ADVANCED FINANCIAL MANAGEMENT

AFM (MAY'24) VS. SFM (NOV'23) NEW TOPICS QUESTIONS COMPILER

ADVANCED CAPITAL BUDGETING DECISIONS (40Q)

1. Illustration (SM)

Determine NPV of the project with the following information:

Initial Outlay of project	₹ 40,000
Annual revenues (Without inflation)	₹ 30,000
Annual costs excl depreciation (Without inflation)	₹ 10,000
Useful life	4 years
Salvage value	Nil
Tax Rate	50%
Cost of Capital (Including inflation premium of 10%)	12%

Solution:

Depreciation:

Initial Investment = 40,000

Life = 4 Years

Salvage Value = Nil

Depreciation per Annum = $\frac{40000}{4}$ = 10,000

Calculation of Profit After Tax:

Particulars	**₹ without inflation**
Revenue	30,000
Less: Cost	10,000
EBITDA	20,000
Less: Depreciation	10,000
EBIT/ PBT	10,000
Tax@ 50%	5,000
Profit After Tax (PAT)	5,000

Cash Flow After Tax = (Revenue – Cost- Depreciation) (1– Tax) + Depreciation
= 5,000+ 10,000 = **15,000**

OR

CFAT = (Revenue – Cost) (1–Tax)+ D*T
= (30,000–10,000) (1-50%)+ 10,000* 50% = **15,000** (without inflation)

k_c (incl. inflation) = 12%

Method 1: Discounting Nominal Cash Flows

Year	Real Cash Flow	Inflation	Nominal Cash Flow	Discount Rate @12%	Present Value
1	15,000	=(1+10%)^1= 1.1	16,500	0.892	14,718.000
2	15,000	=(1+10%)^2= 1.21	18,150	0.797	14,465.550
3	15,000	=(1+10%)^3= 1.331	19,965	0.718	14,334.870
4	15,000	=(1+10%)^4= 1.4641	21,961.5	0.636	13,967.514
			76,576.5		**57,485.934**

PVCIF = ₹ 57,486
Initial Investment = ₹ 40,000
NPV = **₹ 17,486**

Method 2: Discounting Real Cash Flows with Real Rate

Real Discount Rate,
1+ Nominal Rate = (1+ Real Rate) (1+ Inflation Rate)

(1+ Real Rate) $= \frac{1+Nominal\ Rate}{1+Inflation\ Rate}$

Real Rate $= \frac{1+Nominal\ Rate}{1+Inflation\ Rate} - 1$

$= \frac{1+12\%}{1+10\%} - 1$

= 1.82%

Year	Real Cash Flow	PVF @ 1.82%
1	15,000	0.982
2	15,000	0.965
3	15,000	0.947
4	15,000	0.930
		3.824

PVCIF (real) = 15,000* 3.824
= ₹ 57,366
PVCOF (real) = ₹ 40,000
NPV = **₹ 17,366**

2. Illustration

XYZ Ltd. requires ₹ 8,00,000 for a unit.
Useful life of project - 4 years.
Salvage value - Nil.
Depreciation Charge ₹ 2,00,000 p.a.
Expected revenues & costs (excluding depreciation) ignoring inflation.

Year	1	2	3	4
Revenues	₹ 6,00,000	₹ 7,00,000	₹ 8,00,000	₹ 8,00,000
Costs	₹ 3,00,000	₹ 4,00,000	₹ 4,00,000	₹ 4,00,000

Tax Rate 60% cost of capital 10% (including inflation premium).
Calculate NPV of the project if inflation rates for revenues & costs are as follows:

Year	Revenues	Costs
1	10%	12%
2	9%	10%
3	8%	9%
4	7%	8%

Solution:

Year	CF	PVF @10%	PVCF
1	2,49,600	0.909	2,26,909
2	2,58,600	0.826	2,13,719
3	3,19,514	0.751	2,40.055
4	3,31,331	0.683	2,26,303

PVCIF = 9,06,987
Cash Outflow = 8,00,000
NPV = ₹ 1,06,987

Working Notes

Cash Flow After Tax = (Revenue – Cost – Depreciation) (1– Tax) + Depreciation

Year	(1+i)	Revenue	R_N
1	(1.10)	6,00,000	6,60,000
2	(1.10) (1.09)	7,00,000	8,39,300
3	(1.10) (1.09) (1.08)	8,00,000	10,35936
4	(1.10) (1.09) (1.08) (1.07)	8,00,000	11,08,451.52
			36,43,687.52

Year	(1+i)	Cost	C_N
1	(1.12)	3,00,000	3,36,000
2	(1.12) (1.1)	4,00,000	4,92,800
3	(1.12) (1.1) (1.09)	4,00,000	5,37,152
4	(1.12) (1.1) (1.09) (1.08)	4,00,000	5,80,124.16
			19,46,076.61

R_N	C_N	$(R_N - C_N)$	A= $(R_N - C_N)$ + DT DT= 2,00,000* 60%	PVF @10%	A *PVF
6,60,000	3,36,000	1,29,600	2,49,600	0.909	2,26,909
8,39,300	4,92,800	1,38,600	2,58,600	0.826	2,13,719
10,35936	5,37,152	1,99,514	3,19,514	0.751	2,40.055
11,08,451.52	5,80,124.16	2,11,331	3,31,331	0.683	2,26,303
					9,06,987

3. Illustration

A firm has projected the following cash flows from a project under evaluation:

Year	₹ lakhs
0	(70)
1	30
2	40
3	30

The above cash flows have been made at expected prices after recognizing inflation. The firm's cost of capital is 10%. The expected annual rate of inflation is 5%.

Show how the viability of the project is to be evaluated.

Solution:

Method 1: Cash Flow and Discount Rate is Nominal

Year	CF	PVF @ 15.5% (WN 1)	PVCF
0	(70)	1	(70)
1	30	0.865	25.97
2	40	0.749	29.98
3	30	0.649	19.47
		NPV	**5.42889 lakhs**

Project is viable as NPV Is positive.

Method 2: Cash Flow is converted to Real and Discount Rate is Real

Year	CF	Inflation Adjustment @5%	Real CF	PVF @10%	PVCF
0	(70)	1	(70)	1	(70)
1	30	0.952	28.57	0.909	25.97

2	40	0.907	36.28	0.826	29.98
3	30	0.8638	25.915	0.751	19.47
				NPV	**5.426 lakhs**

Project is viable as NPV Is **₹ 5.426 lakhs.**

Working Notes:

1. Assuming given Discount rate is real, 10%. Inflation rate is 5%.
Nominal Discount Rate,

(1+n) = (1+r) (1+i)
(1+n) = (1+10%) (1+5%)
(1+n) = 1.155
n = 15.5%

2. Inflation is 5%, so all nominal cash flows are adjusted for inflation to convert them to real.

4. Illustration (SM)

Shashi Co. Ltd has projected the following cash flows from a project under evaluation:

Year	0	1	2	3
₹ (in lakhs)	(72)	30	40	30

The above cash flows have been made at expected prices after recognizing inflation. The firm's cost of capital is 10%. The expected annual rate of inflation is 5%. Show how the viability of the project is to be evaluated. PVF at 10% for 1 -3 years are 0.909, 0.826 and 0.751.

Solution:

Cash Flow is converted to Real and Discount Rate is Real

Year	**CF**	**Inflation Adjustment @5%**	**Real CF**	**PVF @10%**	**PVCF**
0	(72)	1	(72)	1	(72)
1	30	0.952	28.57	0.909	25.97
2	40	0.907	36.28	0.826	29.98
3	30	0.8638	25.915	0.751	19.47
					3.426 lakhs

5. Illustration

KLM Ltd. requires ₹ 15,00,000 for a new project. The useful life of the project is 3 years. Salvage value - NIL. Depreciation is ₹ 5,00,000 p.a. Given below are projected revenues and costs (excluding depreciation) ignoring inflation:

Year	**1**	**2**	**3**
Revenues in ₹	10,00,000	13,00,000	14,00,000
Costs in ₹	5,00,000	6,00,000	6,50,000

Applicable tax rate is 35%. Assume cost of capital to be 14% (after tax). The inflation rates for revenues and costs are as under:

Year	Revenues %	Costs %
1	9	10
2	8	9
3	6	7

PVF at 14%, for 3 years =0.877, 0.769 and 0.675 Show amount to the nearest rupee in calculations. You are required to calculate net present value of the project.

Solution:

Year	Rev_R	$Cost_R$	Inf_R	Inf_C	Inf Factor (Rev)	Inf Factor (Cost)
1	10	5	9%	10%	= 1.09	=1.1
2	13	6	8%	9%	=(1.09)(1.08) = 1.1772	(1.1)(1.09) =1.199
3	14	6.5	6%	7%	=(1.09)(1.08)(1.06) = 1.247	(1.1)(1.09)(1.07) = 1.283

Inflation Factor		Adjustment	
Revenue	Cost	Revenue	Cost
1.09	1.1	10.9	5.5
1.1772	1.199	15.303	7.194
1.247	1.283	17.458	8.3395

Cash Flow After Tax = (Revenue – Cost) (1– Tax) + Depreciation* Tax

Year	CFAT	PVF @ 14%	PVCF
1	5.26	0.877	4.613
2	7.02085	0.769	5.399
3	7.677	0.675	5.182

PVCIF = 15,19,400
Cash Outflow = 15,00,000
NPV = 19,400

Year	Revenue	Revenue Nominal	Cost	Cost Nominal	(R –C) Nominal
1	10,00,000	10,90,000	50,00,000	5,50,000	5,40,000
2	13,00,000	15,30,360	60,00,000	7,19,400	8,10,960
3	14,00,000	17,46,965	65,00,000	8,33,905	9,13,060

Cash Flow After Tax

(Revenue – Cost)	*PV Factor	*(1-Tax)	+Depreciation* Tax	= CFAT
5,40,000	*0.877	*0.65	+ 1,75,000	= 4,61,302
8,10,960	*0.769	*0.65	+ 1,75,000	= 5,39,933
9,13,060	*0.675	*0.65	+ 1,75,000	= 5,18,730
			PVCIF	= 15,19,965.43
			Investment	= 15,00,000
			NPV	**= ₹ 19,965.43**

6. Illustration

Possible net cash flows of Projects A and B at the end of first year and their probabilities are given below. The discount rate is 10 per cent. For both the projects, initial investment is ₹ 10,000. Calculate the expected net present value for each project. State which project is preferable?

Possible Event	Project A		Project B	
	Cash Flow (₹)	Probability	Cash Flow (₹)	Probability
A	8,000	0.10	24,000	0.10
B	10,000	0.20	20,000	0.15
C	12,000	0.40	16,000	0.50
D	14,000	0.20	12,000	0.15
E	16,000	0.10	8 ,000	0.10

Solution:

Project A

Probability	0.1	0.2	0.4	0.2	0.1
Cash Flow	8,000	10,000	12,000	14,000	16,000
Prob* CF	800	2,000	4,800	2,8000	1,600

Expected Value = 800+ 2,000+ 4,800+ 2,800+ 1,600 =12,000

PV of Project A

(@10% discount rate) = $\frac{12,000}{1.1}$ = 10,909

NPV (A) = 10,909 – 10,000 = **₹ 909**

Project B

Probability	0.1	0.15	0.5	0.15	0.1
Cash Flow	24,000	20,000	16,000	12,000	8,000
Prob* CF	2,400	3,000	8,000	1,800	800

Expected Value = 2,400+ 3,000+ 8,000+ 1,800+ 800 =16,000

PV of Project B

(@10% discount rate) = $\frac{16,000}{1.1}$ = 14,545

NPV (B) = 14,545 – 10,000 = ₹ **4,545**

NPV of Project B is greater than NPV of Project A, hence Project B should be preferred.

7. Illustration

Probabilities for net cash flows for 3 years of a project are as follows:

Year 1		Year 2		Year 3	
Cash Flow (₹)	**Probability**	**Cash Flow (₹)**	**Probability**	**Cash Flow (₹)**	**Probability**
2,000	0.1	2,000	0.2	2,000	0.3
4,000	0.2	4,000	0.3	4,000	0.4
6,000	0.3	6,000	0.4	6,000	0.2
8,000	0.4	8,000	0.1	8,000	0.1

Calculate the expected net present value of the project using 10 per cent discount rate if the Initial Investment of the project is ₹ 10,000.

Solution:

Year 1			Year 2			Year 3		
Cf	**P**	**Ex Val**	**Cf**	**P**	**Ex Val**	**Cf**	**P**	**Ex Val**
2,000	0.1	200	2,000	0.2	400	2,000	0.3	600
4,000	0.2	800	4,000	0.3	1,200	4,000	0.4	1,600
6,000	0.3	1,800	6,000	0.4	2,400	6,000	0.2	1,200
8,000	0.4	3,200	8,000	0.1	800	8,000	0.1	800
		6,000			4,800			4,200

Year	**1**	**2**	**3**
A Expected Value	6,000	4,800	4,200
B PVF @10%	$\frac{1}{1.1^1}$	$\frac{1}{1.1^2}$	$\frac{1}{1.1^3}$
PV of Cash Inflow **C=** A*B	$\frac{6000}{1.1}$	$\frac{4800}{1.1^2}$	$\frac{4200}{1.1^3}$

PVCIF = 12,577
PVCOF = 10,000
NPV = 12,577– 10,000 = **₹ 2,577**

Expected NPV of the investment is **₹ 2,577.**

8. Illustration

Cyber Company is considering two mutually exclusive projects. The investment outlay of both the projects is ₹ 5,00,000 and each is expected to have a life of 5 years. Under three possible situations their annual cash flows and probabilities are as under:

		Cash Flow (₹)	
Situation	**Probabilities**	**Project A**	**Project B**
Good	0.3	6,00,000	5,00,000
Normal	0.4	4,00,000	4,00,000
Worse	0.3	2,00,000	3,00,000

Cost of Capital is 7%. Which project should be accepted and why?

Solution:

Project A

Expected Cash Flow = 0.3*6,00,000 + 0.4* 4,00,000 + 0.3*2,00,000
= ₹ 4,00,000 p.a for 5 yrs

PV of Cash Inflow for Project A for 5 yrs
= PVAF (7%, 5 yrs)
= 4.1

PVIF = 4.1* 4,00,000 = ₹ 16,40,000
PVOF = ₹ 5,00,000
NPV = **₹ 11,40,000**

Project B

Expected Cash Flow = 0.3*5,00,000 + 0.4* 4,00,000 + 0.3*3,00,000
= ₹ 4,00,000 p.a for 5 yrs

Since Expected value of Cash Inflow
= **₹ 4,00,000**

NPV of Project B will also be ₹ 11,40,000. When Outflow =₹ 5,00,000, and k_c= 7%.

Project A

Variance = $\sum P_i(x-\bar{x})^2$
$\bar{x}$ = Expected Cash Flow
σ^2 = 0.3* (6,00,000- 4,00,000)^2 + 0.4* (4,00,000- 4,00,0000)^2 + 0.3(6,00,000- 4,00,000)^2
= 2.4
σ = 1.549 = **₹ 1,54,919**

Project B

σ^2 = 0.3* (5,00,000- 4,00,000)^2 + 0.4* (4,00,000- 4,00,0000)^2 + 0.3(3,00,000- 4,00,000)^2
= 0.6
σ = 0.775 = **₹ 77,459.67**

Since the S.D of cash flow of Project B is ₹ 77,459.67 and is much lower than the S.D of Cash Inflow of Project a which is ₹ 1,54,919, **Project B is less risky**. Even though they both have same NPV is ₹ 11,40,000, we choose Project B as it is less risky.

9. Illustration

Calculate Variance and Standard Deviation of Project A and Project B based on following:

Possible Event	Project A		Project B	
	Cash Flow (₹)	Probability	Cash Flow (₹)	Probability
A	8,000	0.10	24,000	0.10
B	10,000	0.20	20,000	0.15
C	12,000	0.40	16,000	0.50
D	14,000	0.20	12,000	0.15
E	16,000	0.10	8,000	0.10

Solution:

Variance, $\sigma^2 = \sum Pi\ (x - \bar{x})^2$

Project A

Cash Flows (Cf)	Probability (Pi)	$Pi * Cf$	Variance Computation
8,000	0.10	800	0.10 (8,000 – 12,000)^2
10,000	0.20	2,000	0.20 (10,000 – 12,000)^2
12,000	0.40	4,800	0.40 (12,000 – 12,000)^2
14,000	0.20	2,800	0.20 (14,000 – 12,000)^2
16,000	0.10	1,600	0.10 (16,000 – 12,000)^2
		$\bar{x}$ = 12,000	**48,00,000**

σ^2 = **48,00,000**

σ = **₹ 2,190.89**

Project B

Cash Flows (Cf)	Probability (Pi)	$Pi * Cf$	Variance Computation
24,000	0.10	2,400	0.10 (24,000 – 16,000)^2
20,000	0.15	3,000	0.15 (20,000 – 16,000)^2
16,000	0.50	8,000	0.50 (16,000 – 16,000)^2
12,000	0.15	1,800	0.15 (12,000 – 16,000)^2
8,000	0.10	800	0.10 (8,000 – 16,000)^2
		$\bar{x}$ = 16,000	**1,76,00,000**

σ^2 = **1,76,00,000**

σ = **₹ 4,195.23**

10. Illustration

Calculate Coefficient of Variation of Project A and Project B based on the following information:

Possible Event	Project A		Project B	
	Cash Flow (₹)	Probability	Cash Flow (₹)	Probability
A	10000	0.10	26,000	0.10
B	12,000	0.20	22,000	0.15
C	14,000	0.40	18,000	0.50
D	16,000	0.20	14,000	0.15
E	18,000	0.10	10,000	0.10

Solution:

	Expected Cash Flow	σ	CV of ECF
Project A (WN 1)	14,000	2,190.89	0.1565
Project B (WN 2)	18,000	4,195.23	0.2331

Project B has lower Coefficient of Variation, hence from a risk perspective project A is chosen.

WORKING NOTES:

1. Expected Cash Flows of the project A

Pi	Cash Flow	Pi * Cash flows	$\sigma^2 = \sum Pi\,(x-\bar{x})^2$
0.1	10,000	1,000	0.1 (10,000 – 14,000)^2
0.2	12,000	2,400	0.2 (12,000 – 14,000)^2
0.4	14,000	5,600	0.4 (14,000 – 14,000)^2
0.2	16,000	3,200	0.2 (16,000 – 14,000)^2
0.1	18,000	1,800	0.1 (18,000 – 14,000)^2
		14,000	**48,00,000**

σ^2 = **48,00,000**
σ = **2,190.89**

2. Expected Cash Flows of the project A

Pi	Cash Flow	Pi * Cash flows	$\sigma^2 = \sum Pi\,(x-\bar{x})^2$
0.1	26,000	2,600	0.1 (26,000 – 18,000)^2
0.15	22,000	3,300	0.15 (22,000 – 18,000)^2
0.5	18,000	9,000	0.5 (18,000 – 18,000)^2
0.15	14,000	2,100	0.15 (14,000 – 18,000)^2
0.1	10,000	1,000	0.1 (10,000 – 18,000)^2
		18,000	**1,76,00,000**

σ^2 = **1,76,00,000**
σ = **4,195.235**

11. Illustration

Skylark Airways is planning to acquire a light commercial aircraft for flying class clients at an investment of ₹ 50,00,000. The expected cash flow after tax for the next three years is as follows:

Year 1		Year 2		Year 3	
CFAT	**Probability**	**CFAT**	**Probability**	**CFAT**	**Probability**
14,00,000	0.1	15,00,000	0.1	18,00,000	0.2
18,00,000	0.2	20,00,000	0.3	25,00,000	0.5
25,00,000	0.4	32,00,000	0.4	35,00,000	0.2
40,00,000	0.3	45,00,000	0.2	48,00,000	0.1

The Company wishes to take into consideration all possible risk factors relating to airline operations. The company wants to know:

(i) The expected NPV of this venture assuming independent probability distribution with 6 per cent risk free rate of interest.
(ii) The possible deviation in the expected value.
(iii) How would standard deviation of the present value distribution help in Capital Budgeting decisions?

Solution:

i) Expected NPV @ 6%

Year	0	1	2	3
Cash Flow (WN1)	(50)	27	29.3	27.9
PVF @6%	1	0.943	0.890	0.840
	(50)			74.97
NPV				**24.974**

ii) Variance and SD of Cash Flow

Year	1	2	3
σ^2	85.4	7.861	74.29
σ	9.2412	9.9302	8.619

iii) Expected Value of the Deviation

$$= \sqrt{\frac{85.4}{(1.06)^2} + \frac{98.61}{(1.06)^4} + \frac{74.29}{(1.06)^6}}$$

$$= \sqrt{206.4855}$$

= **₹ 14.3696 lakhs**

iv) Standard Deviation identifies risk in cash flows. Using this, coefficient of variation can be computed and per unit of cashflows, how much risk is taken can be measured and decision on capital budgeting can be taken.

Working Notes:

1.

Year 1			Year 2			Year 3		
CF	Pi	CF* Pi	CF	Pi	CF* Pi	CF	Pi	CF* Pi
14	0.1	1.4	15	0.1	1.5	18	0.2	3.6
18	0.2	3.6	20	0.3	6	25	0.5	12.5
25	0.4	10	32	0.4	12.8	35	0.2	7
40	0.3	12	45	0.2	9	48	0.1	4.8
		27			29.3			27.9

2. Standard Deviation and Variance

σ^2 = $\sum P_i(x-\bar{x})^2$

Year 1

0.1* (14-27)^2 = 16.9
0.2* (18-27)^2 = 16.2
0.4* (25-27)^2 = 1.6
0.3* (40-27)^2 = 50.7
σ^2 = 85.4
σ = 9.2412

Year 2

0.1* (15-29.3)^2 = 20.449
0.3* (20-29.3)^2 = 25.947
0.4* (32-29.3)^2 = 2.916
0.2* (45-29.3)^2 = 49.298
σ^2 = 78.61
σ = 9.9302

Year 3

0.2* $(18-27.9)^2$ = 19.602
0.5* $(25-27.9)^2$ = 4.205
0.2* $(35-27.9)^2$ = 10.082
0.1* $(48-27.9)^2$ = 40.401
σ^2 = 74.29
σ = 8.619

12. Illustration

A company is considering Projects X and Y with following information:

Project	Expected NPV (₹)	Standard deviation
X	1,22,000	90,000
Y	2,25,000	1,20,000

(i) Which project will you recommend based on the above data?
(ii) Explain whether your opinion will change, if you use coefficient of variation as a measure of risk.
(iii) Which measure is more appropriate in this situation and why?

Solution:

i) Based on the given data, if risk is the sole criterion, Project X has lower risk i.e., S.D and it will be chosen. However, if NPV were to be the criteria, Project Y which has higher NPV will be chosen.

ii) If Coefficient of variation is used then C.V will be as follows;

	X	Y
NPV	1,22,000	2,25,000
S.D	90,000	1,20,000
C.V = σ/ NPV	$\frac{90}{122}$	$\frac{120}{225}$
C.V	0.7377	0.533

Per unit of Expected NPV, **Project Y has lower risk** i.e., lower C.V of 0.533 and hence it will be chosen.

iii) Project Y is significantly better from both NPV and C.V perspectives and hence **Project Y should be recommended.**

iv) The most appropriate method would depend on an organisation's investment policy on the face of it considering NPV of Project Y which is 85% higher, **the NPV method would be the better method** to utilise in this scenario

13. Illustration

KLM Ltd., is considering taking up one of the two projects-Project-K and Project-S. Both the projects having same life require equal investment of ₹ 80 lakhs each. Both are estimated to have almost the same yield. As the company is new to this type of business, the cash flow arising from the projects cannot be estimated with certainty. An attempt was therefore, made to use probability to analyse the pattern of cash flow from other projects during the first year of operations. This pattern is likely to continue during the life of these projects. The results of the analysis are as follows:

Project K		Project S	
Cash Flow (in ₹)	**Probability**	**Cash Flow (in ₹)**	**Probability**
11	0.10	09	0.10
13	0.20	13	0.25
15	0.40	17	0.30
17	0.20	21	0.25
19	0.10	25	0.10

Required:
(i) Calculate variance, standard deviation and co-efficient of variance for both the projects.
(ii) Which of the two projects is riskier?

Solution:

i)

Project K				**Project S**			
C.F	Pi	CF* Pi	$Pi\ (x-\bar{x})^2$	C.F	Pi	CF* Pi	$Pi\ (x-\bar{x})^2$
11	0.1	1.1	0.1 (11-15)^2	9	0.1	0.9	0.1 (9-17)^2
13	0.2	2.6	0.2 (13-15)^2	13	0.25	3.25	0.25 (13-17)^2
15	0.4	6.0	0.4 (15-15)^2	17	0.3	5.1	0.3 (17-17)^2
17	0.2	3.4	0.2 (17-15)^2	21	0.25	4.25	0.25 (21-17)^2
19	0.1	1.9	0.1 (19-15)^2	25	0.1	2.5	0.1(25-17)^2
	1.0	15.0	**4.8**		1.0	17	**20.8**

Project K
σ^2 = 4.8
σ = **2.1909**
Project S
σ^2 = 20.8
σ = **4.5607**

Coefficient of Variation

$$= \frac{\sigma}{ENPV}$$

K $= \frac{2.1909}{15}$ = **0.14606**

S $= \frac{4.5607}{17}$ = **0.28682**

ii) **Project S is risky** considering per unit of Cash Flow, it has higher element of Standard Deviation. Also on an absolute basis, Project S has higher Standard Deviation.

14. Illustration

Project X and Project Y are under the evaluation of XY Co. The estimated cash flows and their probabilities are as below:
Project X : Investment (year 0) ₹ 70 lakhs

Probability weights	**0.30**	**0.40**	**0.30**
Years	**₹lakhs**	**₹lakhs**	**₹lakhs**
1	30	50	65
2	30	40	55

3	30	40	45

Project Y: Investment (year 0) ₹ 80 lakhs

Probability weighted	**Annual cash flows through life**
	₹lakhs
0.20	40
0.50	45
0.30	50

(i) Which project is better based on NPV, criterion with a discount rate of 10%?
(ii) Compute the standard deviation of the present value distribution and analyse the inherent risk of the projects.

Solution:

Assumption: Both projects X and Y have same time frame for Cash Inflows i.e., 3 years.

i) Expected Value of Cash flows across 3 years

Project X

Probability	0.3	0.4	0.3	Expected CF
Year 1	30	50	65	48.5
Year 2	30	40	55	41.5
Year 3	30	40	45	38.5

ECF $= \sum Pi * CF$

Year	0	1	2	3
Cash Flow	(70)	48.5	41.5	38.5
PVF @ 10%	1	0.909	0.826	0.757
PV	(70)	44.09	34.29	28.925
NPV				**₹ 37.314 Lakhs**

Project Y

ECF (for each year) = 0.2*40 + 0.5*45 + 0.3*50
= 8+ 22.5+ 15 = **45.5 lakhs**

Year	0	1	2	3
Cash Flow	(80)	45.5	45.5	45.5
PVF @ 10%	1	0.909	0.826	0.757
PV	(80)	41.36	37.60	34.84
NPV				**₹ 33.15 Lakhs**

Project X is better on NPV criterion as it has higher NPV of ₹ 37.314 lakhs as compared to Project Y with NPV ₹ 33.15 lakhs

ii) Hillers Model,

$\sigma^2 = \sum_{i=0}^{n}(1+r)^{-2i}\sigma_i^2$

Variance: Project X

Year 1, σ^2 $= \sum P_i(x-\bar{x})^2$

$= 0.3\ (30-48.5)^2 + 0.4\ (50-48.5)^2 + 0.3\ (65-48.5)^2$

$= 185.25$

σ = **13.6107**

Year 2, σ^2 $= \sum P_i(x-\bar{x})^2$

$= 0.3\ (30-41.5)^2 + 0.4\ (40-41.5)^2 + 0.3\ (55-41.5)^2$

$= 95.25$

σ = **9.7596**

Year 3, σ^2 $= \sum P_i(x-\bar{x})^2$

$= 0.3\ (30-38.5)^2 + 0.4\ (40-38.5)^2 + 0.3\ (45-38.5)^2$

$= 35.25$

σ = **5.93717**

Standard deviation of the expected Value/ Standard Deviation of the PV Distribution

σ^2 $= \sum_{i=0}^{n}(1+r)^{-2i}\sigma_i^2$

σ $= \sqrt{\frac{185.25}{(1.1)^2} + \frac{95.25}{(1.1)^4} + \frac{35.25}{(1.1)^6}}$

$= \sqrt{238.05}$

= **15.43**

Variance: Project Y

Year 1, σ^2 $= \sum P_i(x-\bar{x})^2$

$= 0.2\ (40-45.5)^2 + 0.5\ (45-45.5)^2 + 0.3\ (50-45.5)^2$

$= 12.25$

σ = **3.5**

Standard deviation of the expected Value/ Standard Deviation of the PV Distribution

σ^2 $= \sum_{i=0}^{n}(1+r)^{-2i}\sigma_i^2$

σ $= \sqrt{\frac{12.25}{(1.1)^2} + \frac{12.25}{(1.1)^4} + \frac{12.25}{(1.1)^6}}$

$= \sqrt{25.4056}$

= **5.04**

Project X has higher Standard Deviation about the Expected Value and hence **has the higher risk** than Project Y which has Standard Deviation of 5.04.

15. Illustration

Shivam Ltd. is considering two mutually exclusive projects, A and B. Project A costs ₹ 36,000 and project B ₹30,000. You have been given below the net present value probability distribution for each project.

Project A		Project B	
NPV estimates (₹)	**Probability**	**NPV estimates (₹)**	**Probability**
15,000	0.2	15,000	0.1
12,000	0.3	12,000	0.4
6,000	0.3	6,000	0.4
3,000	0.2	3,000	0.1

(i) Compute the expected net present values of projects A and B.
(ii) Compute the risk attached to each project i.e. standard deviation of each probability distribution.
(iii) Compute the profitability index of each project.
(iv) Which project do you recommend? State with reasons

Solution:

i)

A			B		
P_i	CF	P_i *CF	P_i	CF	P_i *CF
0.2	15,000	3,000	0.1	15,000	1,500
0.3	12,000	3,600	0.4	12,000	4,800
0.3	6,000	1,800	0.4	6,000	2,400
0.2	3,000	600	0.1	3,000	300
1.0		**9,000**			**9,000**

Expected NPV of Project A and B are ₹ 9,000 each.

ii) σ_A^2 = 0.2(15,000-9,000)^2 + 0.3(12,000-9,000)^2 +0.3(6,000-9,000)^2 +0.2(3,000-9,000)^2
= 19,80,00,000
σ = **4,449.72**

σ_B^2 = 0.1(15,000-9,000)^2 + 0.4(12,000-9,000)^2 +0.4(6,000-9,000)^2 +0.1(3,000-9,000)^2
= 14,40,00,000
σ = **3,794.73**

iii) Profitability Index of each project

	A	B
NPV	9,000	9,000
Add: Outflow	36,000	30,000
Inflow	45,000	39,000

Profitability Index = $\frac{Discounted\ Cash\ Inflow}{Intial\ Outflow}$

A = $\frac{45,000}{36,000}$ = **1.25**

B = $\frac{39,000}{30,000}$ = **1.30**

iv) Project B has lower standard Deviation of Cash Flow at ₹ 3,794.73 for the same NPV of ₹ 9,000 and for a lower investment of ₹ 30,000 implying a higher Profitability Index of 1.3, hence **project B should be chosen**.

16. Illustration

An enterprise is investing ₹100 lakhs in a project. The risk-free rate of return is 7%. Risk premium expected by the Management is 7%. The life of the project is 5 years. Following are the cash flows that are estimated over the life of the project:

Year	Cash flows (₹ in lakhs)
1	25
2	60
3	75
4	80
5	65

Calculate Net Present Value of the project based on Risk free rate and also on the basis of Risks adjusted discount rate.

Solution:

R_f = 7%

RADR = R_f + Risk Premium

= **14%**

Year	Cash Flow	PVF @7%	PVF@14%	Discount @7%	Cash Flow @14%
1	25	0.9346	0.8772	23.265	21.93
2	60	0.8734	0.7695	52.404	46.17
3	75	0.8163	0.6750	61.2225	50.625
4	80	0.7629	0.5921	61.032	47.368
5	65	0.7130	0.5194	46.345	33.761
				244.3685	**199.854**

Projected NPV	R_f Rate	RADR
PVCIF	244.3685	199.854
Less: PVCOF	(100)	(100)
NPV	**144.3685**	**99.54**

17. Illustration

If Investment proposal costs ₹ 45,00,000 and risk-free rate is 5%, calculate net present value under certainty equivalent technique:

Year	Expected cash flow (₹)	Certainty Equivalent coefficient
1	10,00,000	0.90
2	15,00,000	0.85
3	20,00,000	0.82
4	25,00,000	0.78

Solution:

Cash outflow for Year 0 is ₹ 45 Lakhs

Year	0	1	2	3	4
Cash Flow (in ₹ Lakhs)	(45)	10	15	20	25
PVF @5%	1	0.952	0.907	0.864	0.823
DCF		9.524	13.605	17.277	20.568
∝		0.90	0.85	0.82	0.78
DCF * ∝		8.5716	11.56425	14.16714	16.04304

PVCIF = 50.34603 Lakhs
Less: COF = 45 Lakhs
NPV = **₹ 5.34603 Lakhs**

18. Illustration

X Ltd. is considering its new project with the following details:

Sr. No.	Particulars	Figures
1	Initial capital cost	₹ 400 Cr
2	Annual unit sales	5 Cr
3	Selling price per unit	₹ 100
4	Variable cost per unit	₹ 50
5	Fixed costs per year	₹ 50 Cr
6	Discount Rate	6%

Required:
1. Calculate the NPV of the project.
2. Compute the impact on the project's NPV considering a 2.5 per cent adverse variance in each variable. Which variable is having maximum effect? Consider Life of the project as 3 years

Solution:

1. Project NPV

Particulars	
Units Sold	5 Cr
Selling Price/ Unit	100
Variable Cost/ Unit	50
Contribution Per Unit	50
Total Contribution (D*A)	250 Cr
Less: Fixed Cost	50 Cr
Profit	200 Cr

Cash Flows

Year	0	1	2	3
Cash Flow	(400)	200	200	200
PVF @ 6%	1	0.943	0.8899	0.8396
PVCOF	(400)			
PVCIF	534.60			
NPV	**₹ 134.602 Cr**			

2. Impact on NPV for 2.5% adverse variance in each factor

Initial Investment	400	410	400	400	400	400	400
Units Sold	5 Cr	5 Cr	5 Cr	5 Cr	4.875 Cr	5 Cr	5 Cr
Selling Price/ Unit	100	100	97.5	100	100	100	100
Variable Cost/ Unit	50	50	50	51.25	50	50	50
Contribution / Unit	50	50	47.5	48.75	50	50	50
Total Contribution (D*A)	250 Cr	250 Cr	237.5 Cr	243.75 Cr	243.75 Cr	250 Cr	250 Cr
Less: Fixed Cost	50 Cr	50 Cr	50 Cr	50 Cr	50 Cr	51.25 Cr	50 Cr
Profit	200 Cr	200 Cr	187.5 Cr	193.75	193.75	198.75	200
Discount	6%	6%	6%	6%	6%	6%	6.15%
PVF	2.673	2.673	2.673	2.673	2.673	2.673	2.6656
PVIF	534.6	534.6	501.875	517.893	517.893	531.25	533.122
NPV	134.60	124.6	101.18	117.89	117.89	131.35	133.122
Sensitivity		7.43%	24.82%	12.41%	12.41%	2.48%	1.1%

Factors	**Sensitivity of 5% Adverse Change on NPV**
Investment	7.43%
Selling Price	24.82%
Units	12.41%
VC	12.41%
FC	2.48%
Discount Rate	1.1%

Selling Price variable has the maximum impact, and is the most sensitive factor.

19. Illustration

XYZ Ltd. is considering a project "A" with an initial outlay of ₹ 14,00,000 and the possible three cash inflow attached with the project as follows:

Particulars	**Year 1**	**Year 2**	**Year 3**
Worst case	450	400	700
Most likely	550	450	800
Best case	650	500	900

Assuming the cost of capital as 9%, determine NPV in each scenario. If XYZ Ltd is certain about the most likely result in first two years but uncertain about the third year's cash flow, analyze what will be the NPV expecting worst scenario in the third year.

Solution:

Cash Flows

Year	**PVF**	**Worst**	**Most Likely**	**Best**	**Mix**
0	1	(14)	(14)	(14)	(14)
1	0.9174	4.5	5.5	6.5	5.5
2	0.8417	4	4.5	5	4.5
3	0.7722	7	8	9	7

Scenario	1	2	3	4
PVCIF	12,90,044.5	15,01,089.94	17,12,135.40	14,23,871.59
PVCOF	(14,00,000)	(14,00,000)	(14,00,000)	(14,00,000)
NPV	**(1,09,955.5)**	**1,01,089.94**	**3,12,135.40**	**23,871.59**

Assuming cash inflows of most likely case in Year 1 and 2, and worst case in Year 3.

20. Illustration

Following are the estimates of the net cash flows and probability of a new project of M/s X Ltd.:

	Year	P = 0.3	P = 0.5	P = 0.2
Initial investment	0	4,00,000	4,00,000	4,00,000
Estimated net after tax cash inflows per year	1 to 5	1,00,000	1,10,000	1,20,000
Estimated salvage value (after tax)	5	20,000	50,000	60,000

Required rate of return from the project is 10%. Find:

(i) The expected NPV of the project.

(ii) The best case and the worst case NPVs.

(iii) The probability of occurrence of the worst case if the cash flows are perfectly dependent overtime and independent overtime.

(iv) Standard deviation and coefficient of variation, assuming that there are only three streams of cash flow, which are represented by each column of the table with the given probabilities.

(v) Coefficient of variation of X Ltd. on its average project which is in the range of 0.95 to 1.0. If the coefficient of variation of the project is found to be less risky than average, 100 basis points are deducted from the Company's cost of Capital

Should the project be accepted by X Ltd?

Solution:

Initial Investment = ₹ 4,00,000

Cash Flows (Year 1–5)

(Expected)

0.3* 1,00,000

0.5* 1,10,000

0.2* 1,20,000 = ₹ 1,09,000

Salvage Value

0.3* 20,000

0.5* 50,000

0.2* 60,000 = ₹ 43,000

Present Value Factor @ 10% Cost of Capital

PVF_A (5,10%)	= 3.7908* 1,09,000	= ₹ 4,13,195.76
PVF (5,10%)	= 0.6209* 43,000	= ₹ 26,699.61
	Total	= **₹ 4,39,895.37**

i) Net Present Value = PV of Cash Inflows – PV of Cash Outflows

= 4,39,895.37– 4,00,000

= **₹ 39,895.37**

ii) NPV in best and worst cases

	Best Case	Worst Case
Per year Cash flow **(a)**	1,20,000	1,00,000
PVF_A **(b)**	3.7908	3.7908
A= (a)* (b)	**4,54,896**	**3,79,080**
Salvage Value **(c)**	60,000	20,000
PVF (Year 5) **(d)**	0.6209	0.6209
B= (c)* (d)	**37,254**	**12,418**
A+B	4,92,150	3,91,498
Less: Investment	4,00,000	4,00,000
NPV	**92,150**	**(8,502)**

iii) Cash flows perfectly Dependent Overtime
First year cash flows determine cash flow of all subsequent years, of which probability is provided 0.3.

Cash flows are Independent Overtime.
Probability of worst case in all 5 years,
= 0.3*0.3*0.3*0.3*0.3 = **0.00243**

iv) Computation of Most Likely NPV
= -4,00,000+1,10,000*3.7908 + 50,000*0.6209
= 48,033

σ^2 = 0.3(-18,502-39,895)^2 + 0.5(48,033-39,895)^2 + 0.2(92,150-39,895)^2
= 1,28,19,11,409.4

σ = 35,803.79

Coefficient of Variation = $\frac{\sigma}{ENPV}$

= $\frac{35803.79}{39895}$ = 0.897

v) Because Coefficient of Variation is 0.897, which is less than 0.95, the cost of capital will be 100 bps lower than 10% i.e it will 9%. 9% is RADR.
ENPV of project at 9% Cost of Capital:

Year	**PVF@ 9%**	
Year 0	1	4,00,000 Investment
Year 1-5	3.889	1,09,000 Inflow
Year 5	0.6499	43,000 Salvage
	NPV	**₹ 51,919.03**

21. Illustration

XY Ltd. has under its consideration a project with an initial investment of ₹ 1,00,000. Three probable cash inflow scenarios with their probabilities of occurrence have been estimated as below:

Annual cash inflow (₹)	20,000	30,000	40,000
Probability	0.1	0.7	0.2

The project life is 5 years and the desired rate of return is 20%. The estimated terminal values for the project assets under the three probability alternatives, respectively, are ₹ 0, 20,000 and 30,000.
You are required to:

(i) Find the probable NPV;
(ii) Find the worst-case NPV and the best-case NPV; and
(iii) State the probability occurrence of the worst case, if the cash flows are perfectly positively correlated over time.

Solution:

i)

		S1		S2		S3		EV
Year	PVF @20%	Pi	CF	Pi	CF	Pi	CF	CF
0	1	0.1	(1)	0.7	(1)	0.2	(1)	1
1	0.833	0.1	0.2	0.7	0.3	0.2	0.4	0.31
2	0.694	0.1	0.2	0.7	0.3	0.2	0.4	0.31
3	0.579	0.1	0.2	0.7	0.3	0.2	0.4	0.31
4	0.482	0.1	0.2	0.7	0.3	0.2	0.4	0.31
5	0.402	0.1	0.2	0.7	0.3	0.2	0.4	0.31
5 (TV)	0.402	0.1	0	0.7	0.2	0.2	0.3	0.2
1-5	2.9906							

Expected Value for years 1-5
= 2.9906* 0.31 lakhs = ₹ 0.927086 lakhs
Expected Value for Terminal Value
= 0.402 * 0.2 lakhs = ₹ 0.080375 lakhs
NPV = **₹ 746.151**

ii) Worst Case NPV = Annual Cash Inflow* PVFA (20%, 5 years) + TV* PVF – Investment
= 20,000*2.9906 + 0 – 1,00,000
= **– ₹ 40,188**
Best Case NPV = 40,000*2.9906 + 30,000*0.4018 – 1,00,000
= **₹ 31,680.33**

iii) If cash flows are perfectly positively correlated over time, first year cash flow will determine the subsequent year cash flows. Probability of worst case in first year is 0.1 or **10%.** That will be the probability of worst-case scenario throughout.

22. Illustration

XYZ Ltd. is considering a project for which the following estimates are available:

	₹
Initial Cost of the project	10,00,000
Sales price/unit	60
Cost/unit	40
Sales volumes	
Year 1	20000 units
Year 2	30000 units
Year 3	30000 units

Discount rate is 10% p.a.
You are required to measure the sensitivity of the project in relation to each of the following parameters:

(a) Sales Price/unit
(b) Unit cost
(c) Sales volume
(d) Initial outlay and
(e) Project lifetime Taxation may be ignored.

Solution:

a) Project NPV for the given data

Year	0	1	2	3
Cash Flow (WN1)	– 10,00,000	4,00,000	6,00,000	6,00,000
PVF @10%	1	0.909	0.8264	0.7513
PVCIF	-10,00,000	3,63,636.36	4,95,867.76	4,50,788.8
NPV	-10,00,000			13,10,293
NPV	**3,10,293**			

Method 1: Sensitivity, when Selling Price reduces by 10%

Year	0	1	2	3
Cash Flow (WN1)	– 10,00,000	2,80,000	4,20,000	4,20,000
PVF @10%	1	0.909	0.8264	0.7513
PVCIF	-10,00,000	2,54,545.45	3,47,107.44	3,15,552.22
NPV	-10,00,000			9,17,205.11
NPV	**-82,794.89**			

From a 10% reduction in selling price, the NPV fell by ₹ 3,93,087.89 to ₹ –82,794 from 3,10,293 i.e., a reduction of **126.68%**

Method 2: At what Standard Deviation Selling Price will NPV be zero.

Let S be the sale price,

$$\{\frac{(S-40)*20,000}{1.1}+\frac{(S-40)*30,000}{1.1^2}+\frac{(S-40)*30,000}{1.1^3}\}-10,00,000=0$$

-36,20,586 +65,514.65 S = 0

S = 55.26

At price of ₹ 55.26, i.e., a reduction in Selling Price by ₹ 4.74, i.e., a **reduction of 7.89%** the NPV reduces by 100% to zero.

Change in Unit Cost:

Method 1: Unit Cost Increases by 10% from 40 to 44

Year	0	1	2	3
Cash Flow (WN 3)	-10,00,000	3,20,000	4,80,000	4,80,000
PV @10%	-10,00,000			10,48,234.41
NPV				**₹ 48,234.41**

Reduction in NPV = 3,10,293 – 48,234
= ₹ 2,62,058 = 84.45%

For 10% increase in cost, the NPV reduces by 84.45%

Method 2: At what cost, NPV will be zero

Year	0	1	2	3
Cash Flow (WN 3)	-10,00,000	(60-c)*20,000	(60-c)*30,000	(60-c)*20,000
		1.1	1.21	1.31

29,30,879 – 65,514.6 C = 0
C = 44.736
If Cost increases by **11.84%** i.e., ₹ 4.736 from ₹ 40 to ₹ 44.736, the NPV will reduce by 100% to zero.

Method 1: Reduction in Sales Volume by 10%

Year	0	1	2	3
C.F (WN 4)	= (10,00,000)	= 20* 18,000 = 3,60,000	=20* 27,000 = 5,40,000	=20*27,000 = 5,40,000

PV @ 10% = ₹ 1,79,263
Reduction from original NPV ₹ 3,10,293
= ₹ 1,31,029
= 42.22%
10% reduction in volume, reduces NPV by 42.22%

Method 2: At what volume will NPV be zero.

Since volume reduction is common across all 3 yrs, the reduced NPV equation for NPV @ 0
- Outflow+ current Inflow (1-x)= D
- 10,00,000+ 13,10,293(1-x) = 3,10,293- 3,10,293
3,10,293 = 13,10,293 x
x $= \frac{310293}{1310293}$
x = **23.68%**

b) Method 1: 10% increase in Outlay (₹ 1,00,000)
₹ 1,00,000 increase in outlay, reduces NPV by ₹ 1,00,000 i.e., 10% for ₹ 3,10,293 to ₹ 2,10,293 i.e., a reduction of **32.2276%**

Method 2: Increase in Outflow to be zero
Current Outflow = ₹ 10,00,000
Current Inflow = ₹ 13,10,293
Increase in outflow for NPV to be zero, is increase of ₹ 3,10,293 i.e., **31.03%** for NPV to reduce by 100%.

c) Method 1: If project life time reduces by 10%, i.e., from 3years to 2.7 years, last year reduction from 1 year to 0.7 years or 8.4 months or 255.5 days

Year	0	1	2	3
CF	(10,00,000)	4,00,000	6,00,000	6,00,000
Period	1	1	1	0.7
PV @ 10%	(10,00,000)	11,75,056		
NPV	**1,75,056**			

Reduction for 3,10,293 to 1,75,056 = **43.58%**

Method 2: At what period will NPV be zero.
Amount received in years 1 and 2
= 4,00,000/1.1 + 6,00,000/1.21
= ₹ 8,59,504

	Impact on NPV of a 10% Reduction	**Change required for NPV= 0**
Selling Price	126.68	7.89
Cost	84.45	11.84
Volume	42.22	23.68
Outlay	32.23	31.03
Time	43.58	22.92

Working Notes

1. Selling Price = ₹ 60/ unit
Cost = ₹ 40/ unit
Contribution = ₹ 20/ unit
Year 1 = 20,000 units, Cash flow = ₹ 4,00,000
Year 2 = 30,000 units, Cash flow = ₹ 6,00,000
Year 3 = 30,000 units, Cash flow = ₹ 6,00,000

2. Selling Price reduces by 10%
Selling Price = ₹ 54/ unit
Cost = ₹ 40/ unit
Contribution = ₹ 14/ unit
Year 1 = 20,000 units, Cash flow = ₹ 2,80,000
Year 2 = 30,000 units, Cash flow = ₹ 4,20,000
Year 3 = 30,000 units, Cash flow = ₹ 4,20,000

3. Unit Cost increase by 10%
Selling Price = ₹ 60
Cost = ₹ 40+ 10% = ₹ 44
Profit = ₹ 16
Year 1 = 20,000 units*16 Cash flow = ₹ 3,20,000
Year 2 = 30,000 units*16 Cash flow = ₹ 4,80,000
Year 3 = 30,000 units*16 Cash flow = ₹ 4,80,000

4. Reduction in Sales Volume

Year	Volume	%	Revised Volume
Year 1	20,000 units	10%	18,000
Year 2	30,000 units	10%	27,000
Year 3	30,000 units	10%	27,000

23. Illustration

From the following details relating to a project, analyze the sensitivity of the project to changes in initial project cost, annual cash inflow and cost of capital:

Initial Project Cost (₹)	1,20,000
Annual Cash Inflow (₹)	45,000
Project Life (Years)	4
Cost of Capital	10%

To which of the three factors, the project is most sensitive? (Use annuity factors: for 10% 3.169 and 11% 3.103).

Solution:

Initial Cash Flow = (1,20,000)
Annual Cash Flow = 45,000
k_c = 10%
No. of Years = 4

NPV of the project at 10% k_c
= PVFAC (10%, 4years) * 45,000 – 1,20,000
= 3.169*45,000 -1,20,000
= 1,42,605 – 1,20,000
NPV = ₹ **22,605**

Sensitivity computation for a 10% adverse change

	Current	**Initial COF**	**ACF**	**k_c**
Initial Cash Outflow (A)	(1,20,000)	(1,32,000)	(1,20,000)	(1,20,000)
CFA (B)	45,000	45,000	40,500	45,000
Disc. Factor (C)	10%	10%	10%	11%
PVA	3.169	3.169	3.169	3.103
PVIF (D) = (B)*(C)	1,42,606	1,42,606	1,28,344.5	1,42,606
NPV (A)+(D)	22,605	10,605	8,344.5	19,635
% change in NPV	**0%**	**– 53.08%**	**– 60.41%**	**– 13.14%**

Based on the above, **cash inflow is the most sensitive factor** as 10% change in it leads to 60.41% change in the NPV.

24. Illustration

Red Ltd. is considering a project with the following Cash flows:

₹

Years	Cost of Plant	Recurring Cost	Savings
0	10,000		
1		4,000	12,000
2		5,000	14,000

The cost of capital is 9%. Measure the sensitivity of the project to changes in the levels of plant value, running cost and savings (considering each factor at a time) such that the NPV becomes zero. The P.V. factor at 9% are as under:

Year	Factor
0	1.000
1	0.917
2	0.842

Which factor is the most sensitive to affect the acceptability of the project?

Solution:

Year	Outflow	Cost	Saving	PVF	PV
0	(10,000)			1	(10,000)
1		(4,000)	(12,000)	0.917	7,336
2		(5,000)	14,000	0.842	7,578
					4,914

If initial cash outflow/ investment increases to 14,914 from 10,000, i.e., an increase of 49.14%, NPV becomes zero.

Change in recurring cost such that total recurring cost increases by 4,914
Current PV of recurring cost = 4,000* 0.917 + 5,000*0.842
= **7,878**

4,914+ 7,8,78= 12,792
Let x be the change %
4000(1+x)*0.917+ 5000(1+x)*0.842
= 12,792
x = 12,792/7878 -1
= **62.37%**

i.e., **increase by 62.37%**

Reduction in saving by ₹ 4,914 to make NPV as zero

(12,000*0.917) (1-x) + (14,000*0.842) (1-x)	= 22,792- 4,914
22,792 (1-x%)	= 17,878
22,792 – 17,878	= 22,792 x
x	= 4914/22,792
x	= **21.56%**

Since the lowest required change for the NPV to become zero is in **savings**, that is the most Sensitive factor.

25. Illustration

The Easygoing Company Limited is considering a new project with initial investment, for a product "Survival". It is estimated that IRR of the project is 16% having an estimated life of 5 years.
The Financial Manager has studied that project with sensitivity analysis and informed that annual fixed cost sensitivity is 7.8416%, whereas cost of capital (discount rate) sensitivity is 60%.
Other information available are:
Profit Volume Ratio (P/V) is 70%, Variable cost ₹ 60/- per unit Annual Cash Flow ₹ 57,500/-
Ignore Depreciation on initial investment and impact of taxation.
Calculate

(i) Initial Investment of the Project
(ii) Net Present Value of the Project
(iii) Annual Fixed Cost
(iv) Estimated annual unit of sales
(v) Break Even Units

Cumulative Discounting Factor for 5 years

8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%
3.993	3.890	3.791	3.696	3.605	3.517	3.433	3.352	3.274	3.199	3.127

Solution:

Given, IRR = -16%
Y = 5 years
FC Sensitivity = 7.8416%
k_c sensitivity = 60%
PV Ratio = 70%
Variable Cost = ₹ 60/ unit
Annual Cash Flow = ₹ 57,500

i) Initial Investment
Annual Cash Flow = ₹ 57,500
PVFA (5 yrs, 16%) = 3.274
PV of all Cash Inflows
(A) = PV of all Cash Outflows (B)
(A) = 57,500* 3.274
= ₹ 1,88,255
So, Initial Investment (B) = **₹ 1,88,255**

ii) k_e is <16% and sensitivity of k_c is 60%
At k_c of x, NPV = Y
At k_c of x(1+60%), NPV = 0 where IRR= 16%

So, x(1+60%) = 16%
x = 10%
So, k_c = 10%

At k_c of 10%, PVFA = 3.791
PVIF = 3.791* 57,500

= ₹ 2,17,982.5 (C)

PVOF = ₹ 1,88,255 (D)

NPV = **₹ 29,727.50** (C) –(D)

iii) Annual Fixed Cost

For a 7.8416% in Fixed Cost, the NPV becomes zero.

Let annual fixed cost be x

Then, PV of Annual Fixed Cost is 3.791x

If 3.791x increases by 7.8416%, then NPV of ₹ 29,727.50 becomes zero.

3.791x* 7.8416% = 29,727.50

Solving for x,

x *7.8416% = $\frac{29727.5}{3.791}$

x = ₹ 99,999.98 = **₹ 1,00,000 approx.**

iv) Estimated sale units

Contribution (PV Ratio) = 70%

Variable Cost (%) = 1- Contribution%

= 1 –70% = 30%

Given, Variable Cost = ₹ 60

$\frac{60}{S.P}$ = 30%

Selling Price = ₹ 200

Annual Cash Flow = Contribution* Units –Fixed Cost

₹ 57,500 = 140*Units –1,00,000

Units Sold =57,500 + $\frac{1,00,000}{140}$ =**1,125 units**

v) Break Even Units

Break Even point at which,

Contribution = Fixed Cost

$\frac{FixedCost}{Contribution\ per\ unit}$ = Break Even Units

Contribution/unit*x–FC = 0

140 x = 1,00,000

x = $\frac{1,00,000}{140}$

x = 714.2857 units

Break Even Units = **715 units**

26. Illustration

Unnat Ltd. is considering investing ₹ 50,00,000 in a new machine. The expected life of machine is five years and has no scrap value. It is expected that 2,00,000 units will be produced and sold each year at a selling price of ₹ 30.00 per unit. It is expected that the variable costs to be ₹ 16.50 per unit and fixed costs to be ₹ 10,00,000 per year. The cost of capital of Unnat Ltd. is 12% and acceptable level of risk is 20%.

You are required to measure the sensitivity of the project's net present value to a change in the following project variables:

(a) sale price;

(b) sales volume;

(c) variable cost;

(d) On further investigation it is found that there is a significant chance that the expected sales volume of 2,00,000 units per year will not be achieved. The sales manager of Unnat Ltd. suggests that sales volumes could depend on expected economic states which could be assigned the following probabilities:

State of Economy	Annual Sales (in Units)	Prob.
Poor	1,75000	0·30
Normal	2,00,000	0·60
Good	2,25,000	0·10

Calculate expected net present value of the project and give your decision whether company should accept the project or not.

Solution:

i) Project NPV @ 2,00,000 units sales volume
Annual Cash Inflow (WN) = ₹ 17,00,000
No. of Years = 5
k_c = 12%
PVFA (5 years, 12%) = 3.6047
NPV = PVCIF –PVCOF
= 17,00,000* 3.6047 – 50,00,000
= **₹ 11,28,119.54**

ii) Sensitivity of NPV for a 10% reduction in Selling Price
Annual Cash Flow when Selling Price reduces by 10%
(WN 2) = ₹ 11,00,000
PVFA (5 years, 12%) = 3.6047
NPV = PVCIF –PVCOF
= 11,00,000* 3.6047 – 50,00,000
= (₹ 10,34,830)
% change in NPV = $\frac{-19,34,830-11,28,120}{11,28,120}$
= **–191.73%**

iii) Sensitivity of NPV for a 10% reduction in Sales Volume
Annual Cash Flow when Selling Price reduces by 10%
(WN 2) = ₹ 14,30,000
PVFA (5 years, 12%) = 3.6047
NPV = PVCIF –PVCOF
= 14,30,000* 3.6047 – 50,00,000
= ₹ 1,54,721
% change in NPV = $\frac{11,28,120-1,54,721}{11,28,120}$
= **86.29%**

iv) Sensitivity of NPV for a 10% reduction in Variable Cost
Annual Cash Flow when Selling Price reduces by 10%
(WN 3) = ₹ 13,70,000
PVFA (5 years, 12%) = 3.6047
NPV = PVCIF –PVCOF
= 13,70,000* 3.6047 – 50,00,000
= (₹ 61,561)
% change in NPV = $\frac{-11,28,120-61,561}{11,28,120}$

= **- 105.46%**

v) Expected NPV of the Project

Selling Price	= ₹ 30
Variable Cost	= ₹ 16.5
Contribution	= ₹ 13.5
Value	= (1,75,000*0.3 + 2,00,000*0.6 + 2,25,000*0.1)
	= 1,95,000
Total Contribution	= 13.5* 1,95,000
	= ₹ 26,32,500
Less: Fixed Cost	= ₹ 10,00,000
Annual Profit	= ₹ 16,32,500
PVAF (5yr, 12%)	= 3.6047
PVCIF	= ₹ 58,84,673
Less: PVCOF	= ₹ 50,00,000
NPV	= **₹ 8,84,673**

vi) There is 30% probability that the economy is in poor state, when NPV will be ₹ 88,596 (WN 6) i.e., negative value and the company's risk tolerance level is 20% and therefore the company should not undertake this project.

Working Notes:

1.

Selling Price	= ₹ 30
Less: Variable Cost	= ₹ 16.5
Contribution	= ₹ 13.5
Units sold	= ₹ 2,00,000
Total Contribution	= 2,00,000* 13.5
	= ₹ 27,00,0000
Less: Fixed Cost	= ₹ 10,00,000
Profit	= ₹ 17,00,000

Since tax details are not give, no tax or tax benefit on depreciation are considered

2.

Selling Price	= ₹ 27
Less: Variable Cost	= ₹ 16.5
Contribution	= ₹ 10.5
Units sold	= ₹ 2,00,000
Total Contribution	= 2,00,000* 10.5
	= ₹ 21,00,0000
Less: Fixed Cost	= ₹ 10,00,000
Profit	= ₹ 11,00,000

3.

Selling Price	= ₹ 30
Less: Variable Cost	= ₹ 16.5
Contribution	= ₹ 13.5
Units sold	= ₹ 1,80,000
Total Contribution	= 1,80,000* 13.5
	= ₹ 24,30,0000
Less: Fixed Cost	= ₹ 10,00,000
Profit	= ₹ 14,30,000

4. Selling Price = ₹ 30

Less: Variable Cost	= ₹ 18.15
Contribution	= ₹ 11.85
Units sold	= ₹ 2,00,000
Total Contribution	= 2,00,000* 11.85
	= ₹ 23,70,0000
Less: Fixed Cost	= ₹ 10,00,000
Profit	= ₹ 13,70,000

5. NPV in poor and good state

	Poor	Good
Cost Per Unit	₹ 13.5	₹ 13.5
Units	1,75,000	2,25,000
Total Contribution	₹ 23,62,500	₹ 30,37,500
Less: Fixed Cost	₹ 10,00,000	₹ 10,00,000
Profit	₹ 13,62,500	₹ 20,37,500
PVFA (5y, 12%)	3.6047	3.6047
PVCIF	₹ 49,11,403.75	₹ 73,44,576
Less: PVCOF	₹ 50,00,000	₹ 50,00,000
NPV	(₹ 88,596)	₹ 23,44,576

27. Illustration

The Textile Manufacturing Company Ltd. is considering one of two mutually exclusive proposals, Projects M and N, which require cash outlays of ₹ 8,50,000 and ₹ 8,25,000 respectively. The certainty-equivalent (C.E) approach is used in incorporating risk in capital budgeting decisions. The current yield on government bonds is 6% and this is used as the risk-free rate. The expected net cash flows and their certainty equivalents are as follows:

Project M			**Project N**	
Year-end	**Cash Flow ₹**	**C.E.**	**Cash Flow ₹**	**C.E.**
1	4,50,000	0.8	4,50,000	0.9
2	5,00,000	0.7	4,50,000	0.8
3	5,00,000	0.5	5,00,000	0.7

Present value factors of ₹ 1 discounted at 6% at the end of year 1, 2 and 3 are 0.943, 0.890 and 0.840 respectively. Required:

(i) Which project should be accepted?

(ii) If risk adjusted discount rate method is used, which project would be appraised with a higher rate and why?

Solution:

NPV of Project M

	Year	**Cash Flow**	**C.E**	**CECF**	**PVF @6%**	**PVCF**
Instalment	0	-8,50,000	1	-8,50,000	1	– 8,50,000
Annual CF	1	4,50,000	0.8	3,60,000	0.943	3,39,480
	2	5,00,000	0.7	3,50,000	0.890	3,11,500
	3	5,00,000	0.5	2,50,000	0.840	2,10,000
			2.0			**10,980**

NPV of Project N

	Year	Cash Flow	C.E	CECF	PVF @6%	PVCF
Instalment	0	-8,25,000	1	–8,25,000	1	–8,25,000
Annual CF	1	4,50,000	0.7	4,05,000	0.943	3,81,915
	2	4,50,000	0.8	3,60,000	0.890	3,20,400
	3	5,00,000	0.7	3,50,000	0.840	2,94,000
			2.4			**1,71,315**

NPV on C.E basis $= \sum PV\ of\ Cfi * CEi$

i) Project N should be accepted as it has higher NPV.

ii) Project M has total C.E of 2.0/3 and Project N has C.E of 2.4/3. So, **Project M is more** uncertain as its C.E quotient is lower and hence is **riskier**, and will have higher discount rate/ Risk adjusted Discount Rate.

28. Illustration

Determine the risk adjusted net present value of the following projects:

	X	Y	Z
Net cash outlays (₹)	2,10,000	1,20,000	1,00,000
Project life	5 years	5 years	5 years
Annual Cash inflow (₹)	70,000	42,000	30,000
Coefficient of variation	1.2	0.8	0.4

The Company selects the risk-adjusted rate of discount on the basis of the coefficient of variation:

Coefficient of Variation	Risk-Adjusted Rate of Return	P.V. Factor 1 to 5 years @ Risk adjusted rate of discount
0.0	10%	3.791
0.4	12%	3.605
0.8	14%	3.433
1.2	16%	3.274
1.6	18%	3.127
2.0	22%	2.864
More than 2.0	25%	2.689

Solution:

	Cash Outflow	Annual Cash Inflow	Coeff. Of Var	Disc. Rate	PVFA	PVCIF	Risk Adjusted NPV
X	(2,10,000)	70,000	1.2	16%	3.274	2,29,180	19,180
Y	(1,20,000)	42,000	0.8	14%	3.433	1,44,186	24,186

Z	(1,00,000)	30,000	0.4	12%	3.605	1,08,150	8,150

29. Illustration

New Projects Ltd. is evaluating 3 projects, P-I, P-II, P-III. Following information is available in respect of these projects:

	P-I	P-II	P-III
Cost	₹ 15,00,000	₹ 11,00,000	₹ 19,00,000
Inflows-Year 1	6,00,000	6,00,000	4,00,000
Year 2	6,00,000	4,00,000	6,00,000
Year 3	6,00,000	5,00,000	8,00,000
Year 4	6,00,000	2,00,000	12,00,000
Risk Index	1.80	1.00	0.60

The minimum required rate of return of the firm is 15% and applicable tax rate is 40%. The risk-free interest rate is 10%.

Required:
(i) Find out the risk-adjusted discount rate (RADR) for these projects.
(ii) Which project is the best?

Solution:

Assumption: Cash flows and Discount Rates are Post Tax and tax need not be adjusted.

i) PART 1: RADR

RADR = Risk free rate + Risk Index * (Expected Return – Risk free rate)

	P1	P2	P3
Rf	10	10	10
Risk Index	1.8	1.0	0.6
Expected Return	15	15	15
Risk Premium	(15 -10) =5	(15 -10) =5	(15 -10) =5
RADR	10+ (1.8*5) = 19%	10+(1*5) = 15%	10+(0.6*5) = 13%

ii) PART 2

NPV of Project 1

PVCOF = –15,00,000
PVCIF = PVF (19%,4y) * 6,00,000
= 2.6385* 6,00,000
= 15,83,151
NPV = 15,83,151 –15,00,000
= **₹ 83,151**

NPV of Project 2

Year	Cash Flow	PVF @ 15%	PVCF
0	-11,00,000	1	- 11,00,000

1	6,00,000	0.8695	5,21,739
2	4,00,000	0.7561	3,02,457
3	5,00,000	0.6575	3,28,758
4	2,00,000	0.57175	1,14,350
			₹ 1,67,305

NPV of Project 3

Year	**Cash Flow**	**PVF @ 13%**	
0	-19,00,000	1	- 19,00,000
1	4,00,000	0.8849	3,53,982
2	6,00,000	0.7831	4,69,888
3	8,00,000	0.6930	5,54,440
4	12,00,000	0.6133	7,35,982
			₹ 2,14,292.91

P3 has higher NPV and hence it should be chosen.

30. Illustration

Jumble Consultancy Group has determined relative utilities of cash flows of two forthcoming projects of its client company as follows:

Cash Flow in ₹	-15000	-10000	-4000	0	15000	10000	5000	1000
Utilities	-100	-60	-3	0	40	30	20	10

The distribution of cash flows of project A and Project B are as follows:

Project A					
Cash Flow (₹)	-15000	- 10000	15000	10000	5000
Probability	0.10	0.20	0.40	0.20	0.10
Project B					
Cash Flow (₹)	- 10000	-4000	15000	5000	10000
Probability	0.10	0.15	0.40	0.25	0.10

Which project should be selected and why ?

Solution:

A				**B**			
Cash Flow	Pi	**Utility**	Pi* **U**	**Cash Flow**	Pi	**Utility**	Pi* **U**
-15,000	0.1	-100	-10	-10,000	0.1	-60	-6
-10,000	0.2	-60	-12	-4,000	0.15	-3	-0.45
15,000	0.4	40	16	15,000	0.4	40	16
10,000	0.2	30	6	5,000	0.25	20	5
5,000	0.1	20	2	10,000	0.1	30	3
	1.0	-70	2		1.0	27	17.55

PW of Project A = -1,500 -2,000 +6,000 +2,000 +500

= 5,000

PW of Project B = -1,000 -600 +6,000 +1,250 +1,000
= 6,650

Evaluation

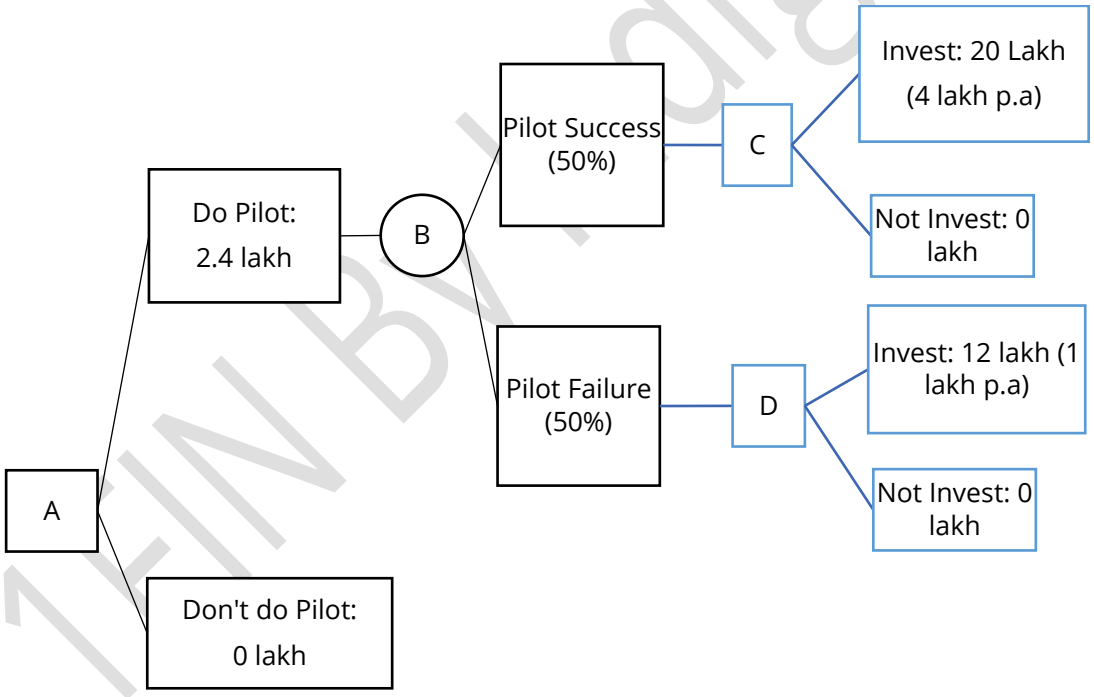
	A	**B**
Expected Cash Flow	5,000	6,650
Probability Weighted Utility Value	2	17.55

The company **should undertake Project B** given that its utility value and expected cash flow are higher than Project A.

31. Illustration

L & R Limited wishes to develop new virus-cleaner software. The cost of the pilot project would be ₹ 2,40,000. Presently, the chances of the product being successfully launched on a commercial scale are rated at 50%. In case it does succeed. L&R can invest a sum of ₹ 20 lacs to market the product. Such an effort can generate perpetually, an annual net after tax cash income of ₹ 4 lacs. Even if the commercial launch fails, they can make an investment of a smaller amount of ₹ 12 lacs with the hope of gaining perpetually a sum of ₹ 1 lac. Evaluate the proposal, adopting decision tree approach. The discount rate is 10%.

Solution:



Value at Node C = ₹ 4,00,000 p.a till perpetuity

Value of ₹ 4,00,000 p.a till perpetuity (@10% discount rate) = $\frac{400000}{0.1}$
= ₹ 40,00,000

Less: Investment = ₹ 20,00,000
NPV = ₹ 20,00,000

Value of Node D = ₹ 1,00,000 p.a till perpetuity

Value of ₹ 1,00,000 p.a till perpetuity (@10% discount rate) = $\frac{100000}{0.1}$
= ₹ 10,00,000
Less: Investment = ₹ 12,00,000
NPV = –₹ 2,00,000

Value at Node B = Probability A* NPV (A) + Probability B* NPV (B)
= 0.5*20,00,000 + 0.5*0
= ₹ 10,00,000

Value at Node A = Invest ₹ 2,40,000 and get ₹ 10,00,000
NPV (When Pilot is done) = ₹ 7,60,000
NPV (No Pilot) = ₹ 0

The company **should go ahead with pilot as investment** of ₹ 2,40,000 will generate ₹ 10,00,000 that is NPV of ₹ 7,60,000

32. Illustration

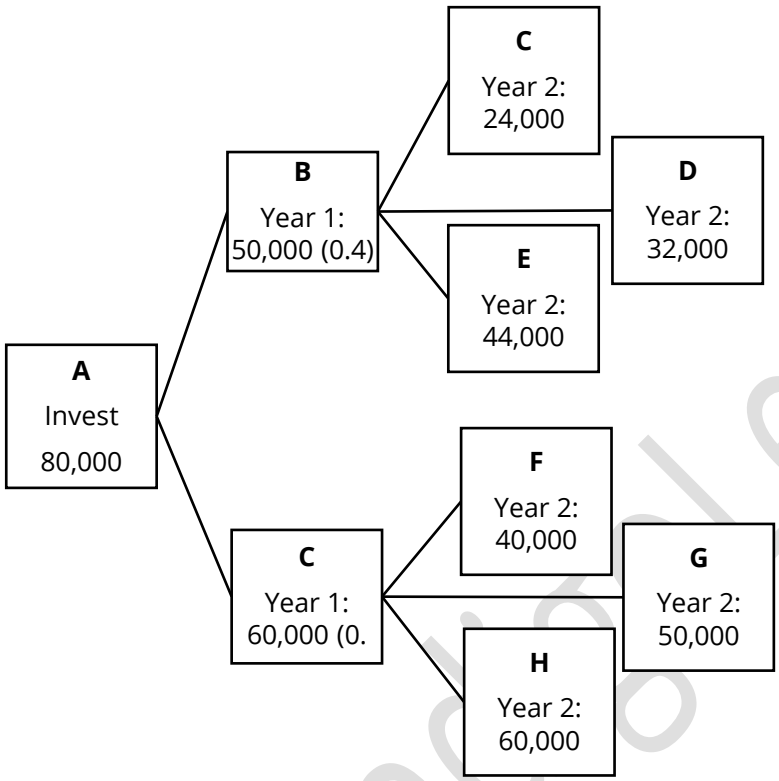
A firm has an investment proposal, requiring an outlay of ₹ 80,000. The investment proposal is expected to have two years economic life with no salvage value. In year 1, there is a 0.4 probability that cash inflow after tax will be ₹ 50,000 and 0.6 probability that cash inflow after tax will be ₹ 60,000. The probability assigned to cash inflow after tax for the year 2 is as follows:

The cash inflow year 1	**₹ 50,000**		**₹ 60,000**	
The cash inflow year 2	**Probability**		**Probability**	
	₹ 24,000	0.2	₹ 40,000	0.4
	₹ 32,000	0.3	₹ 50,000	0.5
	₹ 44,000	0.5	₹ 60,000	0.1

The firm uses a 10% discount rate for this type of investment. Required:
(i) Construct a decision tree for the proposed investment project and calculate the expected net present value (NPV).

(ii) What net present value will the project yield, if worst outcome is realized? What is the probability of occurrence of this NPV?
(iii) What will be the best outcome and the probability of that occurrence?
(iv) Will the project be accepted?
(Note: 10% discount factor 1 year 0.909; 2 year 0.826)

Solution:



i) Expected Value

Part	Year 1 Cf* PV	Year 2 Cf*PV	– Invest	NPV (1)	Joint Probability (2)	(1)* (2)
1	= 50,000*0.909 = 45,450	= 24,000*0.826 = 19,824	– 80,000	–14,726	= 0.4*0.2 = 0.08	–1,178.08
2	45,450	= 32,000*0.826 = 26,432	– 80,000	–8,118	= 0.4*0.3 = 0.12	–974.16
3	45,450	= 44,000*0.826 = 36,344	– 80,000	1,794	= 0.4*0.5 = 0.2	358.8
4	= 60,000*0.909 = 54,540	= 40,000*0.826 = 33,040	– 80,000	7,580	= 0.6*0.4 =0.24	1,819.2
5	54,540	= 50,000*0.826 = 41,300	– 80,000	15,840	= 0.6*0.5 = 0.3	4,752
6	54,540	= 60,000*0.826 = 49,560	– 80,000	24,100	= 0.6*0.1 = 0.06	1,446
						6,223.76

Expected Value = **₹ 6,223.76**

ii) If worst outcome is realised, the NPV will be **–₹ 14,726** and probability will be 0.08

iii) If best outcome is realised, the NPV will be **–₹ 24,100** and probability will be 0.06

iv) Yes, the project is accepted as the expected value of NPV is positive at **₹ 6,223.76.**

33. Illustration

A Company named Roby's cube decided to replace the existing Computer system of their organization. The original cost of the old system was ₹ 25,000 and it was installed 5 years ago. Current market value of old system is ₹ 5,000. Depreciation of the old system was charged with life of 10 years with Estimated Salvage value as Nil. Depreciation of the new system will be charged with life over 5 years. Present cost of the new system is ₹ 50,000. Estimated Salvage value of the new system is ₹ 1,000. Estimated cost savings with the new system is ₹ 5,000 per year. Increase in sales with new system is assumed at 10% per year based on original total sales of ₹ 10,00,00. Company follows straight line method of depreciation. The cost of capital of the company is 10% whereas tax rate is 30%. Evaluate the replacement decision.

Solution:

	Old System	**New System**
Original Cost	25,000	50,000
Original Life	10 years	5 years
Life Expected	5 years	0
Market Value (Today)	5,000	
Salvage Value (10 yrs)	0	1,000
Savings		5,000
Sales	1,00,000	
Increase In Sales		10,00,000 p.a
Depreciation	SLM	SLM
Kc	10%	10%
Tax	30%	30%

STEP 1:
Cash Outflow of New System + Cash Inflow of Old System
= –50,000 + 7,250 = **–42,750**

Calculation:

Book Value of Old System
Depreciation $= \frac{25{,}000-0}{10}$ = ₹ 2,500
Life Exhausted = 5 years
Depreciation for 5 years = 5* 2500
= 12,500
Current Book Value = 25,000– 12,500 = ₹ 12,500

Cash Inflow from Old System
i) Market Value = ₹ 5,000
ii) Tax Benefit on Loss = (Book Value – Market Value) * Tax Rate
= (12,500–5,000)*30% = ₹ 2,250
Cash Inflow = 5,000+ 2,250 = 7,250

Cash Outflow from New System

= – 50,000

STEP 2:

Changes in Annual Cash Flows

= (Change in Sales+ Change in Savings) (1–t) +
(Change in Depreciation)*t

= (15,000) *(1-30%) + 7,300*30%

= 10,500+ 2,190 = **₹ 12,690**

Calculation:

i) Increase in Sales = ₹ 1,00,000* 10% = ₹ 10,000

ii) Savings = ₹ 5,000

iii) Change in Depreciation

= New depreciation – Old Depreciation

= 9,800 – 2,500 = 7,300

New depreciation = (Original cost – Salvage Value)/Life

= 50000-1000/5 = 9,800

STEP 3:

PV of Annual Cash Flow @10% for 5 Years

= (12,690* 3.790786) + (1000*0.62092)

= 48,105.08 + 620.92 = **₹ 48,726**

STEP 4:

PV of Cash Inflow+ PV of Cash Outflow

= 48,726– 42,750 = **₹ 5,976**

STEP 5:

Since NPV>0, the replacement **decision is correct**.

34. Illustration

X Ltd. is a taxi operator. Each taxi cost to company ₹ 4,00,000 and has a useful life of 3 years. The taxi's operating cost for each of 3 years and salvage value at the end of year is as follow s:

	Year 1	Year 2	Year 3
Operating Cost	₹ 1,80,000	₹ 2,10,000	₹ 2,38,000
Resale Value	₹ 2,80,000	₹ 2,30,000	₹ 1,68,000

You are required to determine the optimal replacement period of taxi if cost of capital of X Ltd. is 10%.

Solution:

EAC at the end of Year 1

Period	**0**	**1**
Cash Flow	(4,00,000)	(1,80,000)+ 2,80,000 = 1,00,000
PVF	1	0.909
PVCF	(4,00,000)	90,909.09

PVCF		(3,09,090.9)
EAC	$\frac{3,09,090.9}{0.909}$	**= (3,40,034)**

EAC at the end of Year 2

Period	**0**	**1**	**2**
Cash Flow	(4,00,000)	(1,80,000)	(2,10,000)+ 2,30,000 = 20,000
PVF	1	0.909	0.826
PVCF	(4,00,000)	1,63,636	16,528.92
PVCF		(5,47,107.08)	
EAC	$\frac{5,47,107.08}{1.7353}$	**= 3,15,237.89**	

EAC at the end of Year 3

Period	**0**	**1**	**2**	**3**
Cash Flow	(4,00,000)	(1,80,000)	(2,10,000)	(70,000)
PVF	1	0.909	0.826	0.7513
PVCF	(4,00,000)	(1,63,636.36)	(1,73,553.72)	(52,592.04)
PVCF		(7,89,782.12)		
EAC	$\frac{7,89,782.12}{2.48685}$	**= 3,17,583**		

The optimum replacement period is at **the end of two years.**

35. Illustration

Company X is forced to choose between two machines A and B. The two machines are designed differently but have identical capacity and do exactly the same job. Machine A costs
₹ 1,50,000 and will last for 3 years. It costs ₹ 40,000 per year to run. Machine B is an 'economy' model costing only ₹ 1,00,000, but will last only for 2 years, and costs ₹ 60,000 per year to run. These are real cash flows. The costs are forecasted in rupees of constant purchasing power. Ignore tax. Opportunity cost of capital is 10 per cent. Which machine company X should buy?

Solution:

Machine	**A**	**B**
Cost	1,50,000	1,00,000
Annual Maintenance (p.y)	40,000	60,000
Life	3	2
Cost of Capital	10%	10%

Machine	**A**	**B**
Cost	1,50,000	1,00,000
PVAF	2.48685	1.7355
PV of Maintenance Cost	2.4865* 40,000 = (99,474.08)	1.7355* 1,00,000 = (1,04,132.23)
Total PV of all Cash Flows (PVCF)	(2,49,474.08)	(2,04,132.23)

EAC= $\frac{PVCF}{PVAF}$	1,00,331.42	1,17,621.56

Since **EAC of Machine A is lower**, X Ltd. Is advised to go with it.

36. Illustration

Company Y is operating an elderly machine that is expected to produce a net cash inflow of ₹ 40,000 in the coming year and ₹ 40,000 next year. The current salvage value is ₹ 80,000 and next year's value is ₹ 70,000. The machine can be replaced now with a new machine, which costs ₹ 1,50,000, but is much more efficient and will provide a cash inflow of ₹ 80,000 a year for 3 years. Company Y wants to know whether it should replace the equipment now or wait a year with the clear understanding that the new machine is the best of the available alternatives and that it in turn be replaced at the optimal point. Ignore tax. Take opportunity cost of capital as 10 per cent. Advise with reasons.

Solution:

	Year	**Old Machine**	**New Machine**
Cash Inflow	Year 1	40,000	80,000
	Year 2	40,000	80,000
	Year 3		80,000
Salvage Value	Year 0	80,000	
	Year 1	70,000	
Cash Outflow	Year 0		(15,000)

Alternative 1: Replace Now

Salvage Value of Old Machine = 80,000
Cost of New Machine = (1,50,000)
Cash Flow of Year 0 = (70,000)
Cash Flows Year (1-3) = 80,000 * 2.48685 (PVAF- 3,10%)
= 1,98,948
PVCF = (70,000) + 1,98,948
= **1,28,948**

Alternative 2: Replace after a Year

	Year	**Cash Flow**	**PVF**	**PVCF**
Cash Inflow Old	1	40,000	0.909	36,364
Salvage Old	1	70,000	0.909	63,636
Expense New	1	(1,50,000)	0.909	(1,36,364)
			(A)	**(36,364)**
Year 2-5 Cash flows from New Machine	2-5	80,000	2.48685	1,98,948

PV at Year 0 for New Machine
(cash Inflows) = 1,98,948* 0.909 = 1,80,862 **(B)**

NPV as on day 0 for Replacement
After Year 1 = **(A) + (B)**
= 1,80,862- 36,364 = **₹ 1,44,498**

Hence it is better to **replace after Year 1** as NPV is higher.

37. Illustration

Trouble Free Solutions (TFS) is an authorized service center of a reputed domestic air conditioner manufacturing company. All complaints/service-related matters of Air conditioner are attended by this service center. The service center employs many mechanics, each of whom is provided with a motorbike to attend to the complaints. Each mechanic travels approximately 40000 kms per annum. TFS decides to continue its present policy of always buying a new bike for its mechanics but wonders whether the present policy of replacing the bike every three years is optimal or not. It is believed that as new models are entering into the market on a yearly basis, it wishes to consider whether a replacement of either one year or two years would be a better option than present three-year period. The fleet of bikes is due for replacement shortly soon.
The purchase price of the latest model bike is ₹ 55,000. Resale value of used bike at current prices in market is as follows:

Period	₹
1 Year old	35,000
2-Year-old	21,000
3-Year-old	9,000

Running and Maintenance expenses (excluding depreciation) are as follows.

Year	Road Taxes Insurance etc. (₹)	Petrol Repair Maintenance etc. (₹)
1	3,000	30,000
2	3,000	35,000
3	3,000	43,000

Using opportunity cost of capital as 10% you are required to determine optimal replacement period of bike.

Solution:

Cost of running a bike for years 1,2,3 and salvage value.

Year	Total Expenses	Salvage Value	PVF @ 10%	Cash Flow	PVCF
1	33,000	35,000	0.909	2,000	1,818
2	38,000	21,000	0.8264	(17,000)	(14,049)
3	46,000	9,000	0.7513	(37,000)	(27,798.1)

NPV of Cumulative Cash Flows for the bike

Year	PV of Previous Year Expenses	PV of CY	Cumulative NPV
1	0	1,818	(1,818)
2	(33,000) * 0.909	(14,049)	(44,046)
3	(33,000) * 0.909 (38,000)* 0.8264	(27,798)	(89,198)

Expenses for a new Bike + Cumulative Costs

Year	PV of Previous Year Expenses	Cumulative Cash Flows NPV	Cumulative Cost PV
1	-55,000	(1,818)	53,182
2	-55,000	(44,046)	99,046
3	-55,000	(89,198)	1,44,198

EAC of Cumulative Cost

Year	Cumulative Cost PV	Cumulative Cash Flows NPV	EAC
1	53,182	0.909	58,506
2	99,046	1.7354	57,074
3	1,44,198	2.4867	57,987

As the EAC is lowest for replacement after year 2, it is advised to **replace after year 2.**

38. Illustration

A machine used on a production line must be replaced at least every four years. Costs incurred to run the machine according to its age are:

Age of the Machine (years)					
	0	1	2	3	4
Purchase price (in ₹)	60,000				
Maintenance (in ₹)		16,000	18,000	20,000	20,000
Repair (in ₹)		0	4,000	8,000	16,000
Scrap Value (in ₹)		32,000	24,000	16,000	8,000

Future replacement will be with identical machine with same cost. Revenue is unaffected by the age of the machine. Ignoring inflation and tax, determine the optimum replacement cycle. PV factors of the cost of capital of 15% for the respective four years are 0.8696, 0.7561, 0.6575 and 0.5718.

Solution:

Replacement Cycle:

Repl. Cycle Years		1		2	
Year	PVF 15%	CF	PVCF	CF	PVCF
0	1	-60,000	-60,000	-60,000	-60,000
1	0.896	16,000	13,913.6	-16,000	-13,913.6
2	0.7561			2,000	1,512.2
3	0.6575				

4	0.5718				
			-46,086.4		**-72,401**

Repl. Cycle Years		**3**		**4**	
Year	**PVF 15%**	**CF**	**PVCF**	**CF**	**PVCF**
0	1	-60,000	-60,000	-60,000	-60,000
1	0.896	-16,000	-13,913.6	-16,000	-13,913.6
2	0.7561	-22,000	-16,634.2	-22,000	-16,634.2
3	0.6575	-12,000	-7,890	-28,000	-18,410
4	0.5718			-28,000	-16,010.4
			-98,437.8		**-1,24,968.2**

Optimum Replacement Cycle:

$$\text{EAC} = \frac{CumCF}{PVAF}$$

Replacement Period	**Cum. PV of CF (1)**	**PVAF (2)**	**(1)/ (2)**
1	-46,086.4	0.896	52,997
2	-72,401	1.6257	44,535
3	-98,437.8	2.2832	43,113.96
4	-1,24,968.2	2.855	43,771.7

The optimum replacement cycle is **after 3 years.** When EAC is lowest at ₹ 43,113.96.

Working Notes:

1. **Replacement at the end of Year 1:**

Year	**0**	**1**
Cash Flows:		
Purchase	-60,000	
Maintenance		-16,000
Repairs		0
Scrap		32,000
	-60,000	16,000

2. **Replacement at the end of Year 2:**

Year	**0**	**1**	**2**
Cash Flows:			
Purchase	-60,000		
Maintenance		-16,000	-18,000
Repairs			-4,000
Scrap			24,000
	-60,000	-16,000	2,000

3. **Replacement at the end of Year 3:**

Year	**0**	**1**	**2**	**3**
Cash Flows:				
Purchase	-60,000			

Maintenance		-16,000	-18,000	-20,000
Repairs			-4,000	-8,000
Scrap				16,000
	-60,000	16,000	-22,000	-12,000

4. Replacement at the end of Year 4:

Year	**0**	**1**	**2**	**3**	**4**
Cash Flows:					
Purchase	-60,000				
Maintenance		-16,000	-18,000	-20,000	-20,000
Repairs			-4,000	-8,000	-16,000
Scrap					8,000
	-60,000	**16,000**	**-22,000**	**-28,000**	**-28,000**

39. Illustration

A company has an old machine having book value zero – which can be sold for ₹ 50,000. The company is thinking to choose one from following two alternatives:

(i) To incur additional cost of ₹ 10,00,000 to upgrade the old existing machine.

(ii) To replace old machine with a new machine costing ₹ 20,00,000 plus installation cost ₹ 50,000.

Both above proposals envisage useful life to be five years with salvage value to be nil. The expected after-tax profits for the above three alternatives are as under:

Year	**Old existing Machine (₹)**	**Upgraded Machine (₹)**	**New Machine (₹)**
1	5,00,000	5,50,000	6,00,000
2	5,40,000	5,90,000	6,40,000
3	5,80,000	6,10,000	6,90,000
4	6,20,000	6,50,000	7,40,000
5	6,60,000	7,00,000	8,00,000

The tax rate is 40 per cent. The company follows straight line method of depreciation. Assume cost of capital to be 15 per cent.

P.V.F. of 15%, 5 = 0.870, 0.756, 0.658, 0.572 and 0.497. You are required to advise the company as to which alternative is to be adopted.

Solution:

i) Upgraded Machine

Year	0	1	2	3	4	5
PAT		5,50,000	5,90,000	6,10,000	6,50,000	7,00,000
Add: Depreciation		2,00,000	2,00,000	2,00,000	2,00,000	2,00,000
CFAT		7,50,000	7,90,000	8,10,000	8,50,000	9,00,000
Investment	–10,00,000					

PV Factor	**Cash Flows**	**PV of Cash Flows**
1	-10,00,000	-10,00,000

0.87	7,50,000	6,52,500
0.756	7,90,000	5,97,240
0.658	8,10,000	5,32,980
0.572	8,50,000	4,86,200
0.497	9,00,000	4,47,300
	NPV	**17,16,220**

Depreciation:
No Salvage of Old Machine
Upgradation Cost = 10,00,000
Depreciation = $\frac{10,00,000}{5}$ = 2,00,000

ii) Replacement:

Year	0	1	2	3	4	5
PAT		6,00,000	6,40,000	6,90,000	7,40,000	8,00,000
Scrap (After Tax)	30,000					
Add: Depreciation		4,10,000	4,10,000	4,10,000	4,10,000	4,10,000
CFAT		10,10,000	10,50,000	11,00,000	11,50,000	12,10,000
Investment	–20,50,000					

PV Factor	Cash Flows	PV of Cash Flows
1	-20,20,000	-20,20,000
0.87	10,10,000	8,78,700
0.756	10,50,000	7,93,800
0.658	11,00,000	7,23,800
0.572	11,50,000	6,57,800
0.497	12,10,000	6,01,370
	NPV	**16,35,470**

Replacement Scrap Value = 50,000
Less: Tax = (20,000)
Net Scrap = 30,000

Cost of New = 20,50,000
Depreciable Life = 5 years
Depreciation = 4,10,000

iii) PAT of Old Machine:

Year	0	1	2	3	4	5
PAT	0	5,00,000	5,40,000	5,80,000	6,20,000	6,60,000
PVF	1	0.87	0.756	0.658	0.572	0.497
PVCF	0	4,35,000	4,08,240	3,81,640	3,54,640	3,28,020
NPV	**19,07,540**					

Continuing with old machine is most preferred as it gives the highest NPV of **₹19,07,540.**

40. Illustration

A & Co. is contemplating whether to replace an existing machine or to spend money on overhauling it. A & Co. currently pays no taxes. The replacement machine costs ₹ 90,000 now and requires maintenance of ₹ 10,000 at the end of every year for eight years. At the end of eight years it would have a salvage value of ₹ 20,000 and would be sold. The existing machine requires increasing amounts of maintenance each year and its salvage value falls each year as follows:

Year	Maintenance (₹)	Salvage (₹)
Present	0	40,000
1	10,000	25,000
2	20,000	15,000
3	30,000	10,000
4	40,000	0

The opportunity cost of capital for A & Co. is 15%. Required:
When should the company replace the machine?
(Notes: Present value of an annuity of Re. 1 per period for 8 years at interest rate of 15% : 4.4873; present value of Re. 1 to be received after 8 years at interest rate of 15% : 0.3269).

Solution:

Year	0	1–8	8
Cash Flow	–90,000 Investment	–10,000 Maintenance	20,000 Salvage Value
PV Factor	1	4.4873	0.3269
Present Value	–90,000	–44,873	6538

Net Present Value = –1,28,335 (excluding old machine scrap)

Equivalent Annual Cost = $-\frac{1,28,335}{4.4873}$ = – 28,599.6 = –28,600

Case: Machine Replaced in Year 0		**PVF**	**PVCF**
Cash Flow	40,000	1	40,000
EAC of New Machine	(28,600)	1	(28,600)
			11,400

Case: Machine Replaced in Year 1		**PVF**	**PVCF**
Cash Flow	25,000	0.8695	
	(10,000)	0.8695	
	(28,600)	0.8695	
	(13,600)	0.8695	**(11,825.2)**

Case: Machine Replaced in Year 2			
Year	**Cash Flow**	**PVF**	**PVCF**
1	(10,000)	0.8695	(8,695)
2	(20,000)	0.7561	(25,405)
2	15,000	0.7561	
2	(28,600)	0.7561	
			(34,100)

Case: Machine Replaced in Year 3			
Year	**Cash Flow**	**PVF**	**PVCF**
1	(10,000)	0.8695	(8,695)
2	(20,000)	0.7561	(15,122)
3	(30,000)	0.6575	(19,725)
3	10,000	0.6575	6,575
3	(28,600)	0.6575	(18,804.5)
			(55,771.5)

Case: Machine Replaced in Year 4			
Year	**Cash Flow**	**PVF**	**PVCF**
1	(10,000)	0.8695	(8,695)
2	(20,000)	0.7561	(15,122)
3	(30,000)	0.6575	(19,725)
4	(40,000)	0.5717	(22,868)
4	(28,600)	0.5717	(16,350.62)
			(82,760.62)

It is better to **replace in year 1** itself as NPV of replacement cost is the least.

24. Illustration - Growth Option

ABC Ltd. is a pharmaceutical company possessing a patent of a drug called 'Aidrex', a medicine for aids patient. Being an approach drug ABC Ltd. holds the right of production of drugs and its marketing. The period of patent is 15 years after which any other pharmaceutical company produce the drug with same formula. It is estimated that company shall require to incur $ 12.5 million for development and market of the drug. As per a survey conducted the expected present value of cashflows from the sale of drug during the period of 15 years shall be $ 16.7 million. Cash flow from the previous similar type of drug have exhibited a variance of 26.8% of the present value of cashflows. The current yield on Treasury Bonds of similar duration (15 years) is 7.8%.

Determine the value of the patent.

Given

ln(1.336) =0.2897

e1.0005 = 0.3677 and e -1.17 = 0.3104

Solution

Black Scholes,

C = N(d1) * St - N(d2) * k* e^-rt

d1 = ln(st/k) + (r+ σ^2/2)t /sig root t

d2 = d1- sig root t

C $= N(d_1) * S * e^{-qt} - N(d_2) * k * e^{-rt}$

$d_1 = \frac{ln\left(\frac{S}{k}\right) + \left(r - q + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$

$d_2 = d_1 - \sigma\sqrt{t}$

q, dividend yield rate = 6.67%

k, strike = 12.5

r, risk-free rate = 7.8%

S, spot = 16.7

t, time in years = 15

σ^2, = 26.8%

$$d_1 = \frac{ln\left(\frac{S}{k}\right) + \left(r - q + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$d_1 = \frac{ln\left(\frac{16.7}{12.5}\right) + \left(7.8\% - 6.67\% + \frac{26.8\%}{2}\right)15}{\sqrt{26.8\%}\sqrt{15}}$$

$$= \frac{0.2897 + (0.145333)15}{0.517687 * 3.87298}$$

$$= \frac{2.4696995}{2.00499} = 1.23177$$

$N(d_1)$ = 0.8910

$d_2 = d_1 - \sigma\sqrt{t}$

$= 1.23177 - \sqrt{26.8\%}\sqrt{15}$

= 1.2377 – 2.00499 = –0.7732

$N(d_2)$ = 1– (0.7794,0.7823)
= 0.2196

C $= N(d_1) * S * e^{-qt} - N(d_2) * k * e^{-rt}$
$= 0.8910 * 16.7 * e^{-0.06667*15} - 0.2196 * 12.5 * e^{-0.078*15}$
$= 0.8910 * 16.7 * e^{-1.0005} - 0.2196 * 12.5 * e^{-1.17}$
$= 0.8910 * 16.7 * 0.3677 - 0.2196 * 12.5 * 0.3104$

Value of Option = **$4.6192 million**

25. Illustration - Abandonment option

IPL already in production of Fertilizer is considering a proposal of building a new plant to produce pesticides. Suppose the PV of proposal is ₹100 crore without the abandonment option. However, if market conditions for pesticide turns out to be favourable the PV of proposal shall increase by 30%. On the other hand, market conditions remain sluggish the PV of the proposal shall be reduced by 40%. In case company is not interested in continuation of the project it can be disposed of for ₹ 80 crore.

If the risk-free rate of interest is 8% then what will be value of abandonment option.

Solution

Assume abandonment option exists after a year.

u = 1+30% = 1.3
d = 1- 40% = 0.6
P $= \frac{e^{rt}-d}{u-d}$ $= \frac{1.08-0.6}{1.3-0.6}$ $= \frac{0.48}{0.7}$ = **0.6857**
$1-P$ = 0.3143

Expected Value of abandonment option (Put Option) at the end of year 1;
= 0.6875* 0 + 0.3143*20
= 6.286

PV of the abandonment option;
$= \frac{6.286}{1.08}$ = **$5.8204 million**

26. Illustration - Abandonment option

Airbus is considering a joint venture with Lear Aircraft to produce a small commercial airplane (capable of carrying 40-50 passengers on short haul flights)

Airbus will have to invest $ 500 million for a 50% share of the venture

Its share of the present value of expected cash flows is 480 million.

Lear Aircraft, which is eager to enter into the deal, offers to buy Airbus's 50% share of the investment anytime over the next five years for $ 400 million, if Airbus decides to get out of the venture.

A simulation of the cash flows on this time share investment yields a variance in the present value of the cash flows from being in the partnership is 0.16.

The project has a life of 30 years.

Should Airbus enter into the joint venture?

Solution

S = 480 million
σ^2 = 0.16
t = 30 yrs
r = 6%

k = 400 million

Put Option, Black Scholes Formula,

$= N(-d_2) * k * e^{-rt} - N(d_1) * S * e^{-qt}$

d_1 $= \frac{ln\left(\frac{S}{k}\right) + \left(r - q + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$

$= \frac{ln\left(\frac{480}{400}\right) + \left(6\% - 3.3\% + \frac{16\%}{2}\right)5}{0.8944}$

$= \frac{ln(1.2) + 0.5333}{0.8944}$

$= \frac{0.18232 + 0.5333}{0.8944}$ = 0.8001

$N(d_1)$ = 0.7881

d_2 $= d_1 - \sigma\sqrt{t}$

= 0.8001 – 0.8944 = –0.0943

$N(d_2)$ = 1 – N(0.0943)

= 0.4641

$k * e^{-rt} * N(-d_2) - S * e^{-qt} * N(d_1)$ $= 400 * e^{-6\% * 5}(1 - 0.4641) - 480 * e^{-0.1666}(1 - 0.7881)$

$= 400 * e^{-0.3}(0.5359) - 480 * e^{-0.1666}(0.2119)$

$= 400 * 0.7408 * (0.5359) - 480 * 0.864 * (0.2119)$

= 157.79 – 86.09

PV of Put Option = **$72.7 million**

Project Value Including Option = PV of Outflow+ PV of Inflow+ PV of Option

= –500 + 480 + 72.7

= **$ 52.7 million**

The joint venture is viable.

27. Illustration - Timing option

Suppose MIS Ltd. is considering installation of solar electricity generating plant for light the staff quarters. The plant shall cost ₹ 2.50 crore and shall lead to saving in electricity expenses at the current tariff by ₹ 21 lakh per year forever.

However, with change in Government in state, the rate of electricity is subject to change. Accordingly, the saving in electricity can be of ₹ 12 lakh or ₹ 35 lakh per year and forever.

Assuming WACC of MIS Ltd. is 10% and risk-free rate of rate of return is 8%.

Decide whether MIS Ltd. should accept the project or wait and see.

Solution

Investment = ₹ 2.5 Cr

Current Savings = ₹ 25 lakhs per year (WACC @10%)

PV of Current Savings = $\frac{21}{0.1}$ = ₹ 2.1 Cr

Current NPV = –2.5+ 2.1 = –0.4 Cr

Delay Timing: Options

NPV	$\frac{12}{0.1}$	$\frac{35}{0.1}$
Inflow	120 Lakhs	350 Lakhs
Less: Outflow	250 Lakhs	100 Lakhs
	– 130 Lakhs	100 Lakhs

u = $\frac{350}{250}$ = 1.4

d = $\frac{120}{250}$ = 0.48

P = $\frac{R_f - d}{u - d}$ = $\frac{1.08 - 0.48}{1.4 - 0.48}$ = $\frac{0.6}{0.92}$ = **0.652**

$1 - P$ = 0.348

Pay Off (after 1 year) = 100*0.652 + (–130*0.348)

= 65.2 – 45.24 = **19.96 Lakhs**

Current Value of Option = $\frac{19.96}{1.08}$ = **₹ 18.48 Lakhs**

Option has +ve Value hence the company should wait and decide

28. Illustration - MCQs

Describe each of the following situations in the language of options:

a. Drilling rights to undeveloped heavy crude oil in Northern Alberta. Development and production of the oil is a negative-NPV endeavour. (The break-even oil price is C$32 per barrel, versus a spot price of C$20.) However, the decision to develop can be put off for up to five years. Development costs are expected to increase by 5% per year.

Answer: The case depicts an American call option regarding drilling rights for undeveloped heavy crude oil. With an initial exercise price of $32 per barrel and the ability to delay development for up to five years due to increasing costs, it falls under the category of a timing option.

b. A restaurant is producing net cash flows, after all out-of-pocket expenses, of $700,000 per year. There is no upward or downward trend in the cash flows, but they fluctuate as a random walk, with an annual standard deviation of 15%. The real estate occupied by the restaurant is owned, not leased, and could be sold for $5 million. Ignore taxes.

Answer: This represents an American put option, related to an abandonment option for selling a restaurant's real estate, considering annual cash flows of $700,000, an exercise price of $5 million, and an annual standard deviation of 15% in cash flow variability.

c. A variation on part (b): Assume the restaurant faces known fixed costs of $300,000 per year, incurred as long as the restaurant is operating. Thus,
Net cash flow = revenue less variable costs — fixed costs
$700,000 = 1,000,000 — 300,000
The annual standard deviation of the forecast error of revenue less variable costs is 10.5%. The interest rate is 10%. Ignore taxes.

Answer: The fluctuating annual cash flows of $1,000,000 by 10.5% in a scenario with fixed costs of three lakes annually indicate an American put option, indicating an abandonment option with varying cash flows and an exercise price of $8 million inclusive of property sale and annual savings.

d. A paper mill can be shut down in periods of low demand and restarted if demand improves sufficiently. The costs of closing and reopening the mill are fixed.

Answer: The paper mill's ability to temporarily shut down and restart operations with fixed closure and reopening costs reflects a compound option, a combination of an American put for abandonment and an American call for temporary restart, offering operational flexibility.

e. A real estate developer uses a parcel of urban land as a parking lot, although construction of either a hotel or an apartment building on the land would be a positive-NPV investment.

Answer: In a situation where a real estate developer can choose between building a hotel or an apartment building, both profitable options, it represents an in-the-money American call option, allowing for the deferment of the decision (timing option) to select the most lucrative option during the waiting period.

f. Air France negotiates a purchase option for 10 Boeing 787s. Air France must confirm the order by 2030. Otherwise, Boeing will be free to sell the aircraft to other airlines.

Answer: Air France negotiating a purchase option for 10 Boeing 787 planes by 2030 represents a timing option, specifically an American call option. This grants Air France the flexibility to confirm the purchase at any time until 2030, resembling a timing decision rather than a growth or abandonment choice.

29. Illustration

You own a one-year call option on one acre of Los Angeles real estate. The exercise price is $2 million, and the current, appraised market value of the land is $1.7 million. The land is currently used as a parking lot, generating just enough money to cover real estate taxes. The annual standard deviation is 15% and the interest rate 12%. How much is your call worth?

Solution

$S_0 = 1.7 \mid X = 2 \mid S.D = 0.15 \mid t = 1.0 \mid r_f = 0.12$

$$d1 = \frac{\text{Ln } (1.7/2.0) + (0.12 + (0.15)^2 / 2) \times 1}{(0.15) \times 1}$$

$$d1 = \frac{-0.16252 + (0.13125)}{0.15}$$

$d1 = -0.20846;$

$d2 = d1 - 0.15 \times 1$
$d2 = -0.20846 - 0.15$

$N(d1) = 0.41744$
$N(d2) = 0.36000$

Price of call $= 1.7 \times 0.41744 - 2.0 \times e^{-0.12} \times 0.3600$
C = ₹ 71,057

30. Illustration

Consumers appear to require returns of 25 percent or more before they are prepared to make energy-efficient investments, even though a more reasonable estimate of the might be around 15 percent.

Suppose you have the opportunity to invest $1,000 in new space-heating equipment that would generate fuel savings of $250 a year forever given current fuel prices. What is the PV of this investment if cost of capital is 15 percent? What is the NPV?

Now recognize that fuel prices are uncertain and that the savings could well turn out to be $50 a year or $450 a year. If the risk-free interest rate is 10 percent, would you invest in the new equipment now or wait and see how fuel prices change? Explain.

Solution

Investment	= $1000	
Current Savings	= $ 250 per year (Cost of Capital @10%)	
PV of Current Savings	$= \frac{250}{0.15}$	= $ 1,666.67
Current NPV	= 1,666.67-1,000	= $ 666.67

u	$= \frac{3000}{1667}$	= 1.8
d	$= \frac{333}{1667}$	= 0.20

P $= \frac{k-d}{u-d}$ $= \frac{1.1-0.2}{1.8-0.2}$ $= \frac{0.9}{1.6}$ = **0.5625**

$1-P$ = 0.4375

Pay Off $= \frac{2{,}000*0.5625 + (0*0.4375)}{1.1}$

= **₹ 1023**

It is **advisable to wait** as option to wait is worth $1023 vs current NPV value of the project is at $667.