock:

(i) Purchased one 3-month call option with a premium of Rs. 6.5 and an Exercise price of Rs. 110.

(ii) Purchased one 3-month put option with a premium of Rs. 10 and an Exercise price of Rs. 90.

Coastal Corporation's stock is currently selling at Rs. 100.
Determine profit or loss, if the price of Coastal Corporation's stock:

(i) Remains at Rs. 100 after 3 months.

(ii) Falls at Rs. 70 after 3 months.

(iii) Rises to Rs. 138 after 3 months.

Assume the size of option is 1,000 shares of Coastal Corporation.

[RTP May'22, Nov'20 QP (Old)]

Solution:

Company	Coastal Corp.		
Investor	Mr. P		
Option	Call	Put	
Action	Buy	Buy	
X	₹ 110.00	₹ 90.00	
Expiry	3 months	3 months	
Premia	₹ 6.50	₹ 10.00	
Spot	₹ 100.00	₹ 100.00	
Lot Size	1000	1000	
Scenario	A	B	C
Stock Price	100	70	138

Call Option			
Pay Off	₹ 0	₹ 0	₹ 28
Premia Paid	-₹ 6.5	-₹ 6.5	-₹ 6.5
Net Pay off	-₹ 6.5	-₹ 6.5	₹ 21.50
Lot Size	1000	1000	1000
Total Net Pay off from Call Option	-₹ 6,500.00	-₹ 6,500.00	₹ 21,500.00
Scenario	A	B	C
Closing Price	₹ 100	₹ 70	₹ 138
Put Option			
Pay Off	₹ 0	₹ 20	₹ 0
Premia Paid	- ₹ 10	- ₹ 10	- ₹ 10
Net Pay Off Per Single Share	- ₹ 10	₹ 10	- ₹ 10
Lot Size	1000	1000	1000
Total Net Pay off from Put Option	-₹ 10,000.00	₹ 10,000.00	-₹ 10,000.00
Total Pay off from 1 Call+ 1 Put of 1000 shares each of Coastal Corporation	-₹ 16,500.00	₹ 3,500.00	₹ 11,500.00

21. Illustration

The following information is available for X company's share and Call option

Current Share Price	Rs. 185
Option Exercise Price	Rs. 170
Risk free Interest Rate	7%
Time of the expiry of the option	3 years
Standard deviation	0.18

Calculate the value of the option using Black Scholes formula. *(Old PM)*

Solution:

S_0	**185.00**
K	170.00
R	7%
T	3.00
σ	0.18

Value of Call Option

c	$S_0 * N(d_1) - Ke^{-rT}N(d_2)$		
d_1	$\ln\left(\frac{S_0}{k}\right) + \left(r + \frac{\sigma^2}{2}\right)T$	0.3432	
	---------	---------	1.10071
	$\sigma\sqrt{T}$	0.3118	

$N(d_1)$	0.86448		
d_2	$d_1 - \sigma\sqrt{T}$		0.78894
$N(d_2)$	0.7849		

c	$S_0 * N(d_1) - Ke^{-rT}N(d_2)$
Price of call option	₹ 51.7636

Working Notes:

Ln (185/170) = ln (1.088235) = 0.077 + 0.0076 = 0.08456

Step 1: on Calculator – Type 1.088235
Step 2: Square root 12 times =
Step 3: Subtract 1
Step 4: Divide by 0.000244172
Ans: 0.08456

$\left(r + \frac{\sigma^2}{2}\right)T$ = (7% +(0.18)^2 /2) = 0.2586

$\ln\left(\frac{S_0}{k}\right) + \left(r + \frac{\sigma^2}{2}\right)T$ = 0.08456 + 0.2586 = 0.34316 : RO 0.3432

$\sigma\sqrt{T}$ = 0.18 x (3)$^{1/2}$ = 0.18 x 1.732 = 0.31177 : RO 0.3118

N(d1) = N(1.10071)
N(1.10) = 0.8643
N(1.11) = 0.8665
For 0.01 change on LHS , change in RHS is 0.0022 (0.8665-0.8643)
For 0.00071 change in LHS change in RHS (using interpolation) = 0.0022/0.01 x 0.00071 = 0.0001562
N(d1) = N(1.10071) = 0.8643 + 0.0001562 = 0.8644562: RO 0.86446

d2 = $d_1 - \sigma\sqrt{T}$ = 1.10071 - 0.3118 = 0.78894
N(d2) = N(0.78894)
N(0.78) = 0.7823
N(0.79) = 0.7852
For 0.01 change on LHS , change in RHS is 0.0029 (0.7852-0.7823)
For 0.00894 change in LHS change in RHS (using interpolation) = 0.0029/0.01 x 0.00894 = 0.0026
N(d2) = N(0.78894) = 0.7823 + 0.0026 = 0.7849

C = $S_0 * N(d_1) - Ke^{-rT}N(d_2)$
C = $\mathbf{185 * 0.86446 - 170 * 0.8106 * 0.7849}$
C = 159.924 - 108.160
C = 51.7636

$e^{(-0.21)}$ = 1 / $e^{(0.21)}$

To find out value of $e^{0.21}$
In Calculator

Step 1: Enter 0.21

Step 2: Divide by 4096

Step 3: Add 1

Step 4: x = 12 times

Step 5: Answer = 1.23367

$e^{(-0.21)}$ = 1/1.23367 = 0.8106

22. Illustration

(i) The shares of TIC Ltd. are currently priced at Rs. 415 and call option exercisable in three months' time has an exercise rate of Rs. 400. Risk free interest rate is 5% p.a. and standard deviation (volatility) of share price is 22%. Based on the assumption that TIC Ltd. is not going to declare any dividend over the next three months, is the option worth buying for Rs. 25?

(ii) Calculate value of aforesaid call option based on BlackScholes valuation model if the current price is considered as Rs. 380.

(iii) What would be the worth of put option if current price is considered Rs. 380.

(iv) If TIC Ltd. share price at present is taken as Rs. 408 and a dividend of Rs. 10 is expected to be paid in the two months' time, then, calculate value of the call option.

(ICAI SM, MTP Oct'22)

Solution:

Part 1

	Case 1	**Case 2**
Company	TIC Limited	TIC Limited
S_0	₹ 415.00	₹ 380.00
K	₹ 400.00	₹ 400.00
R_f	5%	5%
Σ	22%	22%
T	3 months	3 months
T	0.25	0.25

c	$S_0 * N(d_1) - Ke^{-rT}N(d_2)$			
d_1	$\dfrac{\ln\left(\frac{S_0}{k}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$	$\dfrac{0.05536}{0.1100}$	0.50331	
$N(d_1)$	0.69266		0.50000	0.69150
			0.51000	0.69500
d_2	$d_1 - \sigma\sqrt{T}$		0.01000	0.00350
d_2	0.39331		0.00331	0.00116
$N(d_2)$	0.65292			
			0.39000	0.65170
c	₹ 29.53		0.4000	0.65540
			0.01000	0.00370
			0.00331	0.00122

The value of the call option as per the Black Scholes Merton Method is ₹29.53. Whereas in the market it is trading at ₹25. So, the option is under-priced in the market and hence worth buying

Part 2

c	$S_0 * N(d_1) - Ke^{-rT}N(d_2)$			
d_1	$\frac{\ln\left(\frac{S_0}{k}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$	$\frac{-0.03274}{0.1100}$	(0.29767)	
d_2	$d_1 - \sigma\sqrt{T}$	(0.40767)		
$N(d_1)$	N (-0.29767)			
	1-N (0.29767)			
N (0.29767)	0.61701		0.29000	0.61410
1-N (0.29767)	0.38299		0.30000	0.61790
N (-0.29767)	0.38299		0.01000	0.00380
			0.00767	0.00291
N (-0.40767)	1-N (0.40767)			
N (0.40767)	0.65824		0.40000	0.65540
1-N (0.40767)	0.34176		0.41000	0.65910
N (-0.40767)	0.34176		0.01000	0.00370
			0.00767	0.00284
c	₹ 10.53			

Part 3

Value of Put option		
$c + Ke^{-rT}$	=	P+ S_0
₹ 405.56	=	P+₹380
P	=	₹405.56-₹380
P	=	₹25.56

Part 4

S_0	₹ 408.00			
Dividend	₹ 10.00			
Time	2 months			
t	16.67%			
PV of dividend	₹ 9.92			
Adjusted S_0	₹ 398.08			
K	₹ 400.00			
R_f	5%			
Σ	22%			
T	0.25			
c	$S_0 * N(d_1) - Ke^{-rT}N(d_2)$			
d_1	$\ln\left(\frac{S_0}{k}\right)+\left(r+\frac{\sigma^2}{2}\right)T$	0.01375		
	--------	--------	0.12496	
	$\sigma\sqrt{T}$	0.11000		
$N(d_1)$	0.54974		0.12000	0.54780

			0.13000	0.55170
d_2	$d_1 - \sigma\sqrt{T}$		0.01000	0.0039 0
d_2	0.01496		0.00496	0.00194
$N(d_2)$	0.50599		0.01000	0.5040 0
			0.02000	0.5080 0
c	₹ 18.96		0.01000	0.0040 0
			0.00496	0.00199

23. Illustration

From the following data for certain stock, find the value of a call option:

Price of stock now	Rs. 80
Exercise price	Rs. 75
Standard deviation of continuously compounded annual return	0.40
Maturity period	6 Months
Annual interest rate	12%

Given,

Number of S.D. from Mean, (z)	Area of the left or right (one tail)
0.25	0.4013
0.30	0.3821
0.55	0.2912
0.60	0.2743

$e^{0.12 \times 0.5}$=1.062; In 1.0667= 0.0646

(ICAI SM, Similar MTP May'20, RTP May'20 New & Old, Old PM)

Solution:

S_0	₹ 80.00	
K	₹ 75.00	1.0667
R_f	12%	
σ	40%	
t	6 months	
t	0.50	
Ln1.0667	0.06460	
$e \quad (0.12 * 0.5)$	1.06200	
$e \quad (-0.12 * 0.5)$	0.942	
c	$S_0 * N(d_1) - Ke^{-rT}N(d_2)$	

d_1	$\frac{\ln\left(\frac{S_0}{k}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$	$\frac{(0.0646) + (12\% + ((40\%)\ \hat{}\ 2)/2) * 0.5}{0.40 * \sqrt{(0.5)}}$		$\frac{0.16460}{0.28}$	0.58195
d_2	$d_1 - \sigma\sqrt{T}$				0.29911
$N(d_1)$	N (0.58195)	0.71960			
$N(d_2)$	N (0.29911)	0.61755			

No of Sd from Mean	Area of left or right	Cumul. Area			
0.25	0.40130	0.59870	0.01920	0.01885	0.04911
0.30	0.38210	0.61790		0.61755	
0.55	0.29120	0.70880			
0.60	0.27430	0.72570	0.01690	0.01080	
				0.03195	
c	$S_0 * N(d_1) - Ke^{-rT}N(d_2)$		₹ 13.96		

24. Illustration - Growth Option

ABC Ltd. is a pharmaceutical company possessing a patent of a drug called 'Aidrex', a medicine for aids patient. Being an approach drug ABC Ltd. holds the right of production of drugs and its marketing. The period of patent is 15 years after which any other pharmaceutical company produce the drug with same formula. It is estimated that company shall require to incur $ 12.5 million for development and market of the drug. As per a survey conducted the expected present value of cashflows from the sale of drug during the period of 15 years shall be $ 16.7 million. Cash flow from the previous similar type of drug have exhibited a variance of 26.8% of the present value of cashflows. The current yield on Treasury Bonds of similar duration (15 years) is 7.8%.

Determine the value of the patent.

Given

ln(1.336) =0.2897

e1.0005 = 0.3677 and e -1.17 = 0.3104

(ICAI SM)

Solution

Black Scholes,

C = N(d1) * St - N(d2) * k* e^-rt

d1 = ln(st/k) + (r+ σ^2/2)t / $\sigma\sqrt{t}$

d2 = d1- $\sigma\sqrt{t}$

C = $N(d_1) * S * e^{-qt} - N(d_2) * k * e^{-rt}$

$$d_1 = \frac{ln\left(\frac{s}{k}\right) + \left(r - q + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

q, dividend yield rate = 6.67%

k, strike = 12.5

r, risk-free rate = 7.8%

S, spot = 16.7

t, time in years = 15

σ^2, = 26.8%

$$d_1 = \frac{ln\left(\frac{s}{k}\right) + \left(r - q + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$d_1 = \frac{ln\left(\frac{16.7}{12.5}\right) + \left(7.8\% - 6.67\% + \frac{26.8\%}{2}\right)15}{\sqrt{26.8\%}\sqrt{15}}$$

$$= \frac{0.2897 + (0.145333)15}{0.517687 * 3.87298}$$

$$= \frac{2.4696995}{2.00499} = 1.23177$$

$N(d_1)$ = 0.8910

d_2 $= d_1 - \sigma\sqrt{t}$

$= 1.23177 - \sqrt{26.8\%}\sqrt{15}$

= 1.2377 - 2.00499 = -0.7732

$N(-d_2)$ = 1-N(d2)

= 0.2196

C $= N(d_1) * S * e^{-qt} - N(d_2) * k * e^{-rt}$

$= 0.8910 * 16.7 * e^{-0.06667*15} - 0.2196 * 12.5 * e^{-0.078*15}$

$= 0.8910 * 16.7 * e^{-1.0005} - 0.2196 * 12.5 * e^{-1.17}$

$= 0.8910 * 16.7 * 0.3677 - 0.2196 * 12.5 * 0.3104$

Value of Option = **$4.6192 million**

25. Illustration - Abandonment option

IPL already in production of Fertilizer is considering a proposal of building a new plant to produce pesticides. Suppose the PV of proposal is ₹100 crore without the abandonment option. However, if market conditions for pesticide turns out to be favourable the PV of proposal shall increase by 30%. On the other hand, market conditions remain sluggish the PV of the proposal shall be reduced by 40%. In case company is not interested in continuation of the project it can be disposed of for ₹ 80 crore. If the risk-free rate of interest is 8% then what will be value of abandonment option.

(ICAI SM, Nov 24 Similar 4M)

Solution

Assume abandonment option exists after a year.

u = 1+30% = 1.3

d = 1- 40% = 0.6

P $= \frac{e^{rt}-d}{u-d}$ $= \frac{1.08-0.6}{1.3-0.6}$ $= \frac{0.48}{0.7}$ = **0.6857**

$1 - P$ = 0.3143

Expected Value of abandonment option (Put Option) at the end of year 1;

= 0.6875* 0 + 0.3143*20

= 6.286

PV of the abandonment option;

$= \frac{6.286}{1.08}$ = **$5.8204 million**

26. Illustration - Abandonment option

Airbus is considering a joint venture with Lear Aircraft to produce a small commercial airplane (capable of carrying 40-50 passengers on short haul flights)

Airbus will have to invest $ 500 million for a 50% share of the venture

Its share of the present value of expected cash flows is 480 million.

Lear Aircraft, which is eager to enter into the deal, offers to buy Airbus's 50% share of the investment anytime over the next five years for $ 400 million, if Airbus decides to get out of the venture.

A simulation of the cash flows on this time share investment yields a variance in the present value of the cash flows from being in the partnership is 0.16.

The project has a life of 30 years and risk free rate is 6% p.a

Should Airbus enter into the joint venture?

Solution

S	= 480 million
σ^2	= 0.16
t	= 30 yrs
r	= 6%
k	= 400 million

Put Option, Black Scholes Formula,

$$= N(-d_2) * k * e^{-rt} - N(d_1) * S * e^{-qt}$$

$$d_1 = \frac{ln\left(\frac{s}{k}\right) + \left(r - q + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$= \frac{ln\left(\frac{480}{400}\right) + \left(6\% - 3.3\% + \frac{16\%}{2}\right)5}{0.8944}$$

$$= \frac{ln(1.2) + 0.5333}{0.8944}$$

$$= \frac{0.18232 + 0.5333}{0.8944} \quad = 0.8001$$

$N(d_1)$ = 0.7881

$d_2 = d_1 - \sigma\sqrt{t}$

= 0.8001- 0.8944= -0.0943

$N(d_2)$ = 1- N(0.0943)

= 0.4624

$$k * e^{-rt} * N(-d_2) - S * e^{-qt} * N(d_1) = 400 * e^{-6\%*5}(1 - 0.4624) - 480 * e^{-0.1666}(1 - 0.7881)$$

$$= 400 * e^{-0.3}(0.5376) - 480 * e^{-0.1666}(0.2119)$$

$$= 400 * 0.7408 * (0.5376) - 480 * 0.8465 * (0.2119)$$

= 159.42 - 86.10

PV of Put Option = **$73.316 million**

Project Value Including Option = PV of Outflow+ PV of Inflow+ PV of Option

= -500 + 480 + 73.316

= **$ 53.316 million**

The joint venture is viable.

We have valued the abandonment option i.e American put using Black Scholes Merton Model. However, BSM is used to value European Options only. Given that the value of an American option is at least equal to or greater than the value of a European Option, the abandonment option in the case is worth atleast $ 53.316 Million

27. Illustration - Timing option

Suppose MIS Ltd. is considering installation of solar electricity generating plant for light the staff quarters. The plant shall cost ₹ 2.50 crore and shall lead to saving in electricity expenses at the current tariff by ₹ 21 lakh per year forever.
However, with change in Government in state, the rate of electricity is subject to change. Accordingly, the saving in electricity can be of ₹ 12 lakh or ₹ 35 lakh per year and forever.
Assuming WACC of MIS Ltd. is 10% and risk-free rate of rate of return is 8%.
Decide whether MIS Ltd. should accept the project or wait and see.

(ICAI SM)

Solution

Investment = ₹ 2.5 Cr

Current Savings = ₹ 21 lakhs per year (WACC @10%)

PV of Current Savings = $\frac{21}{0.1}$ = ₹ 2.1 Cr

Current NPV = -2.5+ 2.1 = -0.4 Cr

Delay Timing: Options

PV of Inflows	$\frac{12}{0.1}$	$\frac{35}{0.1}$
PV of Inflow	120 Lakhs	350 Lakhs
Less: Outflow	250 Lakhs	250 Lakhs
NPV	- 130 Lakhs	100 Lakhs

$u = \frac{350}{250} = 1.4$

$d = \frac{120}{250} = 0.48$

$P = \frac{R_f - d}{u-d} = \frac{1.08-0.48}{1.4-0.48} = \frac{0.6}{0.92}$ = **0.652**

$1 - P$ = 0.348

Pay Off (after 1 year) = 100*0.652 + (-130*0.348)
= 65.2 - 45.24 = **19.96 Lakhs**

Current Value of Option = $\frac{19.96}{1.08}$ = **₹ 18.48 Lakhs**

Option has +ve Value hence the company should wait and decide

28. Illustration - MCQs

Describe each of the following situations in the language of options:

a. Drilling rights to undeveloped heavy crude oil in Northern Alberta. Development and production of the oil is a negative-NPV endeavour. (The break-even oil price is C$32 per barrel, versus a spot price of C$20.) However, the decision to develop can be put off for up to five years. Development costs are expected to increase by 5% per year.
Answer: The case depicts an American call option regarding drilling rights for undeveloped heavy crude oil. With an initial exercise price of $32 per barrel and the ability to delay development for up to five years due to increasing costs, it falls under the category of a timing option.

b. A restaurant is producing net cash flows, after all out-of-pocket expenses, of $700,000 per year. There is no upward or downward trend in the cash flows, but they fluctuate as a random walk, with an annual standard deviation of 15%. The real estate occupied by the restaurant is owned, not leased, and could be sold for $5 million. Ignore taxes.
Answer: This represents an American put option, related to an abandonment option for selling a restaurant's real estate, considering annual cash flows of $700,000, an exercise price of $5 million, and an annual standard deviation of 15% in cash flow variability.

c. A variation on part (b): Assume the restaurant faces known fixed costs of $300,000 per year, incurred as long as the restaurant is operating. Thus,
Net cash flow = revenue less variable costs — fixed costs
$700,000 = 1,000,000 — 300,000

The annual standard deviation of the forecast error of revenue less variable costs is 10.5%. The interest rate is 10%. Ignore taxes.

Answer: The fluctuating annual cash flows of $1,000,000 by 10.5% in a scenario with fixed costs of three Lakhs annually indicate an American put option, indicating an abandonment option with varying cash flows and an exercise price of $8 million inclusive of property sale and annual savings.

d. A paper mill can be shut down in periods of low demand and restarted if demand improves sufficiently. The costs of closing and reopening the mill are fixed.

Answer: The paper mill's ability to temporarily shut down and restart operations with fixed closure and reopening costs reflects a compound option, a combination of an American put for abandonment and an American call for temporary restart, offering operational flexibility.

e. A real estate developer uses a parcel of urban land as a parking lot, although construction of either a hotel or an apartment building on the land would be a positive-NPV investment.

Answer: In a situation where a real estate developer can choose between building a hotel or an apartment building, both profitable options, it represents an in-the-money American call option, allowing for the deferment of the decision (timing option) to select the most lucrative option during the waiting period.

f. Air France negotiates a purchase option for 10 Boeing 787s. Air France must confirm the order by 2030. Otherwise, Boeing will be free to sell the aircraft to other airlines.

Answer: Air France negotiating a purchase option for 10 Boeing 787 planes by 2030 represents a timing option, specifically an American call option. This grants Air France the flexibility to confirm the purchase at any time until 2030, resembling a timing decision rather than a growth or abandonment choice.

29. Illustration

You own a one-year call option on one acre of Los Angeles real estate. The exercise price is $2 million, and the current, appraised market value of the land is $1.7 million. The land is currently used as a parking lot, generating just enough money to cover real estate taxes. The annual standard deviation is 15% and the interest rate 12%. How much is your call worth?

Solution

$S_0 = 1.7$ | $X = 2$ | S. D = 0.15 | $t = 1.0$ | $r_f = 0.12$

$$C = N(d_1) * S0 - N(d_2) * k * e^{-rt}$$

$$d_1 = \frac{ln\left(\frac{s}{k}\right) + \left(r - q + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

$$d1 = \frac{\text{Ln } (1.7/2.0) + (0.12 + (0.15)^2 / 2) \times 1}{(0.15) \times 1}$$

(Hint: Log values for numbers less than 1 are not given in the tables. So one can compute Log 3.4 - Log 4 as a substitute for Log 0.85 to get the same answer.)

$$d1 = \frac{-0.16252 + (0.13125)}{0.15}$$

d1 = -0.20846;

d2 = d1 – 0.15 x 1

d2 = -0.20846 – 0.15

N(d1) = 0.41744
N(d2) = 0.36000

Price of call = 1.7 x 0.41744 – 2.0 x $e^{-0.12}$ x 0.3600
C = ₹ 71,057

30. Illustration

Consumers appear to require returns of 25 percent or more before they are prepared to make energy-efficient investments, even though a more reasonable estimate of the might be around 15 percent. Suppose you have the opportunity to invest $1,000 in new space-heating equipment that would generate fuel savings of $250 a year forever given current fuel prices. What is the PV of this investment if cost of capital is 15 percent? What is the NPV?
Now recognize that fuel prices are uncertain and that the savings could well turn out to be $50 a year or $450 a year. If the risk-free interest rate is 10 percent, would you invest in the new equipment now or wait and see how fuel prices change? Explain.

Solution

Investment = $1000
Current Savings = $ 250 per year
Cost of capital = 15%
Rf = 10%
PV of Current Savings = $\frac{250}{0.15}$ = $ 1,667
Current NPV = 1,667-1,000 = $ 667
Savings on upside = 450/15% = 3000 | Payoff on upside = 3000-1000 = 2000
Savings on downside = 50/15% = 333 | Payoff on downside = 333-1000 = -667

$u = \frac{3000}{1000} = 3.0$

$d = \frac{333}{1000} = 0.33$

$P = \frac{k-d}{u-d} = \frac{1.1-0.33}{3.0-0.33} = \frac{0.9}{1.6}$ = **0.2884**

$1 - P$ = 0.7116

Pay Off $= \frac{2{,}000*0.2884 + (-667*0.7116)}{1.1}$
= **$ 92.87**

It is **advisable to wait** as option to wait has +ve value of $92.87 .

31. Illustration

A Rice Trader has planned to sell 22000 kg of Rice after 3 months from now. The spot price of the Rice is Rs. 60 per kg and 3 months future on the same is trading at Rs. 59 per kg. Size of the contract is 1000 kg. The price is expected to fall as low as Rs. 56 per kg, 3 months hence.

What the trader can do to mitigate its risk of reduced profit? If he decides to make use of future market, what would be the effective realized price for its sale when after 3 months, spot price is Rs. 57 per kg and future contract price for 3 months is Rs. 58 per kg?

(Similar May'23 QP, May'19 QP 8 marks, RTP Nov'20, MTP May'20 Old, RTP Nov'20 Old)

Solution:

Commodity	Rice
Action	Plans to sell
Time duration	3 months
S_0	₹ 60.00
Futures price	₹ 59.00
Expected price fall to	₹ 56.00
3 months later	
Spot	₹ 57.00
Futures price	₹ 58.00
Effective realized price after 3 months	
Qty	22000 KG
Lot Size	1000 KG

In order to mitigate the risk, the trader should sell futures today for 22 MT i.e., 22 contracts.

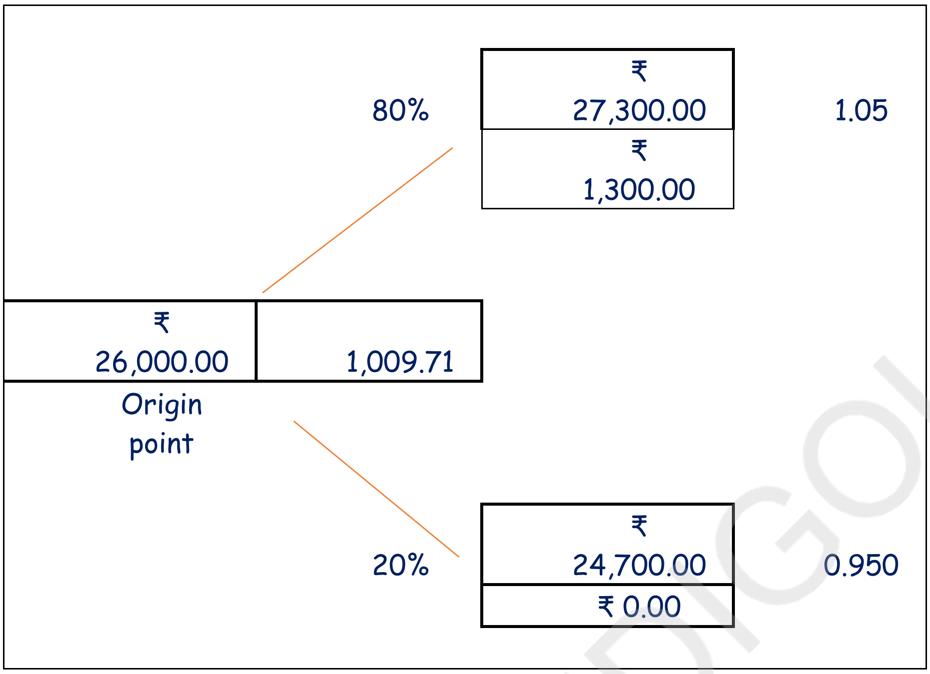
Price of Futures contract on Day 0 per KG	**₹ 59.00**
No of contracts to be sold	22.00
Lot size of each contract in KG	1,000.00
Total Notional value sold	₹ 12,98,000.00
After 3 months	
Futures that are sold are bought back	22.00
@ price of	₹ 58.00
Lot size	1,000
Notional value of Futures purchased	**₹ 12,76,000.00**
Gain from Futures transaction	₹ 22,000.00
Spot rate in market per KG of Rice after 3 months	₹ 57.00
Qty of rice sold in spot	22000
Value realised	₹ 12,54,000.00
Add: Gain on futures	₹ 22,000.00
Total Amount realised	₹ 12,76,000.00
Total Qty sold	22,000.00
Amount Realized per KG	₹ 58.00

32. Illustration

A call option on gold with exercise price of Rs. 26,000 for 10 grams and 3 months to expire is being traded at a premium of Rs. 1,010 per 10 grams. It is expected that 3 months from now, the spot price might change to Rs. 27,300 or Rs. 24,700. At present, the option is At the Money. Rate of interest at simple compounding is 12 percent per annum.
Is the current premium for the option is justified? Evaluate the option and comment.

[Nov'17 QP (Old), Old PM]

Solution:



Commodity	Gold		
Price unit	10 Grams		
Spot	₹ 26,000.00		
U price	₹ 27,300.00	u	1.05
D price	₹ 24,700.00	d	0.95
K / X	₹ 26,000.00		
Premia	₹ 1,010.00		
Expiry	3 months		
Interest rate	12%		
	Is the premia justified?		

P	=	$\frac{R-d}{u-d}$		
R	=	1.03		
P	=	$\frac{1.03-0.95}{1.05-0.95}$	$\frac{0.08}{0.10}$	0.8
Option price as per binomial tree		PV of Expected FV of payoffs from Exercise of option		
Expected FV	=	P x Pay off on up move +(1-P) x Pay off on down move		
Expected FV	=	80% x 1300 +20% x 0		
Expected FV	=	₹ 1,040.00		

PV of expected FV	=	₹ 1,009.71		
The call option for purchase of 10 grams of gold @ ₹26000 is appropriately priced @ ₹1010				

Delta	$\frac{C_U - C_d}{S_U - S_d}$	$\frac{1300 - 0}{27300 - 24700}$	$\frac{1300}{2600}$ =0.5
Delta			0.50
Buy 0.5 x 10 grams of gold + Sell a call	=	R_f rate	
Buy 5 grams of gold + Sell a call	=	R_f rate	
$13000 - c$	=	R_f rate	
Up move value of portfolio		5 grams of gold - call option	
Up move value of portfolio	=	27300 * 0.5 -1300	12,350.00
Down move value of portfolio	=	24700 * 0.5-0	12,350.00
$13000 \; - c$	11,990.29		
C	1,009.71		

33. Illustration

TMC holding limited has a portfolio of shares of diversified companies valued at Rs. 400 crore enters into a swap arrangement with None Bank on the terms that it will get 1.15% quarterly on notional principal of Rs. 400 crores in exchange of return on portfolio which is exactly tracking the Sensex which is presently 21,600.

You are required to determine the net payment to be received or paid if Sensex turns out to be 21,860; 21,780; 22,080; 21,960; at the end of each quarter.

(RTP May'18, MTP Oct'21 New and Old, RTP May'18 Old)

Solution:

Portfolio value in ₹ Crores	400.00
Sensex current value	21,600.00
Quarterly fixed return requirement	1.15%

Q	Sensex	Return on Sensex %	Fixed return for TMC	Fixed return in ₹ Cr	Variable Return on Portfolio in ₹ Cr	Amount to be paid/ received by TMC Holding ltd in ₹ Cr
1.00	21,860.00	1.2037%	1.15%	4.6	4.8148	(0.2148)
2.00	21,780.00	-0.3660%	1.15%	4.6000	(1.4639)	6.0639
3.00	22,080.00	1.3774%	1.15%	4.6000	5.5096	(0.9096)
4.00	21,960.00	-0.5435%	1.15%	4.6000	-2.1739	6.7739
				18.4000	6.6867	11.7133

34. Illustration

You had purchased a 3-month call option on the Equity shares of Satya Ltd for a premium of ₹ 30 each, the current market price of the share is ₹ 560 and the exercise price is ₹ 590. You expect the price range between ₹ 540 to ₹ 640.
The expected share price of Satya Ltd and related probability is given below:

Expected price (₹)	540	560	580	600	620	640
Probability	0.10	0.15	0.05	0.35	0.20	0.15

Compute the following:

ii. Expected share price at the end of 3 months,
iii. Value of call option at the end of 3 months, if the exercise price prevails,
iv. In case the option is held to its maturity, what will be the expected value of the call option?
v. Find out the price of the shares quoted at the stock exchange to get the value of the call option as computed in (iii) above.

(May'22 QP 8 marks)

Solution:

Expected Share price after 3 months

Expected Price (A)	Probability (B)	(A) * (B)
540	0.10	54
560	0.15	84
580	0.05	29
600	0.35	210
620	0.20	124
640	0.15	96
	Expected Share Price after 3 months	597

ii) Value of call option at the end of 3 months, if the exercise price prevails

Exercise Price = ₹ 590

If on maturity, the spot is equal to the exercise price, then value of the option,

= SPOT on Maturity – exercise Price

= 590 – 590= ₹ **0**

iii) The expected value of the call option case the option is held to its maturity

Expected Share Price	Price	Value of Option	Probability	Probability Weighted Value
540	590	-	-	-
560	590	-	-	-
580	590	-	-	-
600	590	10	0.35	3.5
620	590	30	0.20	6
640	590	50	0.15	7.5
				17

Expected Value of the Call Option = ₹ **17**

iv) If the value of the option on Maturity date = ₹ 17

Exercise Price = ₹ 590

Maturity/ Spot price of shares on the date of Maturity

= ₹ 17+ ₹ 590 = **₹ 607**

35. Illustration

Following is the information available pertaining to shares of Omni Ltd.:

Current Market Price (₹)	**₹ 420.00**
Strike Price (₹)	**₹ 450.00**
Maximum Price (₹) expected in next 3 months' time	**₹ 525.00**
Minimum Price (₹) expected in next 3 months' time	**₹ 378.00**
Continuously Compounded Rate of Interest (p.a.) (%)	8.00%
e^{rt}	1.0202

From the above:

- **Calculate the 3 months call option by using Binomial Method and Risk Neutral Method.**
- **Are the calculated values under both the models are same?**
- **State also clearly the basis of Valuation of options under these models**

(Nov 23'QP 8 marks)

Solution:

Given:

S0 = 420

K = 450

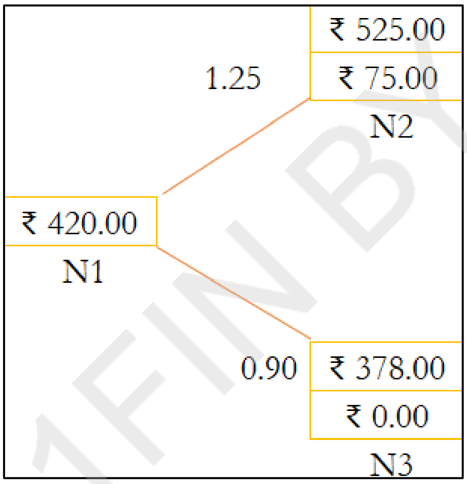
S0u =525

S0d =378

U = 525 / 420 = 1.25

D = 378 / 420 = 0.90

e^{rt} = 1.0202



Binomial Method

Method 1: Delta Hedging i.e using Delta Shares

Delta = Change in option price / Change in Share price

Delta = 75-0 / 525-378

Delta = 75/147

Delta = 0.5102

Create a portfolio of Long Delta Shares and Short 1 call option – This portfolio should generate risk free returns

Present value of this portfolio = 0.5102 x S0 – C0
= 0.5102 x 420 – C0
= 214.284 – C0 ---------------(1)

	When stock Moves up	When stock Moves down
FV of this portfolio	0.5102 X S0U – Cu	0.5102 X S0d – Cd
	= 0.5102 x 525 – 75	0.5102 x 378 – 0
	= 192.855	192.855

The future value of this portfolio is same despite volatility i.e both on upside or downside. Such a portfolio has not risk despite volatility so it generates Rf return
FV of portfolio = 192.855
Assuming Rf of 8% over 3 month of continuous compounding PV of this portfolio
= 192.855/1.0202
= 189.037 --------------(2)

Equating (1) & (2)
214.284 – C0 = 189.037
=> C0 = 214.284 – 189.037
C0 = 25.247

Value of option = 25.247

Method 2 under binomial

Replicating portfolio – a portfolio that generates same returns as other portfolio will have same value as the other portfolio.

Assume a portfolio of Delta shares + Loan which generates same return as call option, then the value of the option is equal to the value of the portfolio.

PV of portfolio = Delta Shares + Loan = 0.5102 Shares + Loan
= 0.5102 X 420 + Loan
= 214.284 + Loan ----------- (A)
Future value of portfolio in Sou = 525 x 0.5102 + Loan
= 267.855 + Loan ---- (B)
Future value of portfolio in Sod = 378 x 0.5102 + Loan
= 192.855 + Loan ---- (C)

If Future value of Portfolio = Future pay off of call then
267.855 + Loan = 75 &
192.855 + Loan = 0

=> Loan = - 192.855 in both the above equations
FV of loan = -192.855 @ Rf of 8% continuous compounding means
PV of loan = -192.855 / 1.0202 = 189.04
Substituting PV of loan in Equation …(A)
214.284 – 189.04 = PV Value of Call

PV Value of Call = Rs.25.244
Risk Neutral method
Method 1

$P = e^{rt} - d / u\text{-}d$
$P = 1.0202 - 0.90 / 1.25\text{-}0.90$
$P = 0.1202 / 0.35$
$P = 34.34\%$
$1\text{-}P = 65.66\%$

Value of call = PV of expected value of future payoffs from call
$C0 = Cu \times P + Cd \times (1\text{-}P) / e^{rt}$
$C0 = 75 \times 34.34\% + 0 \times 65.66\% / 1.0202$

C0 = 25.247

Value of call is Rs.25.25

Method 2:
Expected value of stock based on options volatilities = Expected Future value of stock
$525 \times P + 378\ (1\text{-}P) = 420 \times e^{rt}$
$525P + 378 - 378P = 428.484$
$147P = 50.484$
$P = 50.484 / 147$
$P = 34.34\%$

Value of call = PV of expected value of future payoffs from call
$C0 = Cu \times P + Cd \times (1\text{-}P) / e^{rt}$
$C0 = 75 \times 34.34\% + 0 \times 65.66\% / 1.0202$

C0 = 25.247

Value of call is Rs.25.25

Part 2
Yes the value of the option computed under the binomial method and the Risk Netural Method have same value

Part 3
Under binomial method we use delta shares to hedge and each portfolio has same value – generating Risk free returns. Under Risk Neutral method, risk neutral probabilities are used and based on the principle of No arbitrage the value of the options is arrived at.

36. Illustration – Nov'24 Case Based MCQ

Based on the following information, choose the correct answer from the following questions:

Situation	Action	Exercise Price	Premium	Spot Price	Remarks
I	Exercised	140	20	160	If K < S & option exercised it is Call
II	Exercised	200	15	175	If K > S & option exercised it is Put
III	Lapsed	300	25	400	If K < S & option Not exercised it is Put

(Case Based MCQ Nov'24 QP 2 marks each)

From the information given above, choose the correct answer to the Question no. 1-3:

1. In situation I, the investor's position and amount of profit or loss is:
   A. Put option and ₹ 20
   B. Call option and ₹ 0
   C. Put option and ₹ 0
   D. Call option and ₹ 20

Call Payoff 20 Less Premia 20 – Net pay off Nil – Option B

2. In Situation III, the investor's position and the amount of profit / loss is:
   A. Put option, ₹ (25)
   B. Call option, ₹ 75
   C. Short position, ₹ 100
   D. Long position, ₹ (100)

Put Premia 25 – Net pay off (25) – Option A

3. In situation II, the investor's position and the amount of profit / loss is:
   A. Put option and ₹ 10
   B. Call option and ₹ 10
   C. Put option and ₹ 25
   D. Call option and ₹ 25

Put Payoff 25 Less Premia 15 – Net pay off 10 – Option A

37. Illustration

A firm is considering a proposal to set up a cement manufacturing plant with an initial investment of ₹150 crore. The firm has the option to abandon the project after one year by selling it to a competitor for ₹100 crore if the market conditions are unfavorable and the demand is low. The project's value will decline by 60%. However, if the market conditions turn out to be favorable and the demand for cement is high, the value of the project at the end of year 1 will increase by 50%.
Given that the risk-free rate of interest is 8%, what will be the value of the abandonment option and the value of the project with abandonment option?

(Nov'24 QP 4 marks)

Solution:

Investment Value of the Project in ₹ Lacs = 150
Assume Same is the Value of the Project as of Now as no other details are provided

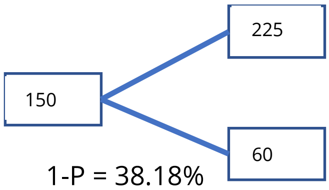
Time period = 1 Year

Sale price on abandonment in ₹ Lacs = 100

Project value if conditions are favourable in ₹ Lacs = 150 x (1+50%) = 225

Project Value if conditions are not favourable in ₹ Lacs = 150 (1-60%) = 60

P = 61.82%



1-P = 38.18%

If PV is>100, Project will not be abandoned

Put Option - So Downside value is relevant.
Payoff = Strike – PV, i.e 100-60 = 40

Using Risk neutral Valuation Methodology to Value the real option

u = 1+50% = 1.5

d = 1- 60% = 0.4

$P = \frac{e^{rt}-d}{u-d} = \frac{1.08-0.4}{1.5-0.4} = \frac{0.68}{1.1}$ = 0.6182

$1-P$ = 0.3818

Expected Value of abandonment option (Put Option) at the end of year 1 = 0.6182* 0 + 0.3818*40 = ₹ 15.272 Lacs

PV of the abandonment option = $\frac{15.272}{1.08}$ = ₹14.14 Lacs

Value of project along with abandonment Option = ₹150 Lacs + ₹ 14.14 Lacs = ₹ 164.14 Lacs

It can also be computed as PV of 0.6182* 225 + 0.3818*60 + ₹14.14 Lacs

= PV of ₹ 162.003 Lacs + ₹14.14 Lacs

= ₹162.003 lacs / 1.08 + ₹14.14 Lacs = ₹150 Lacs + ₹14.14 Lacs

= ₹ 164.14 Lacs

NPV = ₹164.14 Lacs - ₹150 Lacs = ₹ 14.14 Lacs

38. Illustration

Ms. Priya initiated the following option strategy on Omega Industries Limited's equity shares: Transactions executed:

1. Bought one European Call Option with the following terms:
   - Premium paid : ₹ 42 per share
   - Strike Price : ₹ 620
   - Maturity : 3 months
2. Bought one European Put Option with the following terms:
   - Premium paid : ₹ 8 per share
   - Strike Price : ₹ 480
   - Maturity : 3 months

Additional Information:

- Current Market Price (CMP) of Omega Industries: ₹ 550 per share
- Lot size: 150 shares per contract
- Ms. Priya holds the positions until expiration

Required:

(i) Calculate the net profit/loss in the following scenarios at expiration

- Share price remains unchanged at ₹ 550

- Share price declines to ₹ 380
- Share price appreciates to ₹ 680

(ii) Determine the upper and lower breakeven points for this strategy.

(Jan'26 Q 1 (c) 4 Marks)

(Note: Round off all intermediate and final calculations to four decimal places.)

Solution:

	BC			BP		
Premium	42	42	42	8	8	8
Strike	620	620	620	480	480	480
Maturity	3 Months	3 Months	3 Months	3 Months	3 Months	3 Months
CMP	550	550	550	550	550	550
Lot size	150	150	150	150	150	150
Holding period	Till Maturity	Till Maturity	Till Maturity	Till Maturity	Till Maturity	Till Maturity
Market price	550	380	680	550	380	680
BC	620	620	620	-	-	-
BP	-	-	-	480	480	480
Option Exercised	**No**	**No**	**Yes**	**No**	**Yes**	**No**
Payoff	**-**	**-**	**60**	**-**	**100**	**-**
Premia Paid	-42	-42	-42	-8	-8	-8
Net Gain	**-42**	**-42**	**18**	**-8**	**92**	**-8**
Lot Size	150	150	150	150	150	150
Overall Gain	**- 6300 (42 x 150)**	**-6300**	**2700**	**-1200**	**13800**	**-1200**

Combined Gain Loss

MP	*550*	*380*	*680*
Net gain on BC	*-6300*	*-6300*	*2700*
Net Gain on BP	*-1200*	*13800*	*-1200*
Net Gain / Loss	***-7500***	***7500***	***1500***

Breakeven price:
CMP = 550
Total Premia paid for BC & BP (42+8) = 50
Strike of Call 620
Strike of Put 480
Break Even Price (BEP) Upper band = 620 + 50 = 670
Break Even Price (BEP) Lower band = 480 - 50 = 430

39. Illustration

A speculator purchases BFL Ltd. May Futures (lot of 125 shares) at 7750 and chooses to Write BFL 7790 May call option with a premium of ₹ 30 (lot of 125 shares). As on May 18, spot prices rise and so the futures price and call premium. Futures price rise to 7780. Call premium also rises to ₹ 36. Brokerage for the transaction is 0.02% for the transaction value of futures and strike price net of call premium for options.

You are required to calculate:

- **(i)** Profit/Loss on Futures net of transaction costs.
- **(ii)** Profit/Loss on options net of transaction costs.
- **(iii)** Overall profit from both the positions net of costs.
- **(iv)** Total Brokerage cost. (Jan 26, Q 3 (a) 6 Marks)

Solution:

(1)

Buy futures	**Original contract (may end)**	**7750**
CMP of Futures May 18	Increase	7780
Gain		**30**
Lot size		125
Gross Gain	**(125 x 30)**	**3750**
Brokerage on Buying	7750 x 0.02%	1.55
Brokerage on Buying	7780 x 0.02%	1.556
Total Brokerage per lot		3.106
Total brokerage	**125 x 3.106**	**388.25**
Net Gain	**3750 – 388.25**	**3361.75**

(2)

Sell Call @ strike of 7790 at CMP – Premia	**For May end**	**30**
Buy call @ revised futures price of 7780	May 18th	36
Net Loss per call		-6
Lot size		125
Gross Loss	- **6 x 125**	**-750**
Brokerage on Call		
Sell Call	(7790 – 30) x 0.02%	1.5520
Buy call	(7790 – 36) x 0.02%	1.5508
Total Brokerage		3.1028
Lot size		125
Total Brokerage	**125 x 3.1028**	**387.85**
Net Loss on Options	-750-387.85	-1137.85

(3) Combined Profit on Futures + Options	3361.75-1137.85	2223.90
(4) Total Brokerage	**388.25+387.85**	**776.10**

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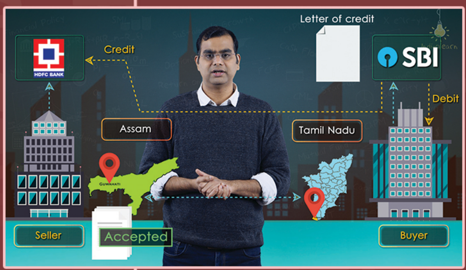

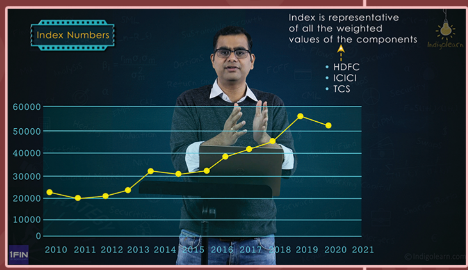

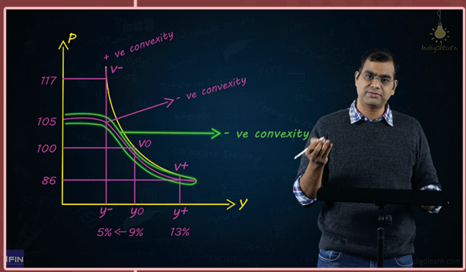

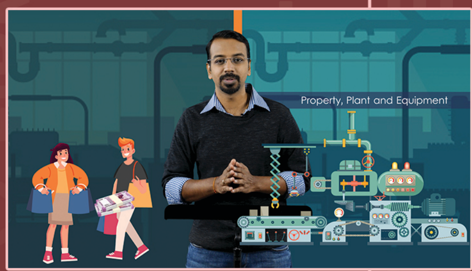

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