

**Paper- 4: FUNDAMENTALS OF BUSINESS MATHEMATICS
AND STATISTICS**

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Full Marks: 100

Time Allowed: 3 Hours

Section – A
(Fundamentals of Business Mathematics)

PART A

1. Answer All objective questions.

(a) Answer Multiple Choice Question

[9×2= 18]

i. 20 litres of a mixture contain milk water in 5 : 3 ratio. If 4 litres of this mixture are replaced by 4 litres of milk, the ratio of milk water in the new mixture will be:

- (a) 2 : 1
- (b) 6 : 3
- (c) 7: 3
- (d) 8 : 3

ii. If $x = \log_{2a} a$, $y = \log_{3a} 2a$, $z = \log_{4a} 3a$, then $xyz + 1$ is equal to

- (a) $2xy$
- (b) $2yz$
- (c) $2zx$
- (d) None of these.

iii. Find the present worth of an ordinary annuity of ₹1,200 p.a. for 10 years at 12% p.a. compounded annually.

[Use $(1.12)^{-10} = 0.3221$]

- (a) ₹6,770
- (b) ₹6,779
- (c) ₹6,805
- (d) None

iv. The sum of money that amounts to ₹1,110 in 10 years at the rate of 5% simple interest will be –

- (a) ₹700
- (b) ₹740
- (c) ₹760
- (d) ₹780

v. The number of ways in which 15 mangoes can be equally divided among 3 students is

- (a) $\frac{15!}{(5!)^4}$
- (b) $\frac{15!}{(5!)^3}$
- (c) $\frac{15!}{(5!)^3}$
- (d) None of these

vi. If p^{th} , q^{th} and r^{th} terms of a G. P. be a , b , c respectively, then $a^{(q-r)} b^{(r-p)} c^{(p-q)} = ?$

- (a) 0
- (b) 1
- (c) -1
- (d) None.

vii. Which term of the A.P. 5, 8, 11, 14, ... is 320?

- (a) 104^{th}
- (b) 105^{th}
- (c) 106^{th}
- (d) 64^{th}

viii. $\log_3 \left(\frac{1}{243} \right) = ?$

- (a) -5
- (b) -6
- (c) -9
- (d) None of these

ix. The number of subsets of a set containing n elements is

- (a) 2^n
- (b) 2^{-n}
- (c) n
- (d) None of these

(b) Answer the following Question True or False

[6×1=6]

i. If $A : B = B : C = C : D = 5 : 6$ then $A : B : C : D = 125 : 150 : 180 : 216$

ii. The S.A. I at $x\%$ for x years will be Rs. X on a sum of x

iii. If the set A has 4 elements, B has 3 elements then the number of elements in $A \times B$ is 12 elements

iv. If $x = 2 + \sqrt{5}$ then $x^3 + 3x^2 - 29x$ is 7

v. The logarithm of 324 to base $\frac{1}{3\sqrt{2}}$ is -4

vi. $X^2 - 4x - 1 = 0$ is the quadratic equation whose roots are $2 + \sqrt{5}$ and $2 - \sqrt{5}$

Answer: 1(a)

- i. c
- ii. b
- iii. b
- iv. b
- v. b
- vi. b
- vii. c
- viii. a

ix. d

Answer: 1(b)

- i. True
- ii. False
- iii. True
- iv. True
- v. True
- vi. True

PART B

4 questions to be answered out of questions [4×4=16]

- 2. In 165 litres of a quantity of milk mixed up with water, the ratio of milk and water is 9: 2. How much water must be added to it so as to make the ratio of milk and water 5 : 3? [4]**

Answer: 2

	Milk	Water	Mixture	
First case	9 litres	2 litres	11 litres	$\frac{9}{x} = \frac{11}{165}$ or, $x = \frac{9 \times 165}{11} = 135$
	x litres	y litres	165 litres	$\frac{9}{y} = \frac{11}{165}$ or, $y = \frac{2 \times 165}{11} = 30$
Second case	5 litres	3 litres	8 litres	$\frac{5}{135} = \frac{3}{z}$
	135 litres	z litres		or $z = \frac{3 \times 135}{5} = 81$

(amount of milk remaining same)

∴ quantity of water to be added = 81 – 30 = 51 litres.

- 3. The difference between the simple and compound interest on a certain sum for 3 years at 5% p. a. is Rs.228.75 P. Find the compound interest on the sum for 2 years at 5% p.**

Answer: 3

Let P = reqd. sum. Here, $i = 5/100 = 0.05$, $n = 3$;

∴ C. I. of P for 3 years = $P(1 + 0.05)^3 - P = P \times 0.157625$,

and S. I. of P for 3 years = $P \times 3 \times 0.05 = P \times 0.15$;

∴ C.I. - S.I. = $P \times 0.007625$,

or, Rs. 228.75 = $P \times 0.007625$; ∴ $P = \text{Rs. } \frac{228.75}{0.007625} = \text{Rs. } 30,000$;

∴ Reqd. C.I. = $\text{Rs. } 30,000 \{(1+0.05)^2 - 1\} = \text{Rs. } 30,000 \times 0.1025 = \text{Rs. } 3,075$.

4. Solve $1 + 6 + 11 + 16 + \dots + x = 148$

Answer: 4

Let n^{th} term be x .

Take $a_1 = 1, d = 6 - 1 = 11 - 6 = 5, S_n = 148, a_n = x$

$$S_n = n/2 [2a_1 + (n - 1) d]$$

$$\Rightarrow 148 = n/2 [2 + (n - 1) \times 5]$$

$$\Rightarrow 148 = n/2 [2 + 5n - 5]$$

$$\Rightarrow 296 = n (5n - 3)$$

$$\Rightarrow 5n^2 - 3n - 296 = 0$$

$$\Rightarrow n = \frac{3 \pm \sqrt{(-3)^2 - 4(5)(-296)}}{2(5)}$$

$$= \frac{3 \pm \sqrt{9 + 5920}}{10}$$

$$= \frac{3 \pm \sqrt{5929}}{10}$$

$$= \frac{3 \pm 77}{10}$$

$$= \frac{80}{10}, \frac{-74}{10}$$

$$= 8 (\because n \geq 0)$$

Now, $a_n = a_1 + (n - 1) d$

$$\Rightarrow x = 1 + (8 - 1) (5)$$

$$\Rightarrow x = 1 + 35$$

$$\therefore x = 36.$$

5. Prove that $\frac{\log \sqrt{27} + \log 8 + \log \sqrt{1000}}{\log 120} = \frac{3}{2}$.

Answer: 5

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$$= \frac{\log (27)^{\frac{1}{2}} + \log 2^3 + \log (1000)^{\frac{1}{2}}}{\log (3 \times 2^2 \times 10)}$$

$$= \frac{\frac{1}{2} \log 27 + 3 \log 2 + \frac{1}{2} \log 1000}{\log 3 + \log 2^2 + \log 10}$$

$$\begin{aligned}
 &= \frac{\frac{1}{2} \log 3^3 + 3 \log 2 + \frac{1}{2} \log 10^3}{\log 3 \times 2 \log 2 + \log 10} \\
 &= \frac{\frac{3}{2} \log 3 + \frac{3}{2} \times 2 \log 2 + \frac{3}{2} \log 10}{\log 3 + 2 \log 2 + \log 10} \\
 &= \frac{\frac{3}{2} (\log 3 + 2 \log 2 + \log 10)}{\log 3 + 2 \log 2 + \log 10} = \frac{3}{2}
 \end{aligned}$$

6. If ${}^{11}P_r = {}^{12}P_{r-1}$ find the value of r.

Answer: 6

$$\begin{aligned}
 \Rightarrow \quad & \frac{11!}{(11-r)!} = \frac{12!}{(12-(r-1))!} \\
 \Rightarrow \quad & \frac{11!}{(11-r)!} = \frac{12!}{(12-r+1)!} \\
 \Rightarrow \quad & \frac{11!}{(11-r)!} = \frac{12 \times 11!}{(13-r)!} \\
 \Rightarrow \quad & \frac{11!}{\cancel{(11-r)!}} \times \frac{1}{11!} = \frac{12}{(13-r) \cdot (12-r) \cdot \cancel{(11-r)!}} \\
 \Rightarrow \quad & 1 = \frac{12}{(13-r)(12-r)} \\
 \Rightarrow \quad & (13-r)(12-r) = 12 \\
 \Rightarrow \quad & 156 - 13r - 12r + r^2 = 12 \\
 \Rightarrow \quad & r^2 - 25r + 144 = 0 \\
 \Rightarrow \quad & r^2 - 9r - 16r + 144 = 0 \\
 \Rightarrow \quad & r(r-9) - 16(r-9) = 0 \\
 \Rightarrow \quad & (r-9)(r-16) = 0 \\
 \Rightarrow \quad & r = 9, 16 \\
 \Rightarrow \quad & \therefore r = 9 \quad (r \leq 11)
 \end{aligned}$$

7. If one root of $px^2 + qx + r = 0$ be square of the other, prove that.

$$q^3 + p^2r + pr^2 = 3pqr.$$

Answer: 7

Let one root be α . Then the other root is α^2 .

$\therefore$ Sum of the roots = $\alpha + \alpha^2 = -q/p$;

And product of the roots = α . $\alpha^2 = r/p$ or $\alpha = \left(\frac{r}{p}\right)^{\frac{1}{2}}$

Substituting these value of α in (1) we get

$$\left(\frac{r}{p}\right)^{\frac{1}{3}} + \left(\frac{r}{p}\right)^{\frac{2}{3}} = \frac{-q}{p} \dots\dots\dots (2)$$

Cubing both sides, $\frac{r}{p} + \frac{r^2}{p^2} + 3\left(\frac{r}{p}\right)^{\frac{1}{3}} \left(\frac{r}{p}\right)^{\frac{2}{3}} \left\{ \left(\frac{r}{p}\right)^{\frac{1}{3}} + \left(\frac{r}{p}\right)^{\frac{2}{3}} \right\} = -\frac{q^3}{p^3}$

Or, $\frac{r}{p} + \frac{r^2}{p^2} + 3 \frac{r}{p} \left\{ \frac{-q}{p} \right\} = \frac{-q^3}{p^3}$ [using (2)]

Or, $p^2r + pr^2 - 3pqr = -q^3$

$\therefore q^3 + p^2r + pr^2 = 3pqr$.

Section – B

PART A

8. Answer All objective questions.

(a) Answer Multiple Choice Question

[12×2= 24]

(i) Frequency density corresponding to a class interval is the ratio of

- (a) Class frequency to the total frequency**
- (b) Class frequency to the class length**
- (c) Class length to the class frequency**
- (d) Class frequency to the total frequency.**

(ii) Mean deviation from the mean for the observations 0, -1, 4 is

- (a) 2**
- (b) 2/5**
- (c) 3/5**
- (d) None of these**

(iii) The quartile deviation of 7, 10, 12, 15, 17, 19, 25 is

- (a) 4.2**
- (b) 4.3**
- (c) 4.5**
- (d) None of these**

(iv) If probability that horse A wins the race is $\frac{1}{5}$ and that horse B wins is $\frac{1}{4}$. What is the probability that at least one of them wins the race?

- (a) $\frac{1}{9}$**
- (b) $\frac{9}{20}$**
- (c) $\frac{1}{20}$**
- (d) $\frac{2}{5}$**

- (v) The rank correlation coefficient for 10 pair of observations is 0.5. Later it was noticed that the ranks to x & y are given as 2 & 9 are wrongly given as 4 & 7. The correct value of correlation coefficient is
- (a) 0.20
 - (b) 0.24
 - (c) 0.26
 - (d) 0.28
- (vi) The two lines of regression become identical when
- (a) $r = 1$
 - (b) $r = -1$
 - (c) $r = 0$
 - (d) Both (a) & (b)
- (vii) Pie-diagram is used for
- (a) Comparing different components and their relation to the total
 - (b) Representing qualitative data in a circle
 - (c) Representing quantitative data in circle
 - (d) (b) or (c) .
- (viii) Which of the following statements is false?
- (a) Statistics is derived from the Latin word 'Status'
 - (b) Statistics is derived from the Italian word 'Statist'
 - (c) Statistics is derived from the French word 'Statistik'
 - (d) None of these
- (ix) The variance of first five even natural numbers is
- (a) $2\sqrt{2}$
 - (b) 8
 - (c) $2\sqrt[3]{2}$
 - (d) $4\sqrt{2}$
- (x) If covariance between two variables x and y is 50 and the variance of x is 25 then the variance of variable y must be
- (a) Less than 100
 - (b) More than 100
 - (c) At least 100
 - (d) At most 10
- (xi) Two unbiased dice are thrown together. The probability that the sum of the digits on the dice is more than 8 is
- (a) $\frac{5}{18}$
 - (b) $\frac{5}{12}$
 - (c) $\frac{13}{18}$
 - (d) $\frac{7}{12}$

(xii) For a normal distribution with mean 150 and S. D. 45; find Q_1 and Q_3 :

- (a) 119.35 and 190.65 respectively
- (b) 180.35 and 119.65 respectively
- (c) 119.65 and 180.35 respectively
- (d) 123.45 and 183.65 respectively

(b) Answer the following Question True or False

[12×1=12]

- (i) Statistics are affected by a small number of causes.
- (ii) A frequency distribution is the arrangement of the given data in the form of a table showing frequency with which each variables occurs.
- (iii) Good average should be unduly affected by the presence of extreme values.
- (iv) The geometric mean is obtained by multiplying the value of the item together and then taking it to its root corresponding to the number of items.
- (v) Mode is the value that has maximum frequency.
- (vi) Dispersion is a measure of variation of the items.
- (vii) Correlation is an analysis of the co-variation between two or more variables.
- (viii) Graphic method is also known as simple graph method.
- (ix) A set of all possible outcomes from an experiment is called a sample space.
- (x) The word odd is frequently used in statistics.
- (xi) Regression analysis is not used in prediction and forecasting problems.
- (xii) If b_{xy} is positive, then b_{yx} will also be positive.

Answer: 8(a)

- (i) (b)
- (ii) (a)
- (iii) (c)
- (iv) (d)
- (v) (c)
- (vi) (d)
- (vii) (a)
- (viii) (c)
- (ix) (b)
- (x) (c)
- (xi) (a)
- (xii) (b)

Answer: 8(b)

- (i) False
- (ii) True
- (iii) False
- (iv) True
- (v) True
- (vi) True
- (vii) True
- (viii) True
- (ix) True
- (x) True
- (xi) False
- (xii) True

Part B

4 questions to be answered out of questions [6×4=24]

9. Explain the Essential parts of a Statistical table.

Answer: 9

A good statistical table should invariably has the following parts:

1. **Table Number:** A table should be numbered for identification, especially, when there are a large number of tables in a study. The number maybe put at the centre, above the title or at the bottom of the table.
2. **Title of the table:** Every table should have a title. It should be clear, brief and self explanatory. The title should be set in bold type so as to give it prominence.
3. **Date:** The date of preparation of a table should always be written on the table. It enables to recollect the chronological order of the table prepared.
4. **Stubs or Row Designations:** Each row of the table must have a heading. The designations of the rows are called stubs or stub items. Stubs clarify the figures in the rows. As far as possible, the items should be considered so that they can be included in a single row.
5. **Captions or Column headings:** A table has many columns. Sub-headings of the columns are called captions or headings. They should be well-defined and brief.
6. **Body of the table:** It is the most vital part of the table. It contains the numerical information. It should be made as comprehensive as possible. The actual data should be arranged in such a manner that any figure may be readily located. Different categories of numerical variables should be set out in an ascending order, from left to right in rows and in the same fashion in the columns, from top downwards.

10. Find the average marks of a student from the follows table

Marks	No. of students
Below 10	25

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Below 20	40
Below 30	60
Below 40	75
Below 50	95
Below 60	125
Below 70	190
Below 80	240

Answer: 10

This cumulative series should be turned into a simple one in order to find out the average:

Calculation of average Marks of 240 Students

Marks	No. of students (f)	Mid Values x	Deviation from assumed average (45) d = (x - a)	Step Deviation $d' = \frac{d}{i}$	Total Division fd
0 – 10	25	5	-40	-4	-100
10 – 20	15	15	-30	-3	-45
20 – 30	20	25	-20	-2	-40
30 – 40	15	35	-10	-1	-15
40 – 50	20	45	0	0	0
50 – 60	30	55	+10	+1	+30
60 – 70	65	65	+20	+2	+130
70 – 80	50	75	+30	+3	+150
	n = 240				$\Sigma fd = +150$

$$\bar{x} = a + \frac{\Sigma fd'}{n} \times i = 45 + \frac{110}{240} \times 10$$

$$= 45 + 4.58 = 49.58 \text{ marks}$$

11. The following table gives the weights of one hundred persons. Compute the coefficient of dispersion by the methods of Limits (i. e. range)

Weight in lbs. Class interval	No. of persons
85-95	4
95-105	13
105-115	8
115-125	14
125-135	9
135-145	16
145-155	17
155-165	9
165-175	8
175-185	2
	100

Answer: 11

Computation of Dispersion by the Method of Limits

Weight in lbs. Class interval	Mid- Values (lbs.)	No. of persons
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MTP_Foundation_Syllabus 2016_Dec19_Set 1

85-95	90	4
95-105	100	13
105-115	110	8
115-125	120	14
125-135	130	9
135-145	140	16
145-155	150	17
155-165	160	9
165-175	170	8
175-185	180	2

Range of dispersion = 185 lbs. – 85 lbs. = 100 lbs.

$$\text{Co-efficient of dispersion} = \frac{185-85}{185+85} = \frac{100}{270} = 370$$

12. Quotations of index number of equity shares price of a certain joint stock company and of prices of preference shares are given below:

Year	1991	1992	1993	1994	1995	1996	1997
Equity Shares	97.5	99.4	98.6	92.2	95.1	98.4	97.1
Preference Shares	75.1	75.9	77.1	78.2	79.0	74.8	76.2

Use the method of rank correlation to determine the relationship between Equity share and preference share prices.

Answer: 12

X	R _x	Y	R _y	(R _x - R _y) = D	D ²
97.5	4	75.1	2	2	4
99.4	7	75.9	3	4	16
98.6	6	77.1	5	1	1
92.2	1	78.2	6	5	25
95.1	2	79.0	7	5	25
98.4	5	74.8	1	4	16
97.1	3	76.2	4	1	1
					$\sum d^2 = 88$

$$R = 1 - \frac{6 \sum D^2}{N^3 - N}$$

$$= 1 - \frac{6 \times 88}{7^3 - 7} = 1 - 1.571 = -0.571$$

13. Find both the regression equations from the following:

$$\sum X = 60$$

$$\sum Y = 40$$

$$\sum XY = 1150$$

$$\sum X^2 = 4160$$

$$\sum Y^2 = 1720$$

$$N = 10$$

Answer: 13

$$b_{xy} = \frac{\sum XY - \frac{\sum X \times \sum Y}{N}}{\sum Y^2 - \frac{\sum Y^2}{N}}$$

$$= \frac{1150 - \frac{60 \times 40}{10}}{1720 - \frac{40^2}{10}}$$

$$= \frac{910}{1560} = 0.58$$

And

$$b_{yx} = \frac{\sum XY - \frac{\sum X \times \sum Y}{N}}{\sum Y^2 - \frac{\sum Y^2}{N}}$$

$$= \frac{910}{4160 - \frac{60^2}{10}}$$

$$= 0.24$$

Here $\bar{X} = \frac{60}{10} = 6$. and $\bar{Y} = \frac{40}{10} = 4$.

Now, the regression equity of Y on X is

$$Y - 4 = 0.24 (X - 6)$$

$$Y = 0.24 X + 2.56$$

And, the regression equitation of X on Y is

$$X - 6 = 0.58 (Y - 4)$$

$$X = 0.58Y + 3.68$$

14. An urn contains 3 white balls, 4 red balls and 5 black balls. Two balls are drawn. What is the probability that

(i) both are red?

(ii) both are white?

(iii) one red and one white?

Answer: 14

$$\text{Total balls} = 3 + 4 + 5 = 12$$

(i) Required probability = $\frac{{}^4C_2}{{}^{12}C_2} = \frac{4 \times 3}{12 \times 11} = \frac{1}{11}$

(ii) Required probability = $\frac{{}^3C_2}{{}^{12}C_2} = \frac{3 \times 2}{12 \times 11} = \frac{1}{22}$

(iii) Required probability = $\frac{{}^4C_1 \cdot {}^3C_1}{{}^{12}C_2} = \frac{4 \times 3 \times 2}{12 \times 11} = \frac{2}{11}$